

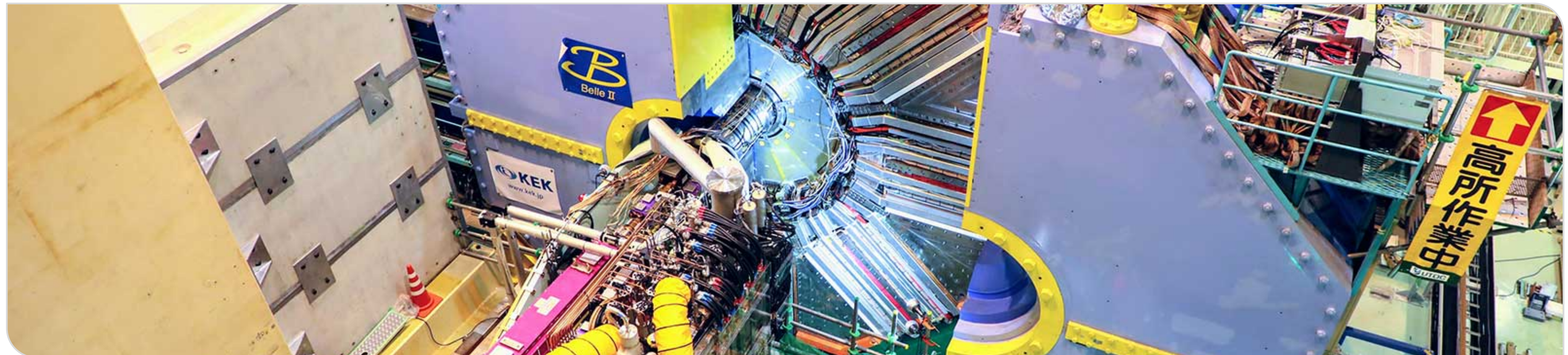
# Systematics in Flavour Physics Analyses (rare decays)

**Systematic Effects and Nuisance Parameters in Particle Physics Data Analyses**

**BIRS, Banff, Canada, April 27, 2023**

**Slavomira Stefkova (KIT)**

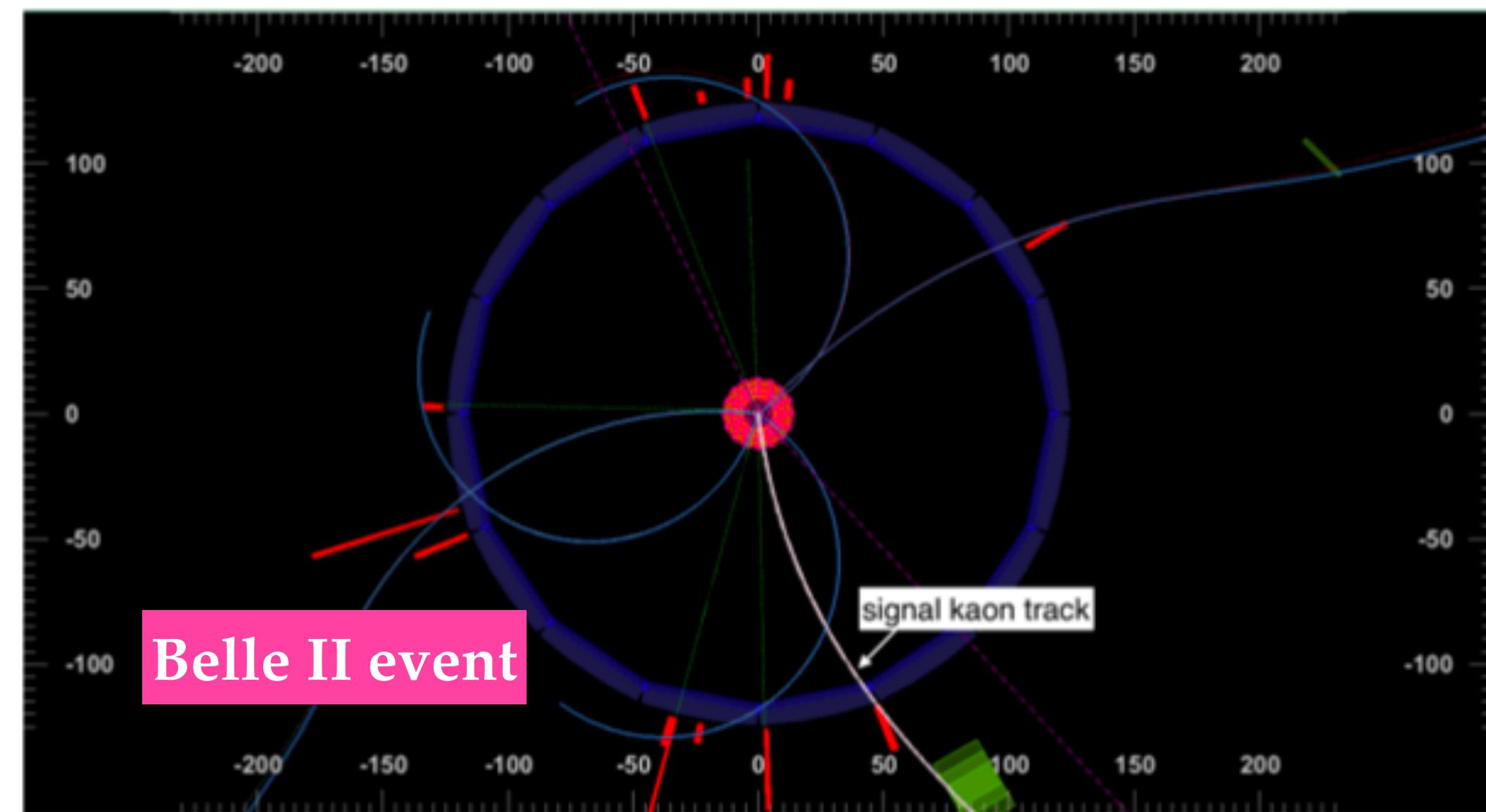
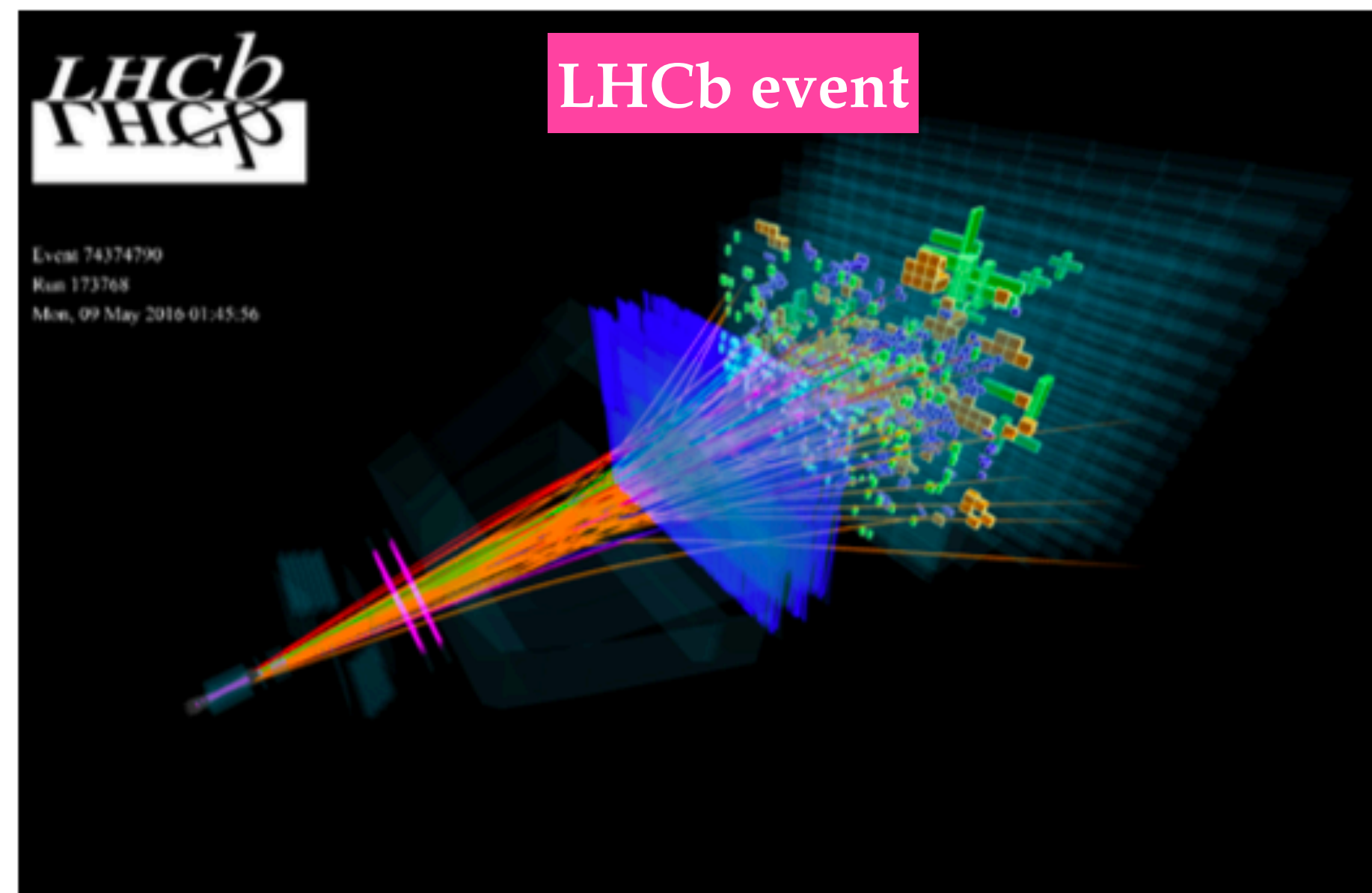
[slavomira.stefkova@kit.edu](mailto:slavomira.stefkova@kit.edu)



# Flavour experiments in nutshell

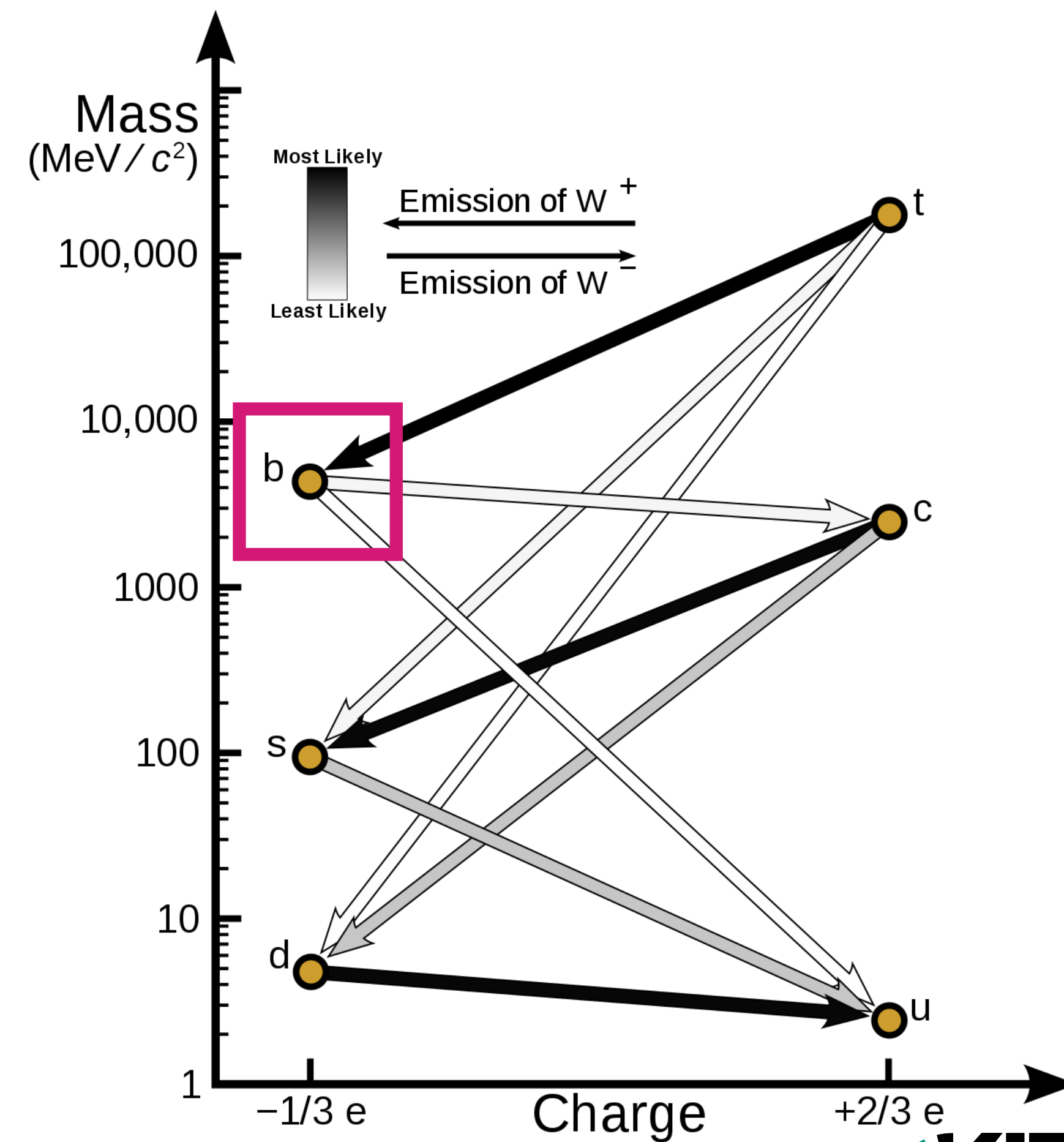
# Collider Flavour Physics: Belle II and LHCb

- Two active collider flavour physics experiments specialising in  $B$ -physics: **LHCb** and **Belle II**
- The **main differences** with respect to other ATLAS and CMS experiments:
  - Generally more sensitive to lower deposits: mass of  $B$ -meson  $\sim 5$  GeV
  - The innermost detector has excellent vertex resolution close to IP



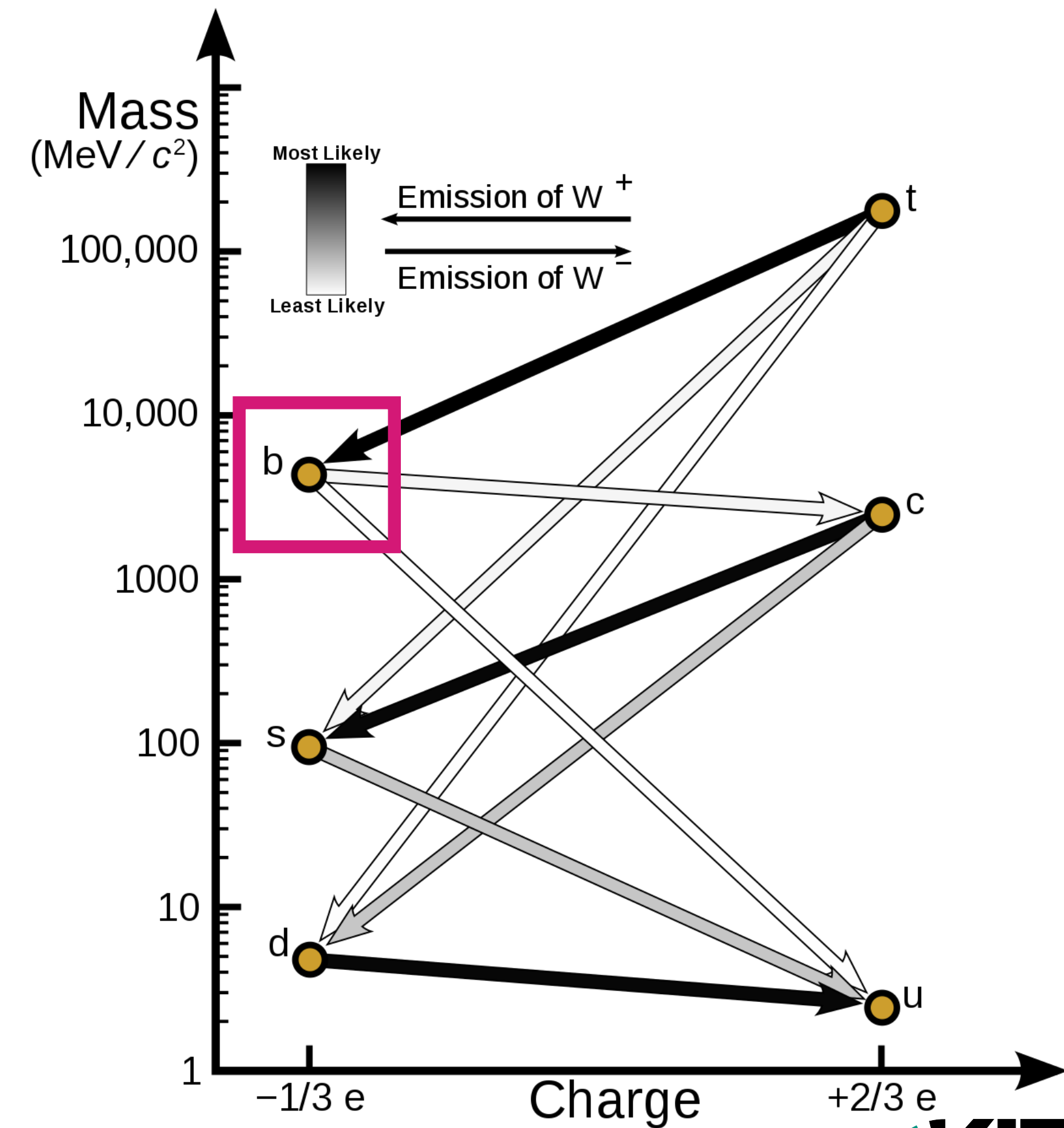
# B-decays measurements (and others)

- $B$ -decay is **abundant** or **rare** depending on the likelihood dictated by CKM matrix:
  - Precision measurements of CP violation (**rare and abundant**)
  - Indirect searches/measurements for NP in rare  $B$ -decays (**rare**)
  - Indirect searches for NP in semileptonic  $B$ -decays (**abundant**)
  - Spectroscopy of  $B$ -decays (**abundant**)
- Belle II and LHCb also well suited for measurements with
  - Decays involving  $c$  and  $\tau$
  - Direct searches for NP, e.g DM, mediators connected to DM, leptoquarks,  $Z'$ , .....

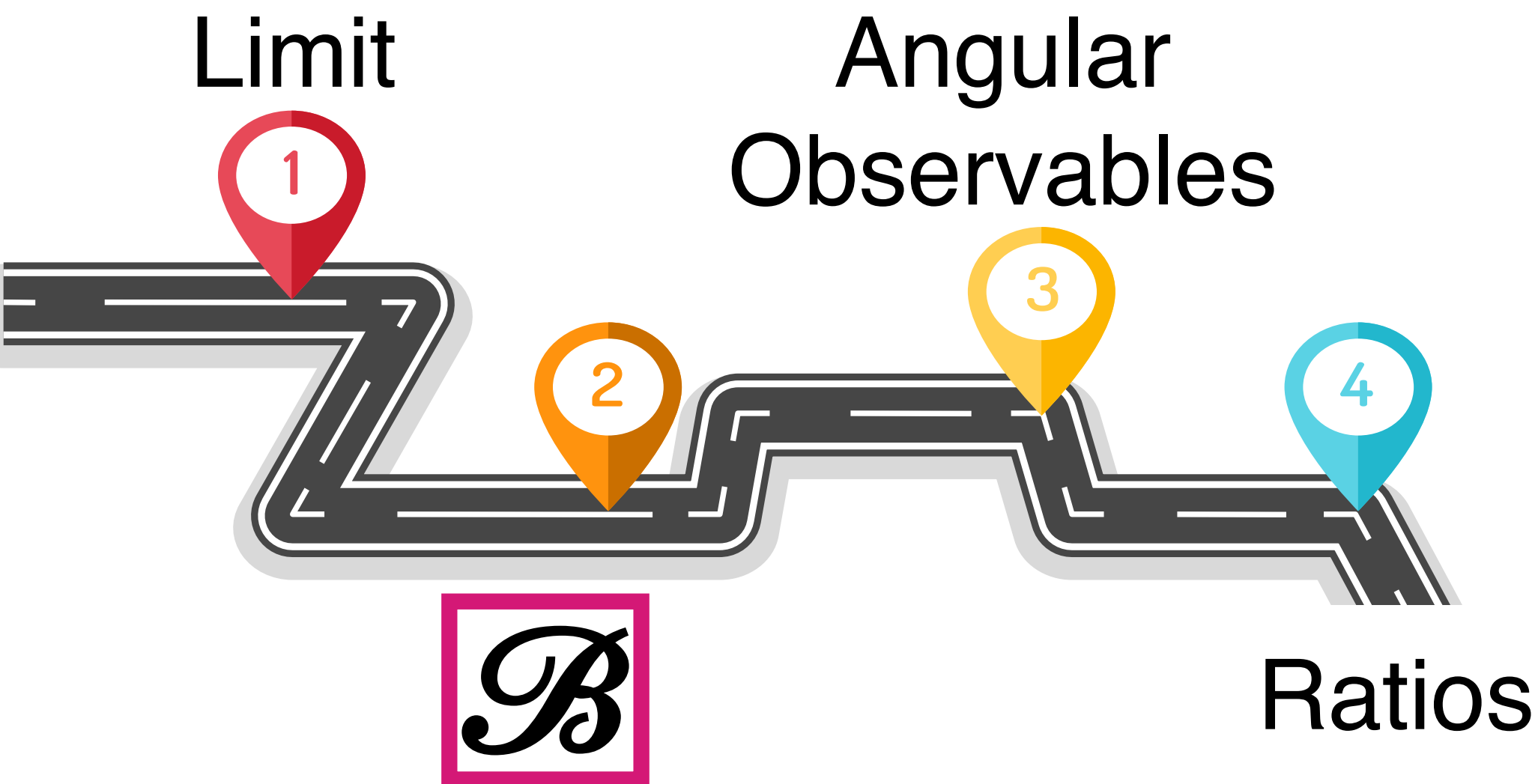


# B-decays measurements (and others)

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  - Direct searches for NP, e.g. **DM**, mediators connected to DM, leptoquarks,  $Z'$ , .....



# What do we measure in rare $B$ -decays?



Parameter of interests:

1.  $\mathcal{B} \rightarrow 1$  parameter of interest  
Signal strength of the process ( $1 \mu = \text{SM expectation}$ )
2. **Angular observables**  $\rightarrow$  **several parameters of interest:**  
e.g: angular observables of  $B \rightarrow K^{*0} \mu^+ \mu^-$ :  $P_5'$ ,  $F_L$ ,  $A_{\text{FB}}$
3. **Ratios**  $\rightarrow$  **2 parameters of interests:**  
e.g: signal strengths of two processes

Theoretically cleaner  $\rightarrow$

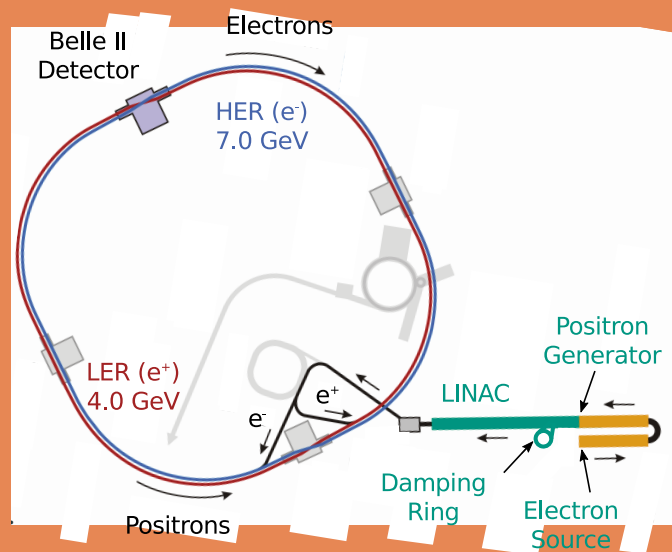
$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} = \frac{\epsilon(B^+ \rightarrow K^+ e^+ e^-)}{\epsilon(B^+ \rightarrow K^+ \mu^+ \mu^-)} \times \frac{N(K^+ B^+ \rightarrow \mu^+ \mu^-)}{N(B^+ \rightarrow K^+ e^+ e^-)}$$

How often does  $B$ -decay into the decay products?

rare  $\rightarrow \mathcal{B} < 10^{-5}$

# Types of Systematic Uncertainties

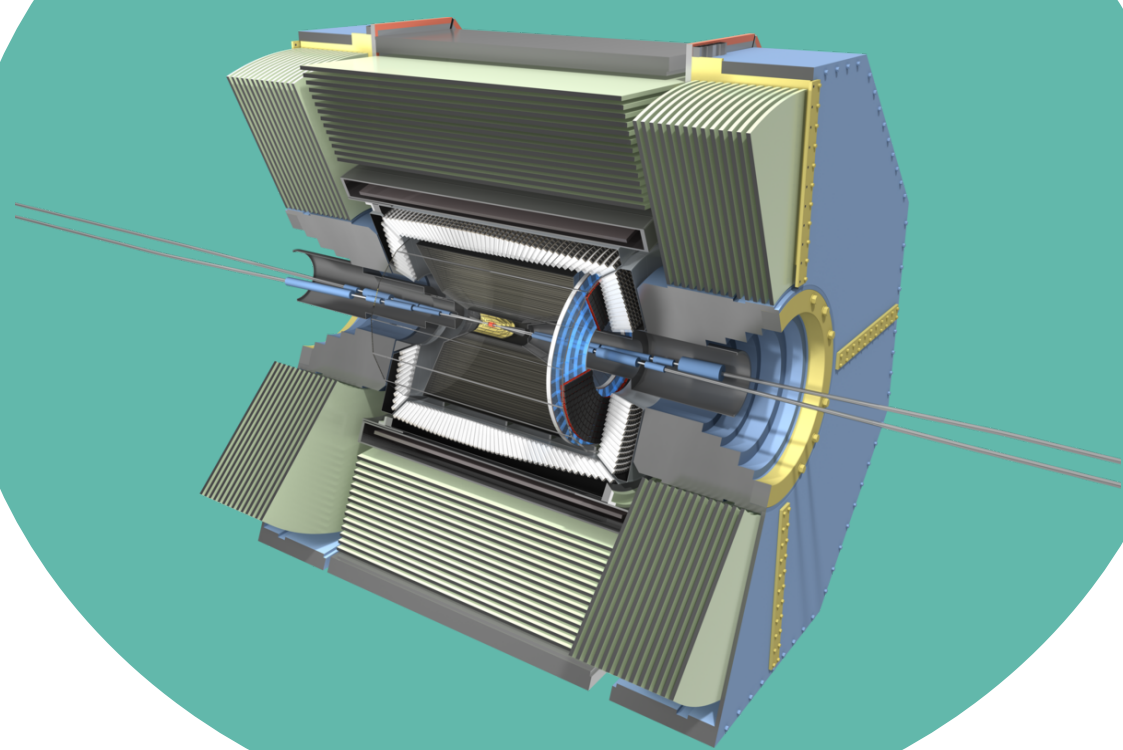
## Accelerator



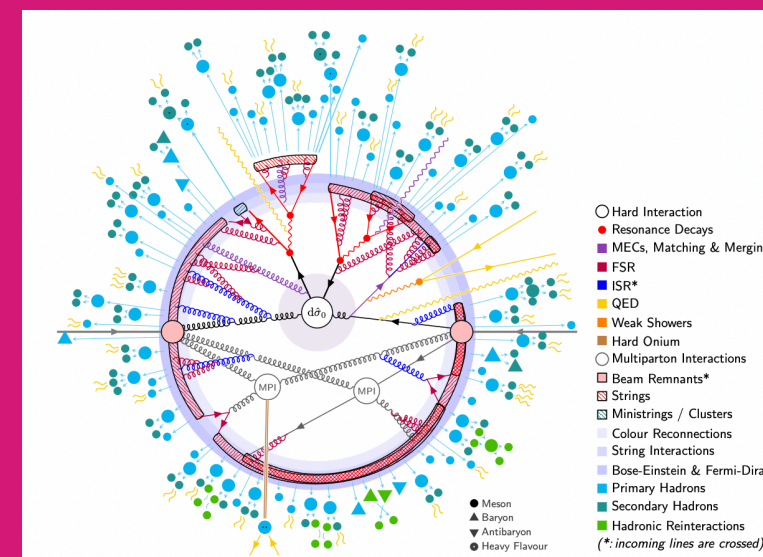
## External Measurement



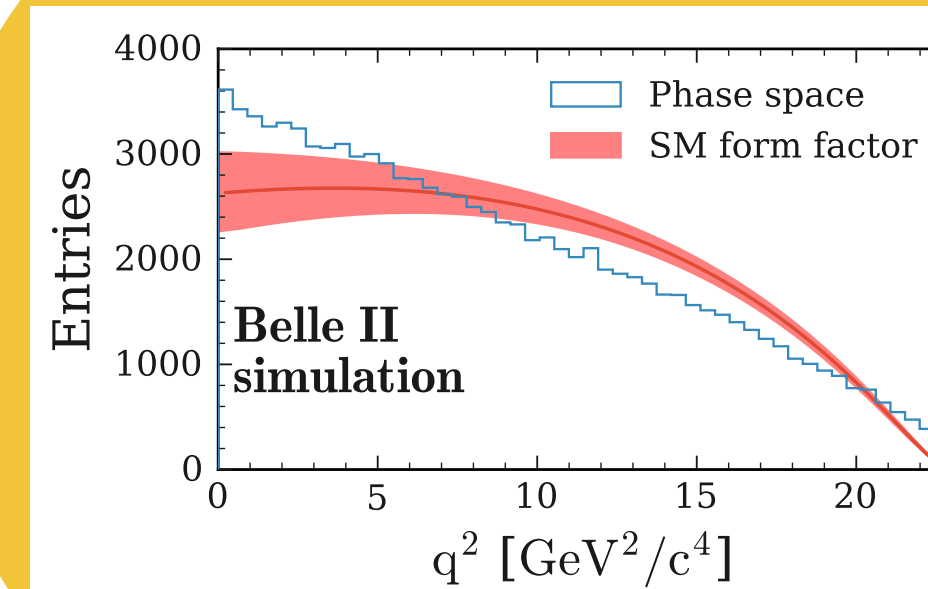
## Detector



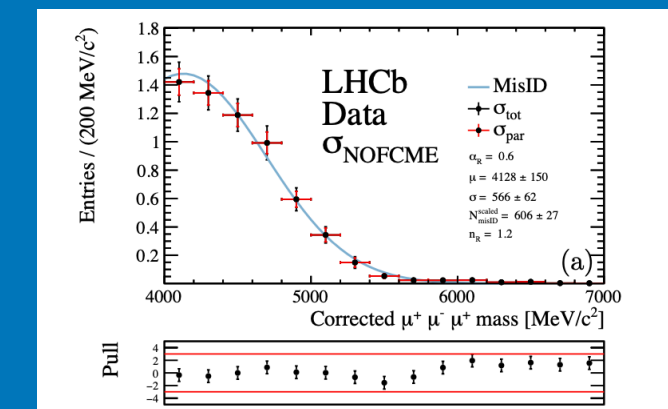
## Simulation



## Theory



## Analysis Technique

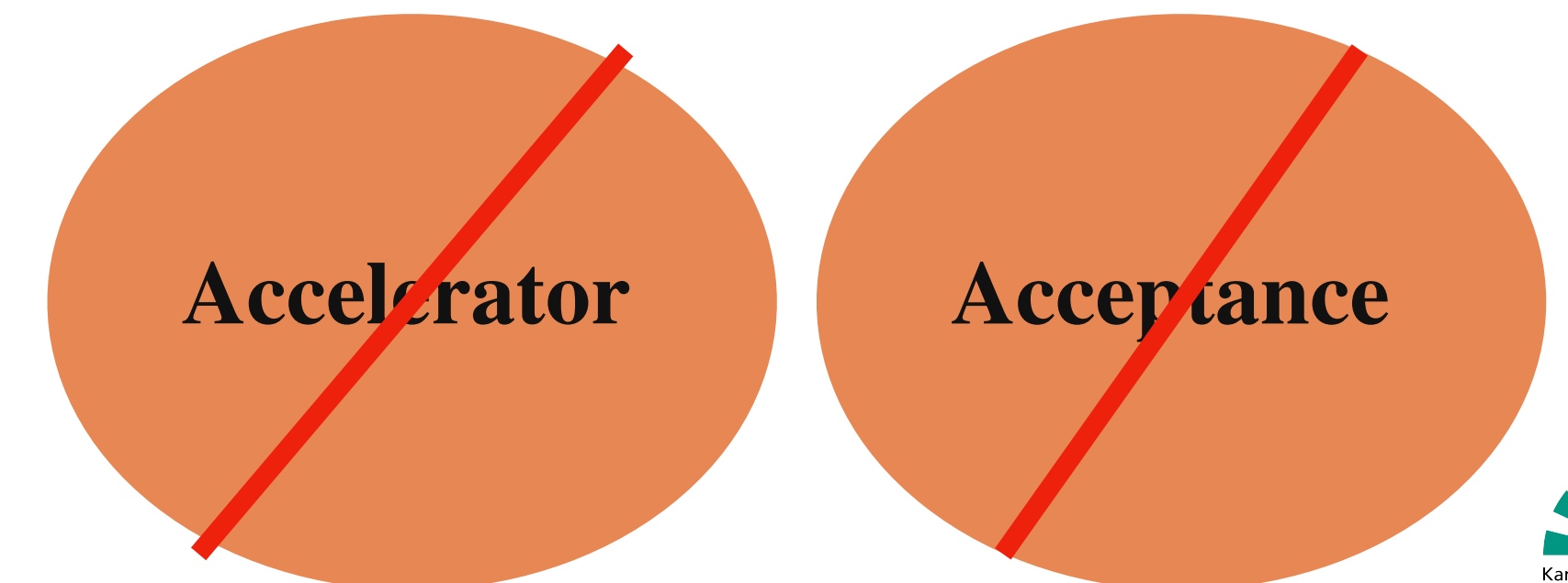
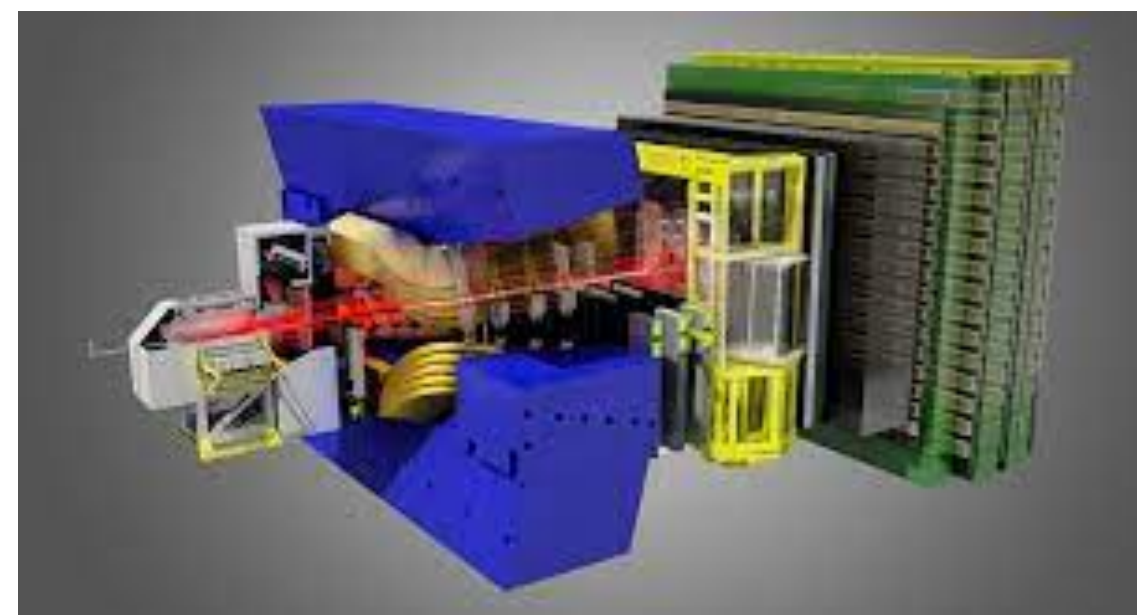


# LHCb and Belle II: Measurement Style

For measuring  $\mathcal{B}$  of rare  $B$ -decays **LHCb uses mostly relative** and **Belle II absolute approach**:

- $$\mathcal{B}(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu) = \mathcal{B}(B^+ \rightarrow J/\psi(\rightarrow \mu^+ \mu^-) K^+) \times \frac{\epsilon(B^+ \rightarrow J/\psi(\rightarrow \mu^+ \mu^-) K^+)}{\epsilon(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)} \times \frac{N(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)}{N(B^+ \rightarrow J/\psi(\rightarrow \mu^+ \mu^-) K^+)}$$
- $$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = \frac{N(B^+ \rightarrow K^+ \nu \bar{\nu})}{\epsilon(B^+ \rightarrow K^+ \nu \bar{\nu})}$$

- Systematics need to be therefore derived either on the efficiencies or numbers of events in Belle II or on relative efficiencies and yields in LHCb:
- Relative measurements cancel uncertainties due to accelerator and acceptance





# Building models and fitting

Unbinned

Fitting

Model Description



Maximum Likelihood Fit  
systematics included as  
nuisances

**Per-event model**

$$p(\{x_i\} | \theta) = \text{Pois}(N | \theta) \prod_{i=0}^N p(x_i | \theta)$$

Parametrised functions:  
(Crystal Balls,  
exponential, Gauss, ...)  
Density Estimates (KDE)

Limit Setting

- o Frequentist
  - o Asymptotic
  - o Toy-based
- o Bayesian

Observation

- o Significance

Binned

**Step-wise per-event model**

$$p(\{n_b\} | \theta) = \prod_b \text{Pois}(n_b | \lambda(\theta) p(b | \theta)) = \prod_b \text{Pois}(n_b | \lambda_b(\theta))$$

Parametrised  
histograms

# How do we get systematic variations?

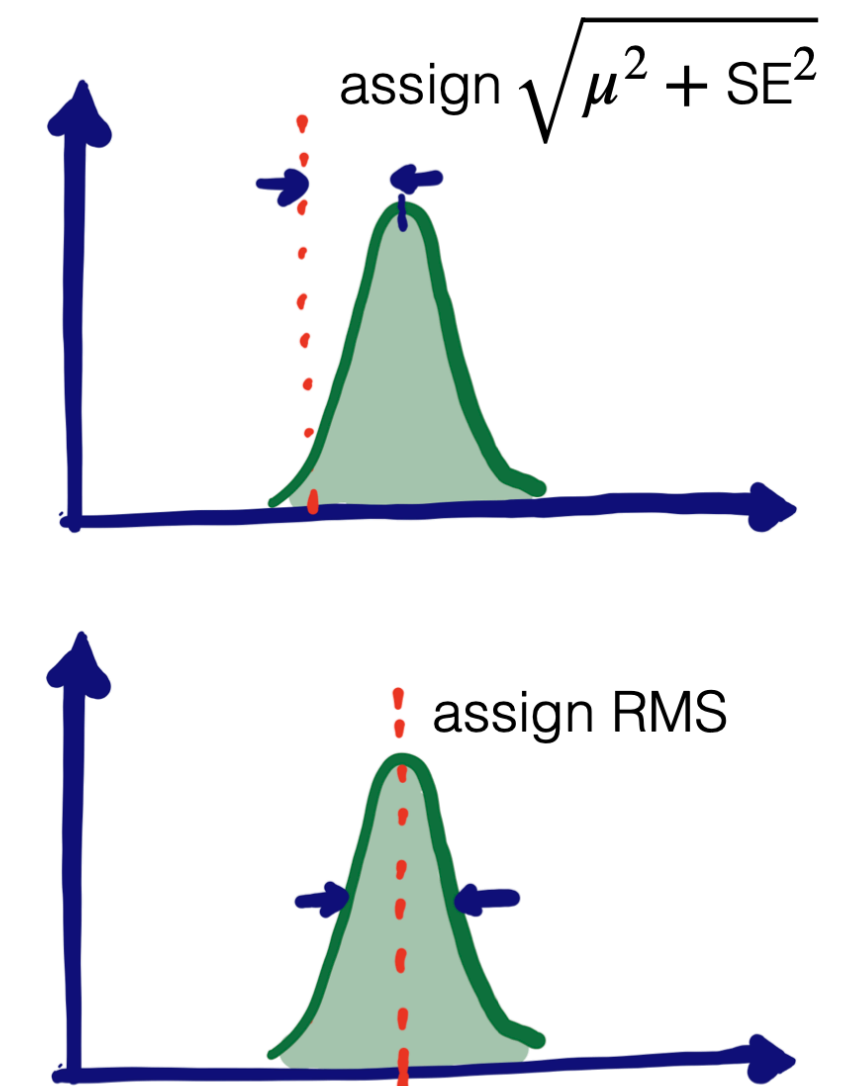
→ Systematic uncertainties can affect shape, normalisation or both

What samples do we use?

1. Alternative models (**simulation**)
2. Calibration samples (mixture of **simulation** and **data**)
3. Signal embedding (mixture of **simulation** and **data**)
4. (Orthogonal) data samples (**data**)

What methods do we use? [[T.Blake \(Phystat 2021\)](#)]:

1. Generate a large number of pseudo-experiments from a varied model and determine observables using the nominal model...(bootstrap method)
2. Repeat the determination of the observables in data using a different set of assumptions...(alternative method)



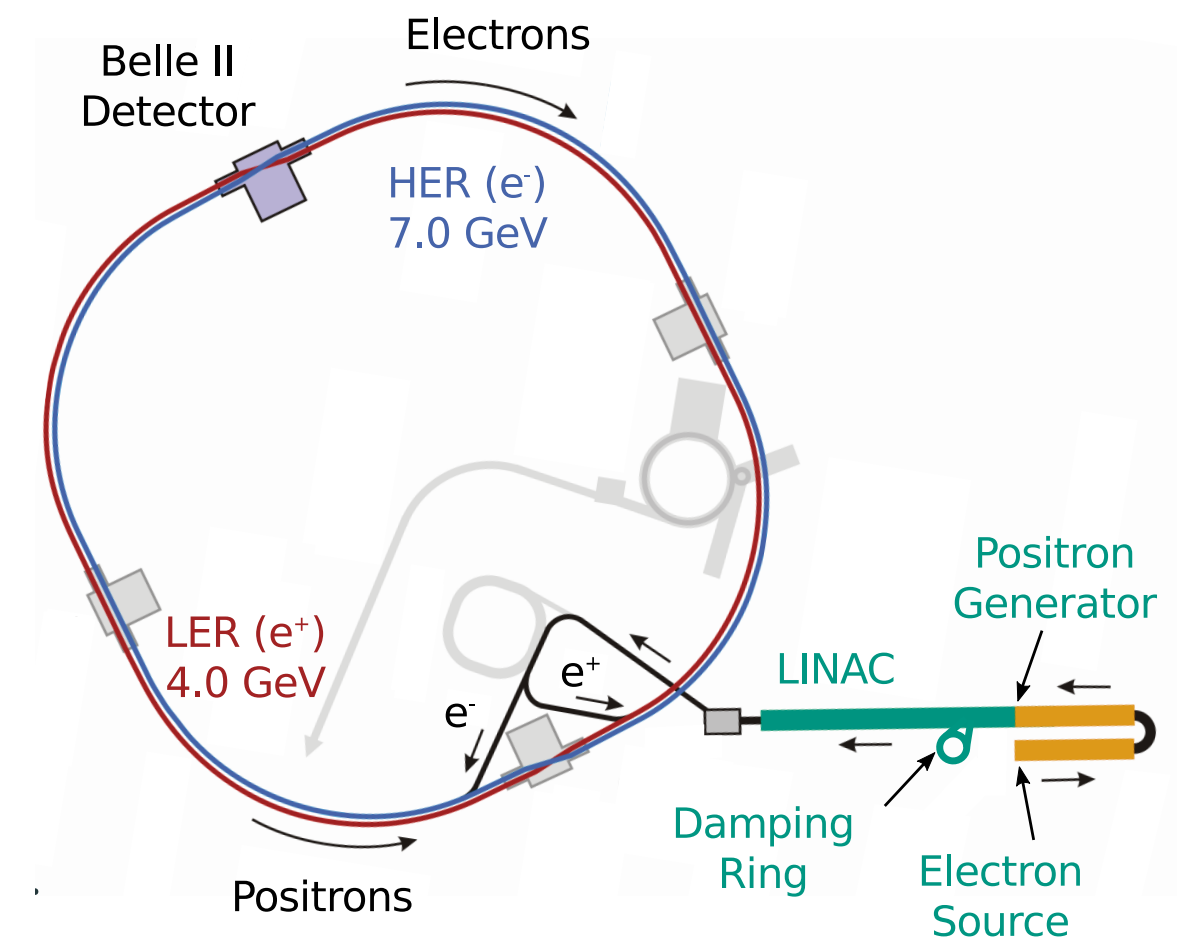
→ After evaluation of systematics we include them in a likelihood as constraints, aka penalty terms

# Practical examples of systematics\*

\*mixture of good, bad and ugly

# Systematic Uncertainties: Accelerator

Accelerator	$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$ (Belle II)	$\mathcal{B}(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)$ (LHCb)
Delivered Luminosity	Integrated Luminosity (Calibration)	
<i>B</i> -dataset	Number of <i>BB</i> (Calibration)	



$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$$

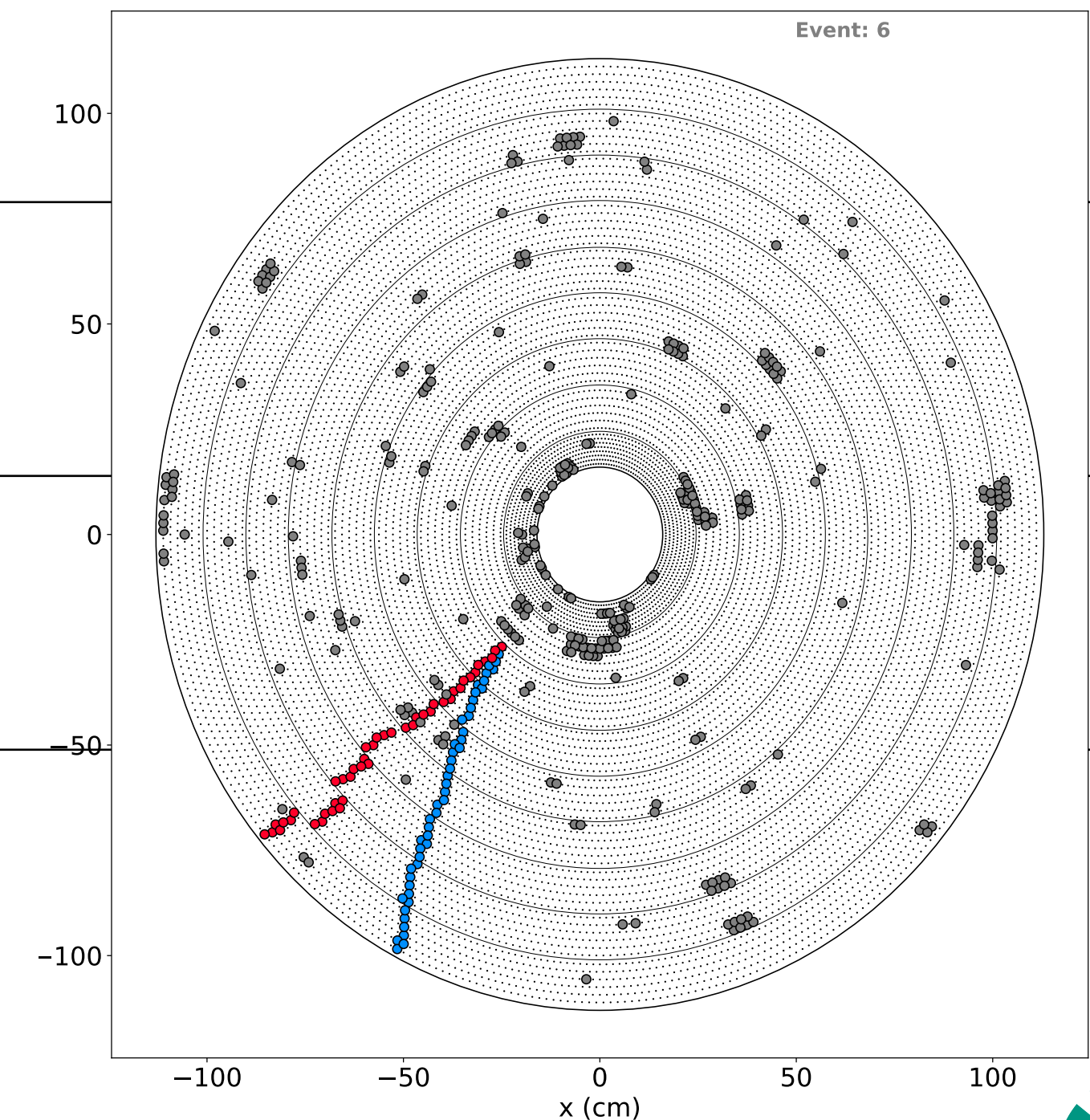
[[Phys. Rev. Lett. 127, 181802 \(2021\)](#)]

$$\mathcal{B}(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)$$

[[Eur. Phys. J. C 79 \(2019\) 675](#)]

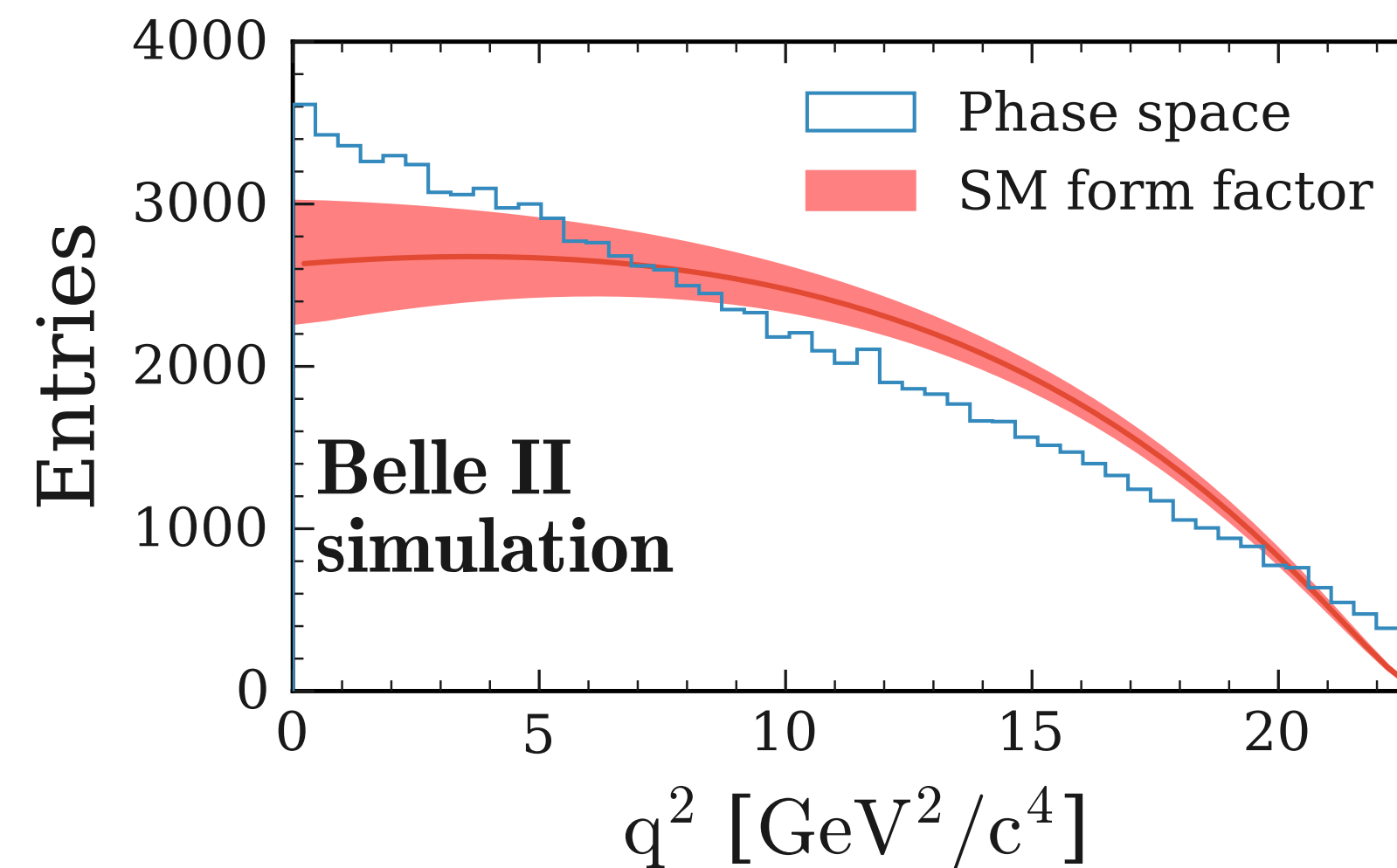
# Systematic Uncertainties: Detector

Detector	$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$ (Belle II)	$\mathcal{B}(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)$ (LHCb)
Tracking of charged particles	Tracking efficiency (Calibration)	Tracking efficiency (Calibration)
Measurement of energy deposit (photons)	Uncertainty on the absolute energy for photons (0.5%) (Calibration)	
Measurement of energy deposit (others)	Uncertainty on the absolute energy for other clusters (5-10%) (Calibration)	
Particle Identification	Uncertainty on the PID corrections (Calibration)	



# Systematic Uncertainties: Theory

Theory	$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$ (Belle II)	$\mathcal{B}(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)$ (LHCb)
Signal shape	Form Factor Uncertainty (theory model)	
Signal model		Signal Model (Alternative simulation model)



# Systematic Uncertainties: Simulation

Simulation	$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$ (Belle II)	$\mathcal{B}(B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu)$ (LHCb)
<b><i>B</i>-production kinematics</b>		Kinematic reweighting (Calibration)
<b>Uncertainty on background BF</b>	Uncertainty on the BF of leading <i>B</i> -background (alternative models)	
<b>Background normalisation</b>	Continuum background (Orthogonal data sample)	
<b>Missing background template</b>		Modelling of $B^+ \rightarrow (D \rightarrow (\eta \rightarrow \mu\mu)\mu\nu)$ (alternative fitting model)

# Systematic Uncertainties: Further Analysis

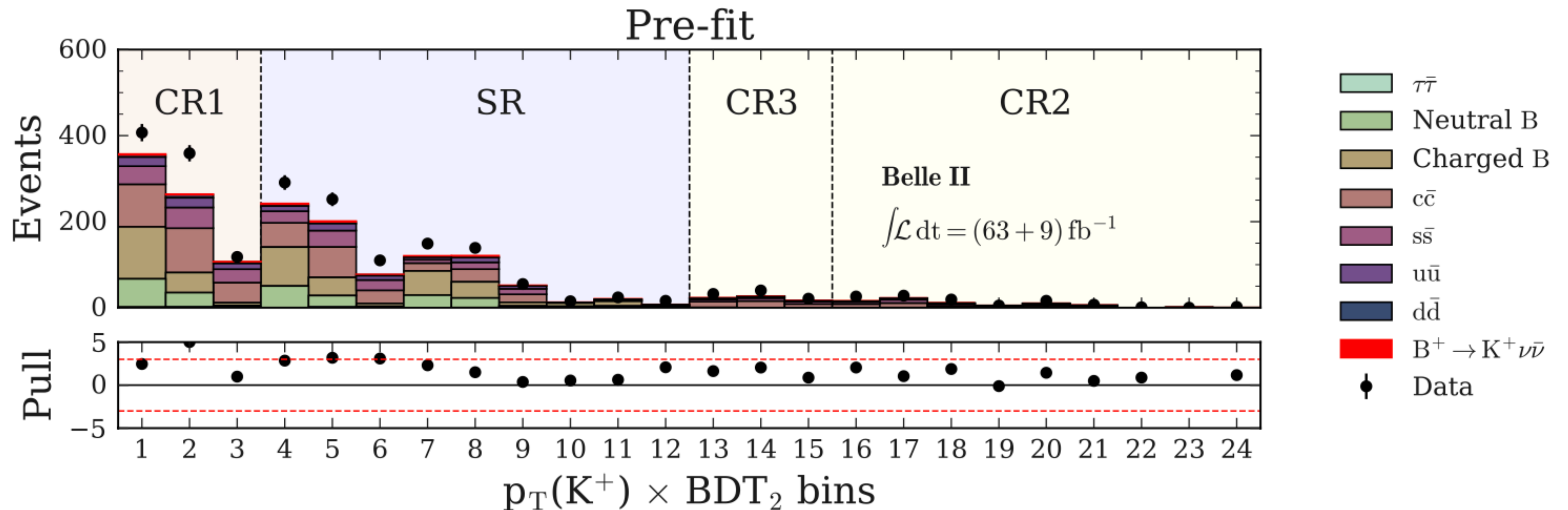
<b>Further Analysis</b>	$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})$ (Belle II)	$\mathcal{B}(B^+ \rightarrow \mu^+\mu^-\mu^+\nu)$ (LHCb)
Background shape		Background shape modelling (Alternative fitting model)
Fitting bias		Fitting bias (Alternative simulation models)



# Methods: capturing correlations, assessing data-model compatibility

# Few words about correlations...

Search for  $B^+ \rightarrow K^+ \nu \bar{\nu}$  decays: binned fit



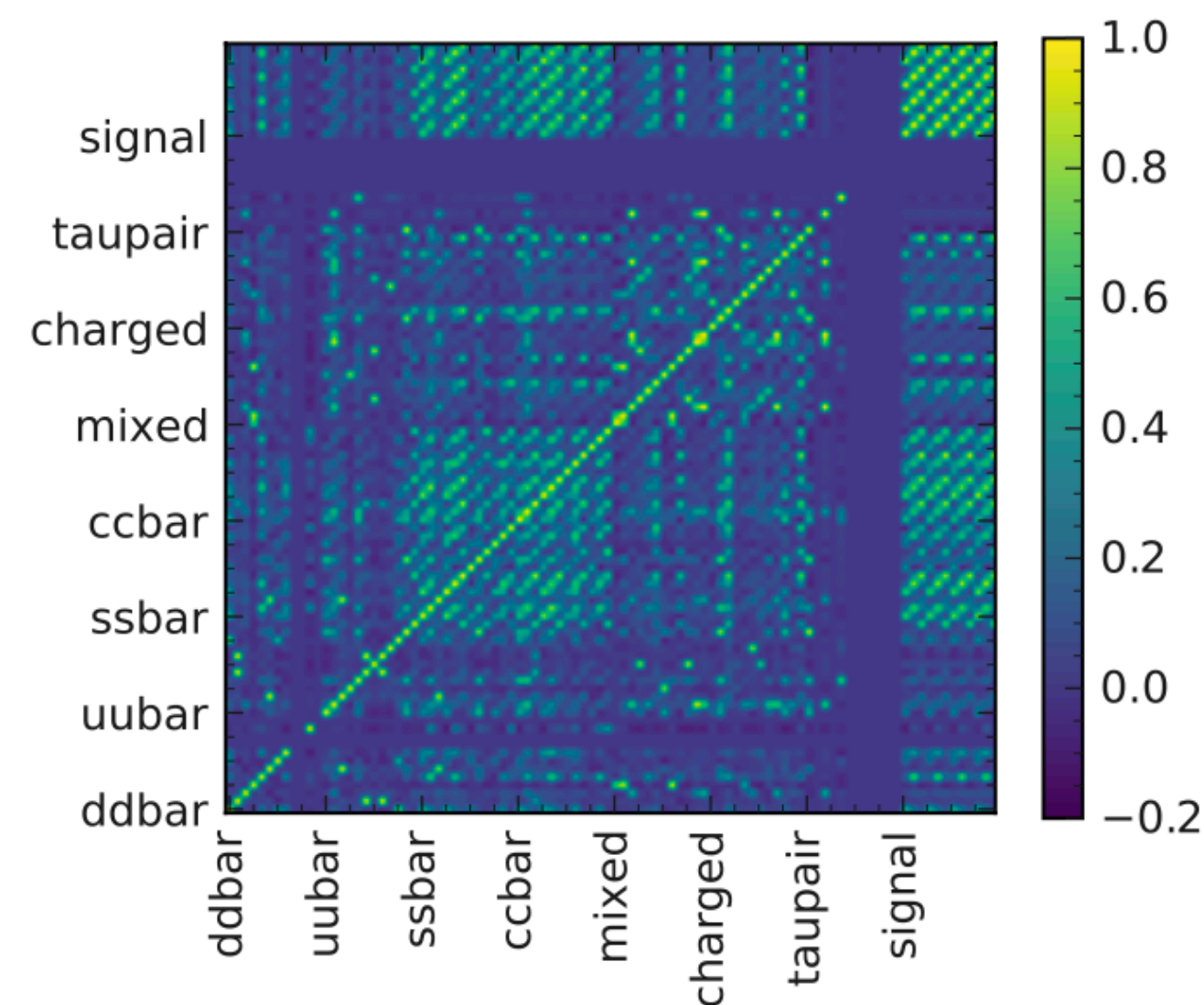
# Correlated systematics (I)

## Bootstrap

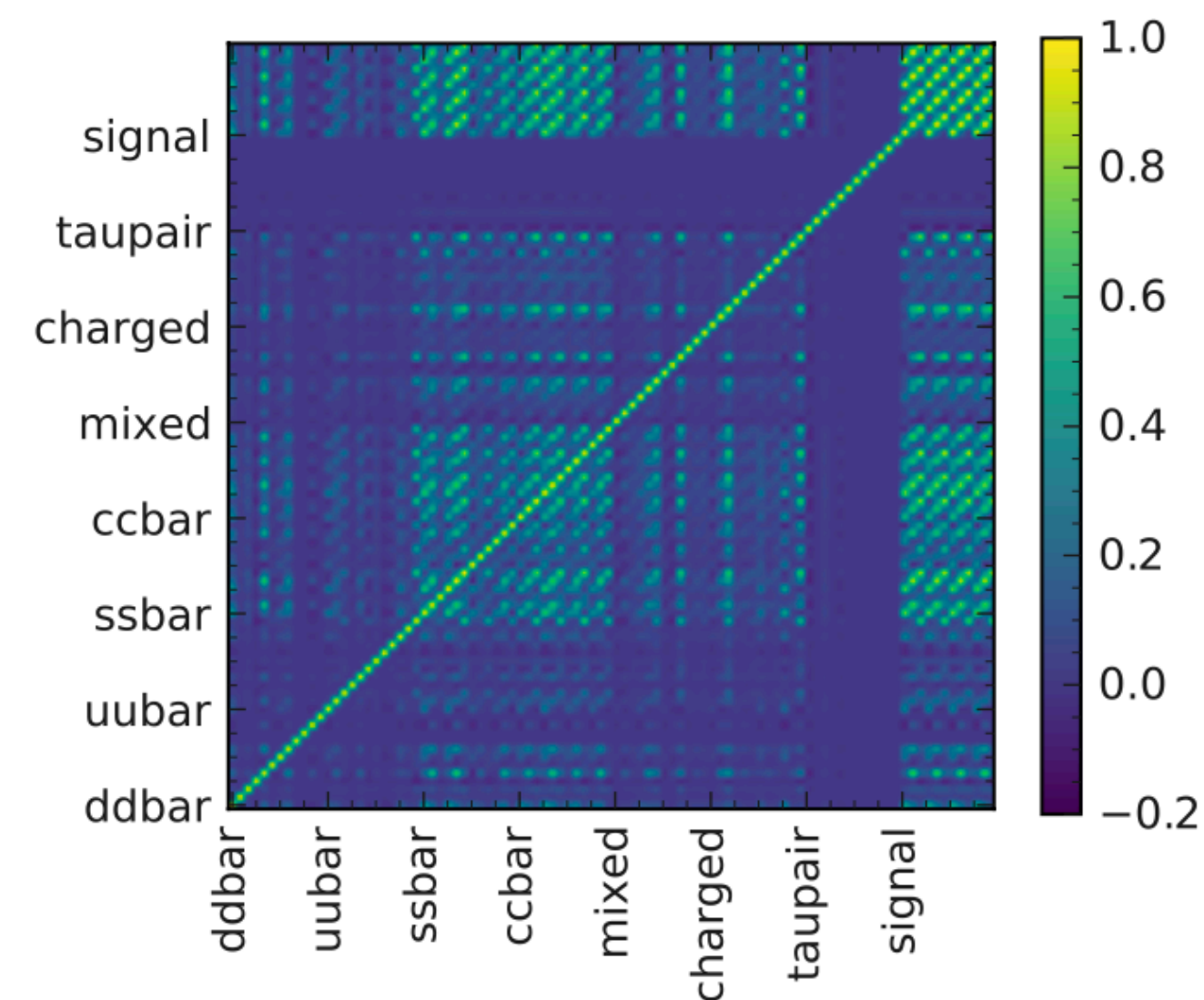
$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$$

- PID (statistical error= $\sigma$ ), uncertainty on the branching fractions (measurement uncertainty= $\sigma$ ):
  - Non-trivial correlations PID corrections computed in bins of  $(p_T, \theta)$
  - Produce alternative simulations with bootstrap method: central values varied with  $\text{lognormal}(0, \sigma)$
  - Reconstruct covariance matrix
  - Translation to nuisance parameters: SVD decomposition + leading eigenvectors as the nuisances

Full covariance



3 leading eigenvectors



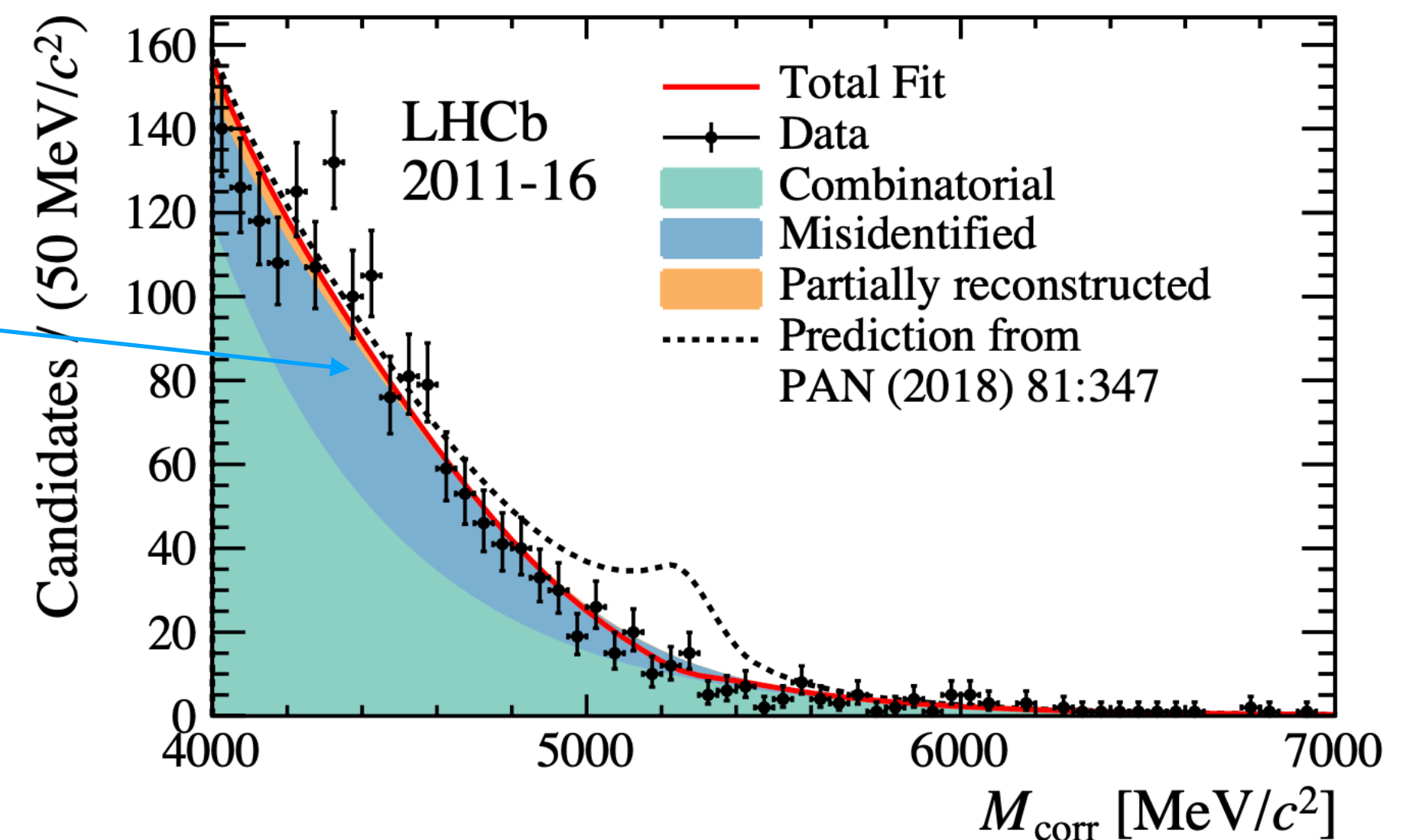
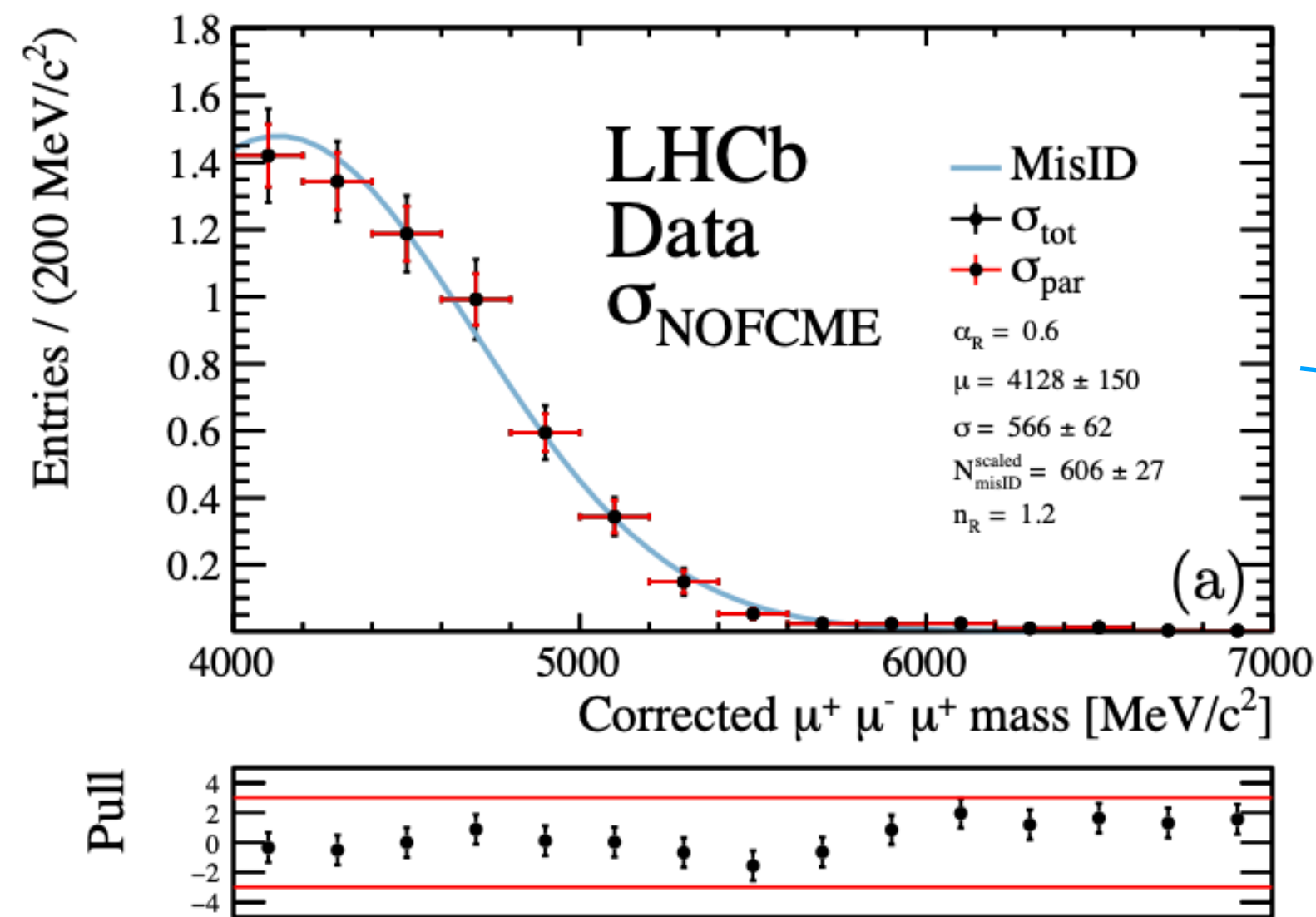
# Correlated systematics (II)

## MVA-Gaussian

Data-driven estimate of a particular background:

- Fit to “sideband” data and then propagate the result of the fit together with the correlation to the main fit
- Using multivariate gaussian constraints to input or preserve the correlation of this systematics

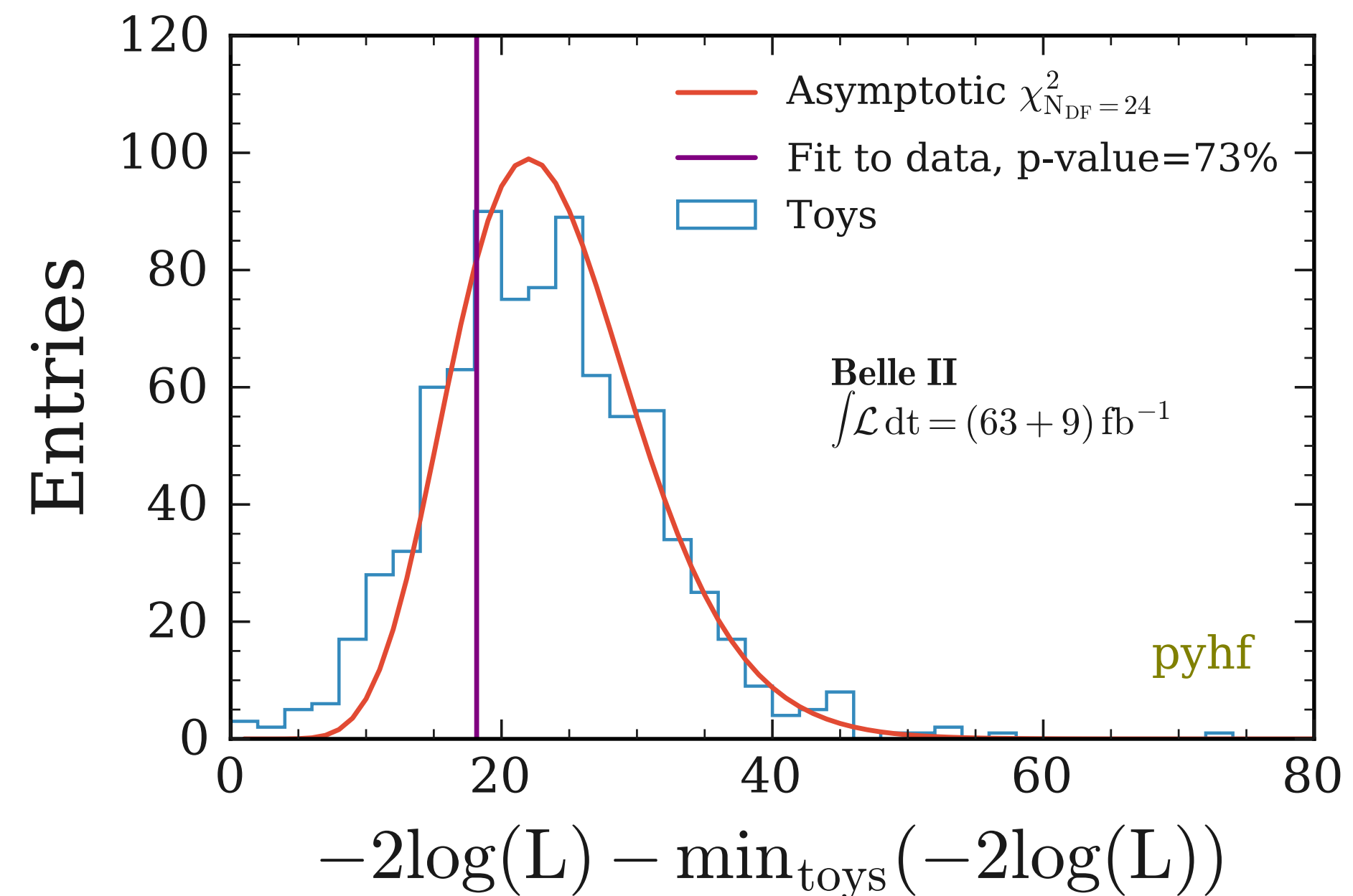
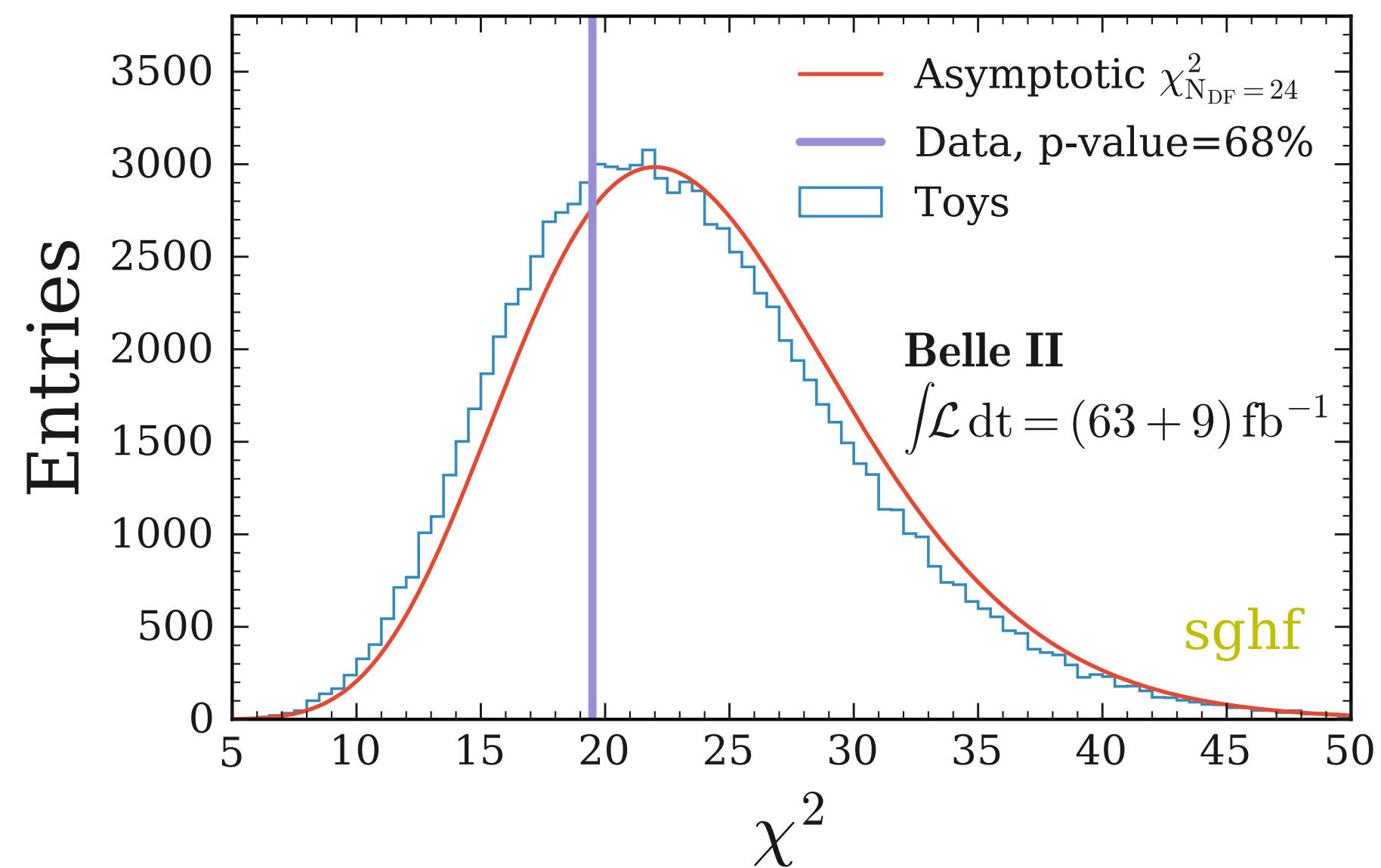
### Misidentified background



# Data-model compatibility

Using toy-based method:

- The toys are generated using observed data counts based on Poisson statistics
- In order to avoid double counting of the fluctuations, the observed counts are subtracted from each toy and the expectation is added instead
- This way the toys are centered around the Asimov dataset which by construction has no fluctuations



# Statistical modelling challenges in rare decays

# Rare-decay searches/measurements

**The main goal is to suppress the background as much as possible**

- Low-statistics samples → trade-off between smoothness and fit stability
- Asymptotic conditions may not be necessarily satisfied

**Control samples are not so easy to get by!**

- Inflates the nuisances (if it does not make them basically unconstrained)

**Rare signal decays can have as background decays that are also rare:**

- Signal rare decays may have backgrounds that have never been measured and are not modelled

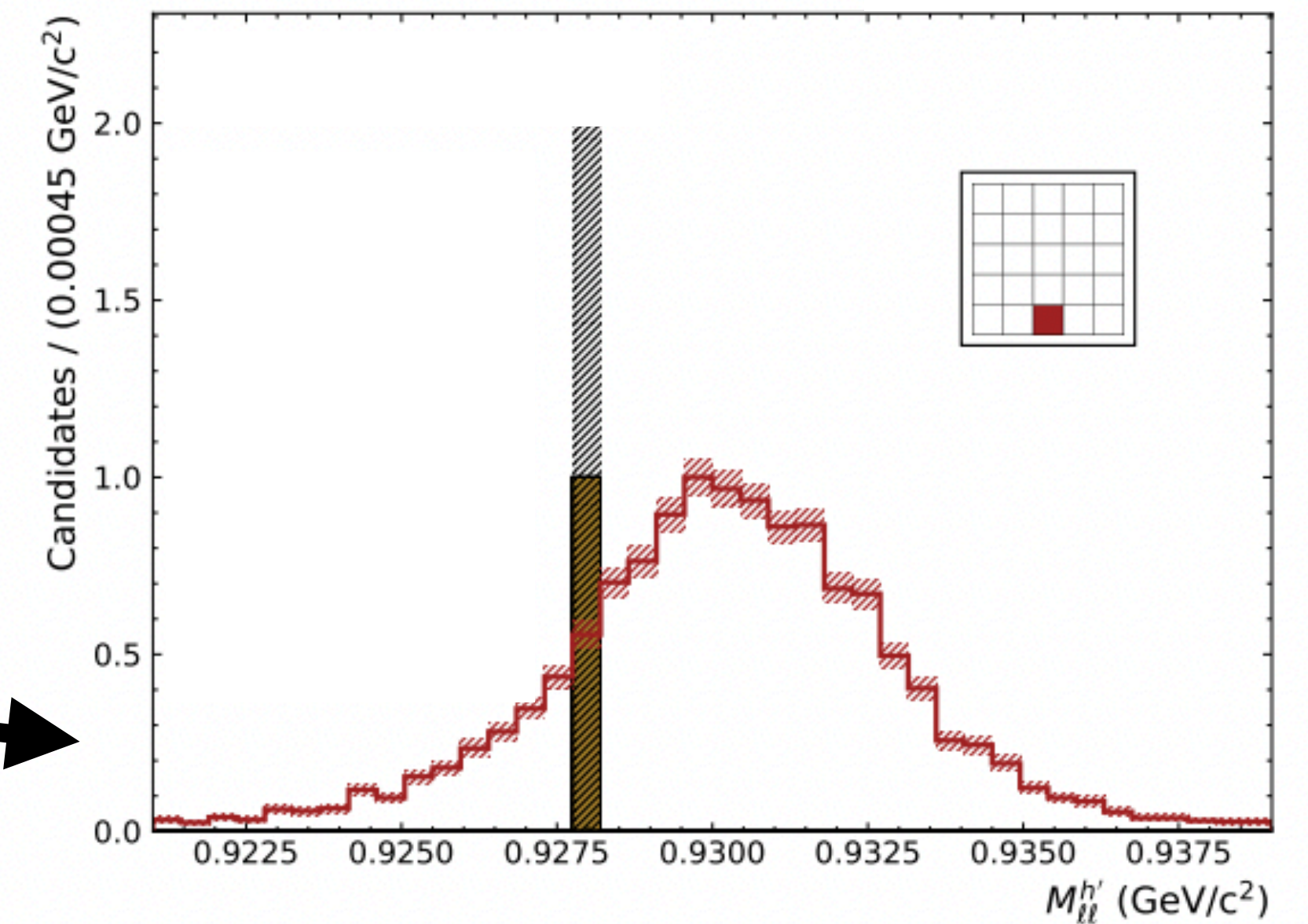
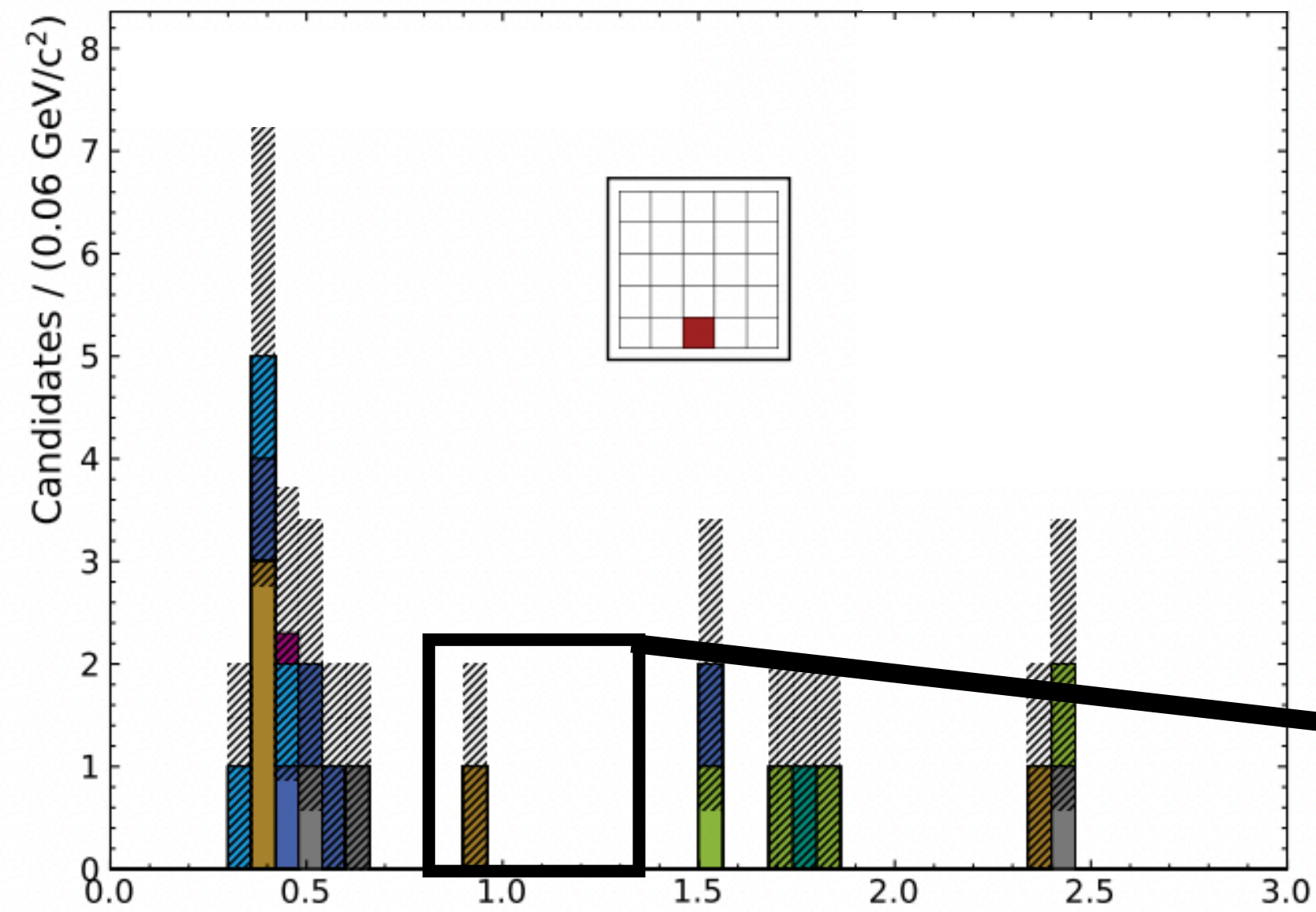
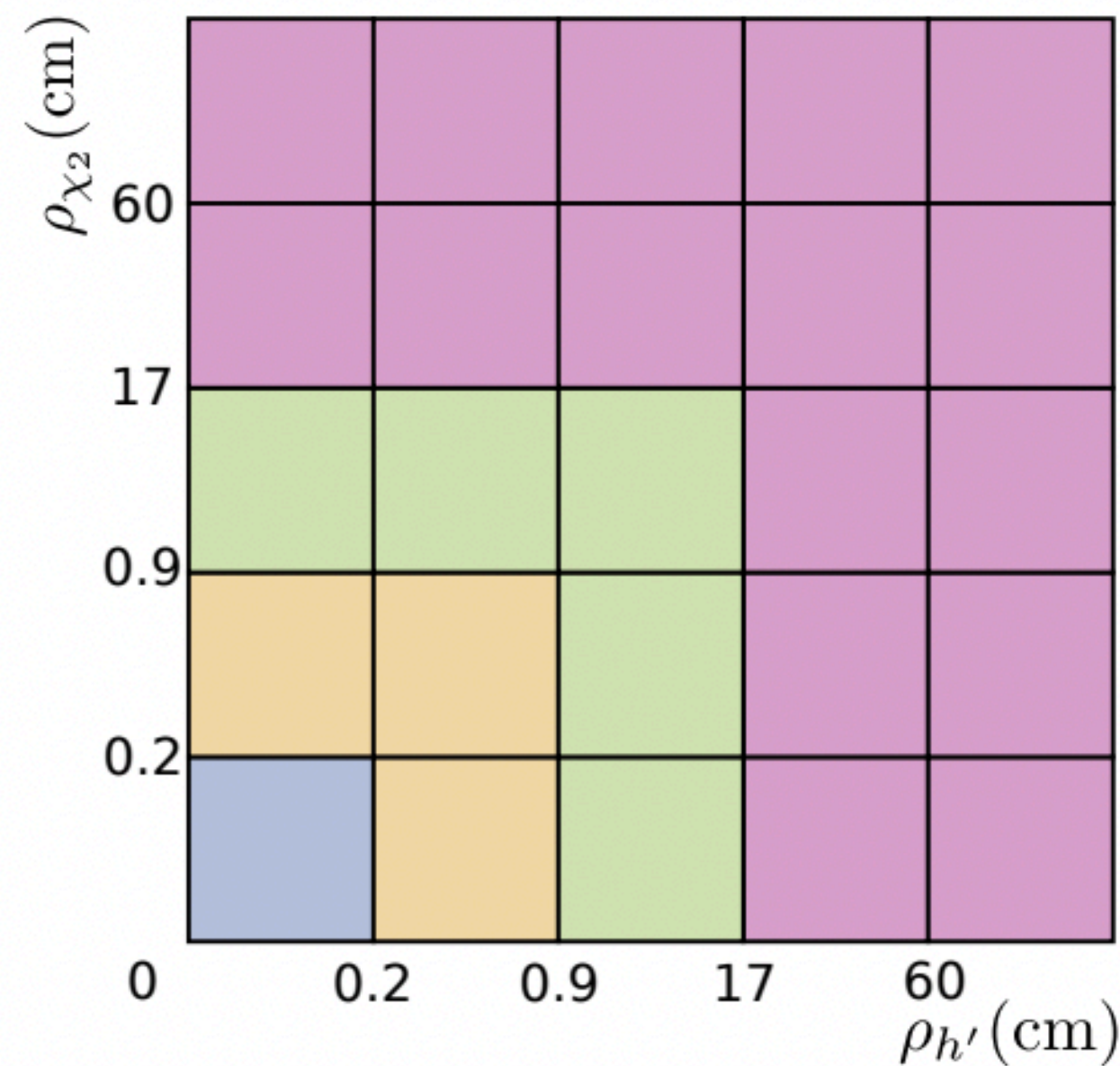
**Calibration channels may not be necessarily very large**

- (Not ultra rare decays) MC statistical error becomes prominent

# Aside: DM searches

The main goal is to suppress the background as much as possible

- Very low-statistics samples  $\rightarrow$  trade-off between smoothness and stability
- Asymptotic conditions may not be necessarily satisfied
- **If signal resolution very narrow sensitive to left-over events**





# Conclusion

**In this presentation I have summarised the**

- Statistical modelling of for rare B-decay searches
- Shown examples of systematic uncertainties that are treated as nuisances
- Show examples of how correlated systematic are computed and propagated into the model
- Highlighted few problems with statistical modelling and propagation of nuisances relevant for rare decay searches and DM


# Thank you!

# Back-up

# Systematic Uncertainties

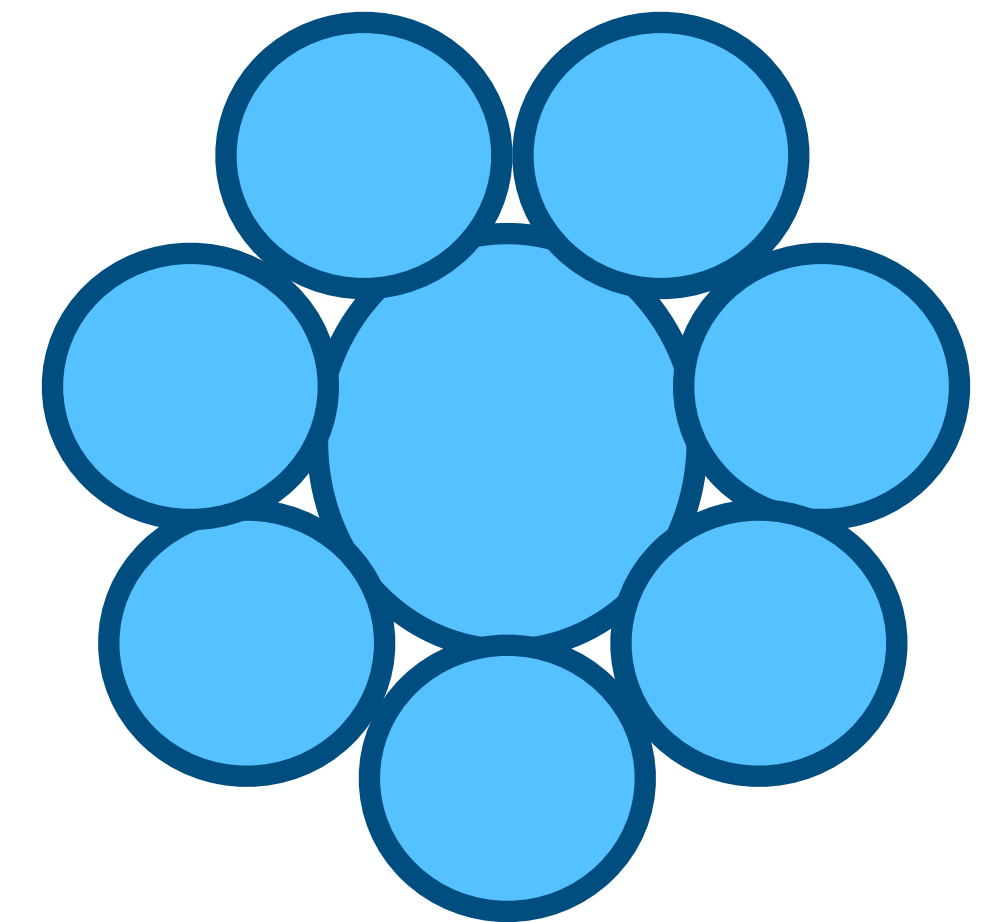
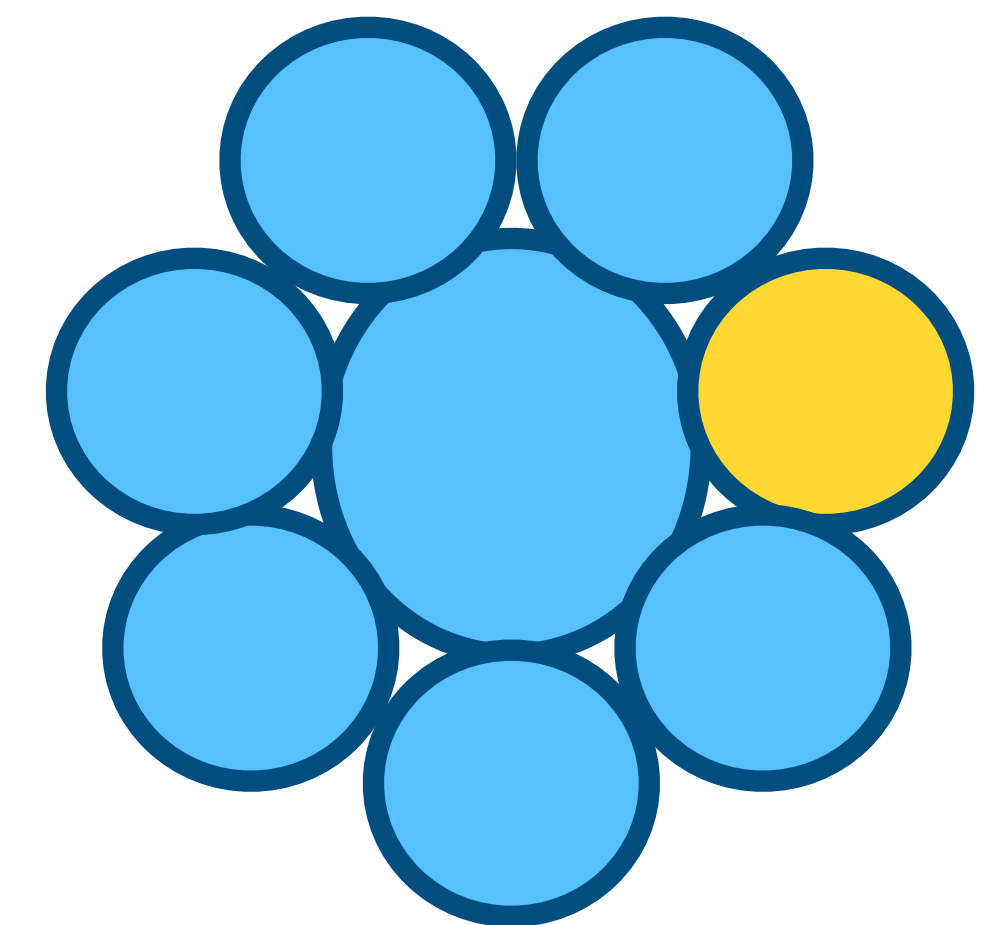
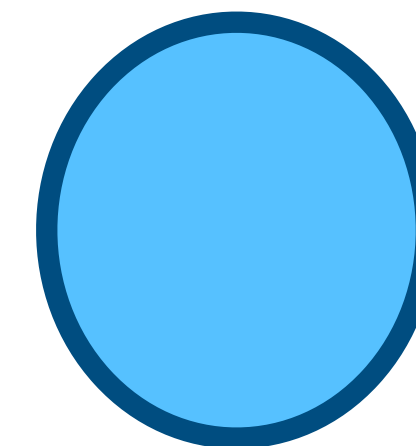
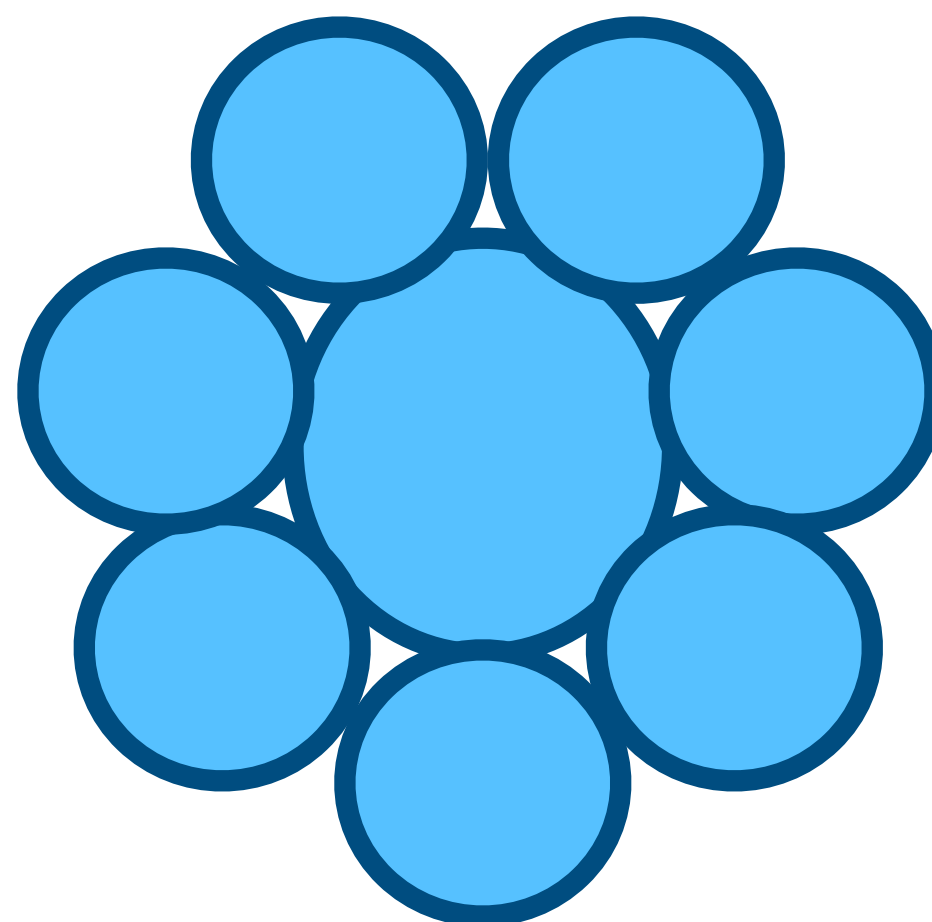
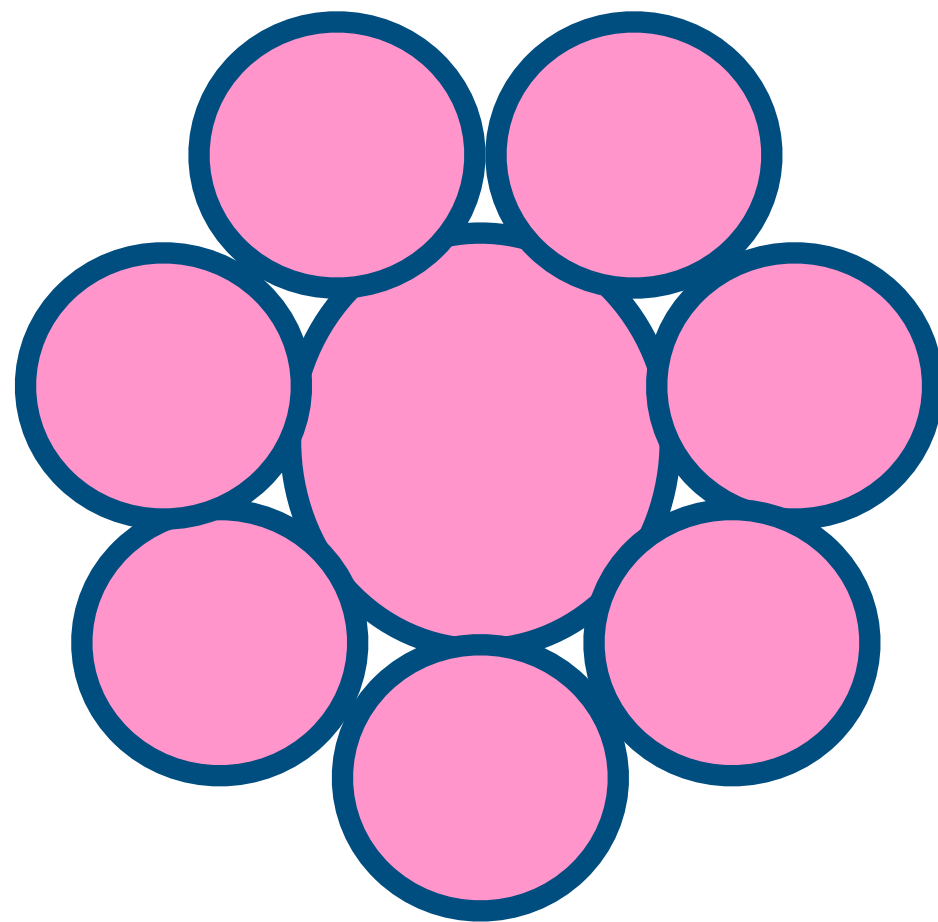
Reconstruction	$B^+ \rightarrow K^+ \nu \bar{\nu}$ (Belle II)	$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ (LHCb)	
		Trigger efficiency	
	Tracking efficiency	Tracking efficiency	

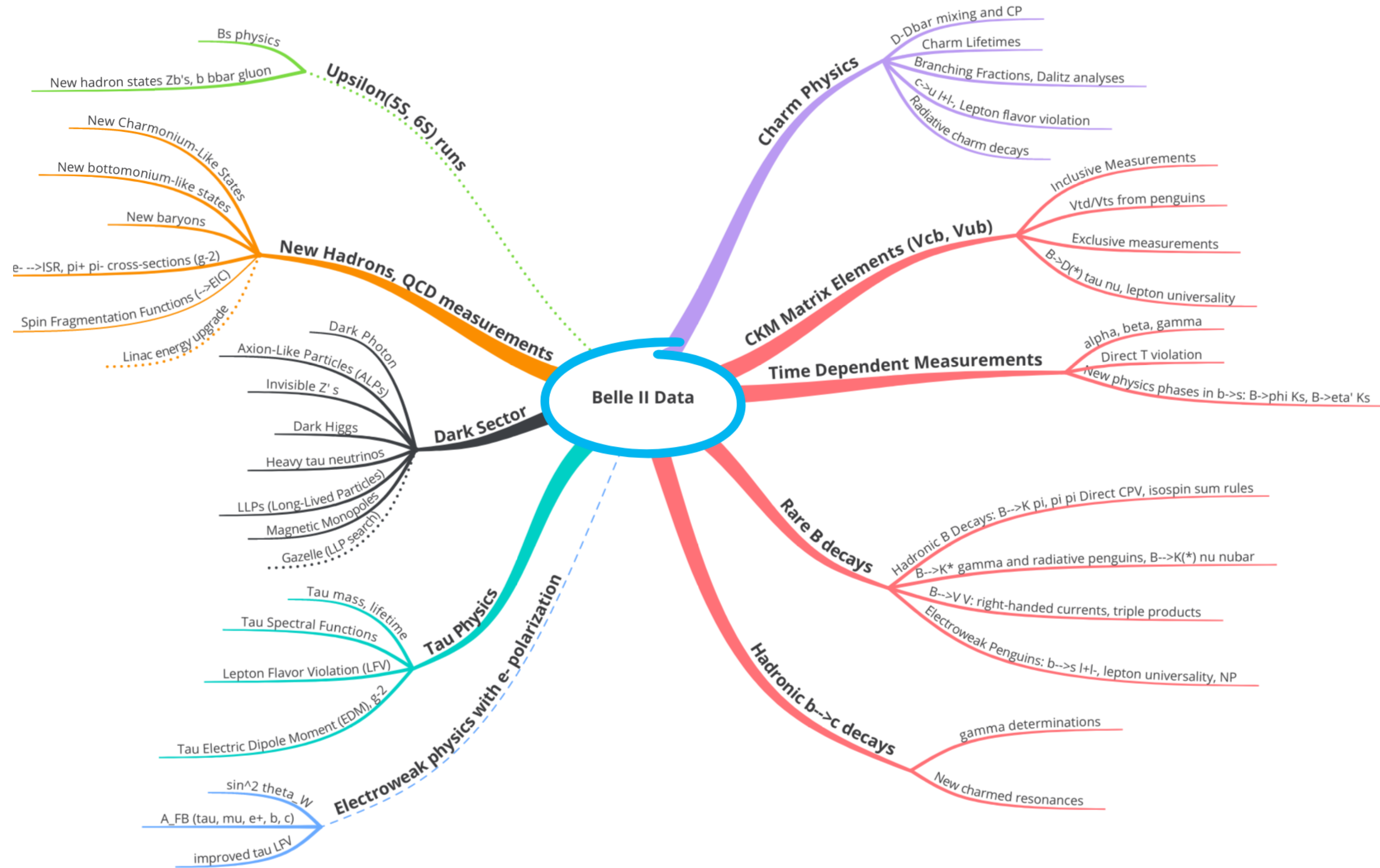
# Systematic Uncertainties

 Further Analysis	$B^+ \rightarrow K^+ \nu \bar{\nu}$ (Belle II)	$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ (LHCb)	
		Trigger efficiency	
	Tracking efficiency	Tracking efficiency	

# When do we integrate systematics?

- When we know something in our analysis chain is maybe incorrect impacting on the measurement:
  1. Wrong → we apply corrections (known as calibrations)
  2. Uncertain → we do cross-checks:
    1. If passed with no major impact on the measurement → no action
    2. If major impact on the measurement → analysis is not robust
    3. If minor impact → **systematics and nuisance parameters**





# HistFactory Template

## Modifiers and Constraints

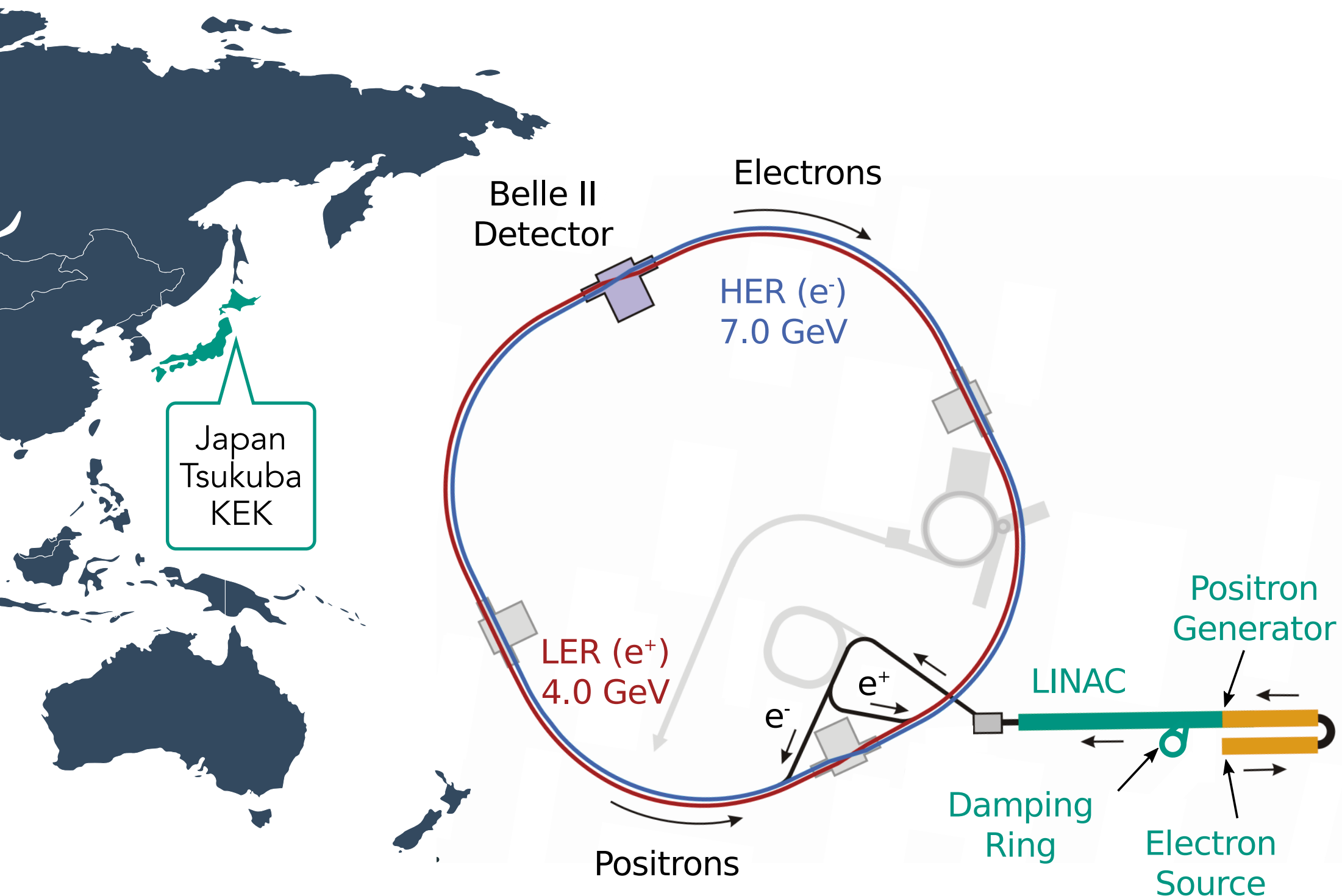
Description	Modification	Constraint Term $c_\chi$	Input	Factor	“Data”
Uncorrelated Shape	$\kappa_{scb}(\gamma_b) = \gamma_b$	$\prod_b \text{Pois}(r_b = \sigma_b^{-2}   \rho_b = \sigma_b^{-2} \gamma_b)$	$\sigma_b$	Bin-wise	Per-bin
Correlated Shape	$\Delta_{scb}(\alpha) = f_p(\alpha   \Delta_{scb,\alpha=-1}, \Delta_{scb,\alpha=1})$	$\text{Gaus}(a = 0   \alpha, \sigma = 1)$	$\Delta_{scb,\alpha=\pm 1}$	Global	Per-bin
Normalisation Unc.	$\kappa_{scb}(\alpha) = g_p(\alpha   \kappa_{scb,\alpha=-1}, \kappa_{scb,\alpha=1})$	$\text{Gaus}(a = 0   \alpha, \sigma = 1)$	$\kappa_{scb,\alpha=\pm 1}$	Global	Per-sample
MC Stat. Uncertainty	$\kappa_{scb}(\gamma_b) = \gamma_b$	$\prod_b \text{Gaus}(a_{\gamma_b} = 1   \gamma_b, \delta_b)$	$\delta_b^2 = \sum_s \delta_{sb}^2$	Bin-wise	Per-bin
Luminosity	$\kappa_{scb}(\lambda) = \lambda$	$\text{Gaus}(l = \lambda_0   \lambda, \sigma_\lambda)$	$\lambda_0, \sigma_\lambda$	Global	Nothing
Normalisation	$\kappa_{scb}(\mu_b) = \mu_b$			Global	Nothing
Data-driven Shape	$\kappa_{scb}(\gamma_b) = \gamma_b$			Bin-wise	Nothing

- Correlated Shape: same source of uncertainty which has a different effect on the various sample shapes (e.g PID, tracking inefficiency, ...)
- MC Stat. uncertainty: uncertainty due to the finite sample size of the datasets
- Luminosity especially useful if cross-section is to be measured

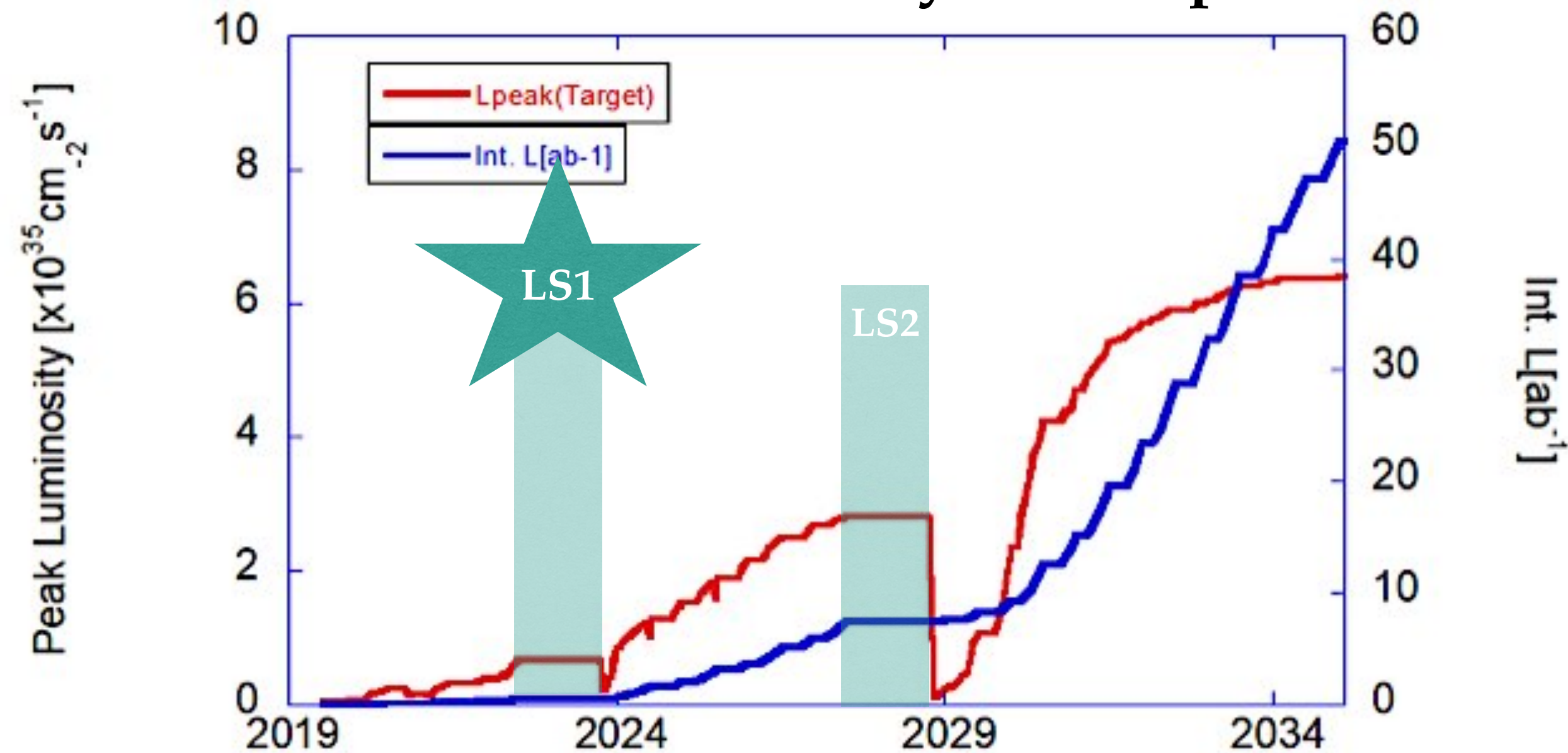
# SuperKEKB

SuperKEKB operates nominally at  $\sqrt{s} = 10.58$  GeV

- $\Upsilon(4S) \rightarrow B\bar{B}$  in 96 %
- Currently  $363 \text{ fb}^{-1} \sim 390$  mil.  $B$ -meson pairs
  - $\sim 1/2$  Belle,  $\sim$  BaBar
- Record-breaking  $\mathcal{L}_{inst} = 4.7 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$



Belle II luminosity roadmap



- Now in long shutdown (**LS1**) until this autumn
- **Final aim:** operate at  **$30 \times$  higher  $\mathcal{L}_{inst}$**  than KEKB at a cost of  **$\mathcal{O}(10) \times$  higher backgrounds**
- **Final luminosity goal:**  $\mathcal{L}_{int} = 50 \text{ ab}^{-1}$

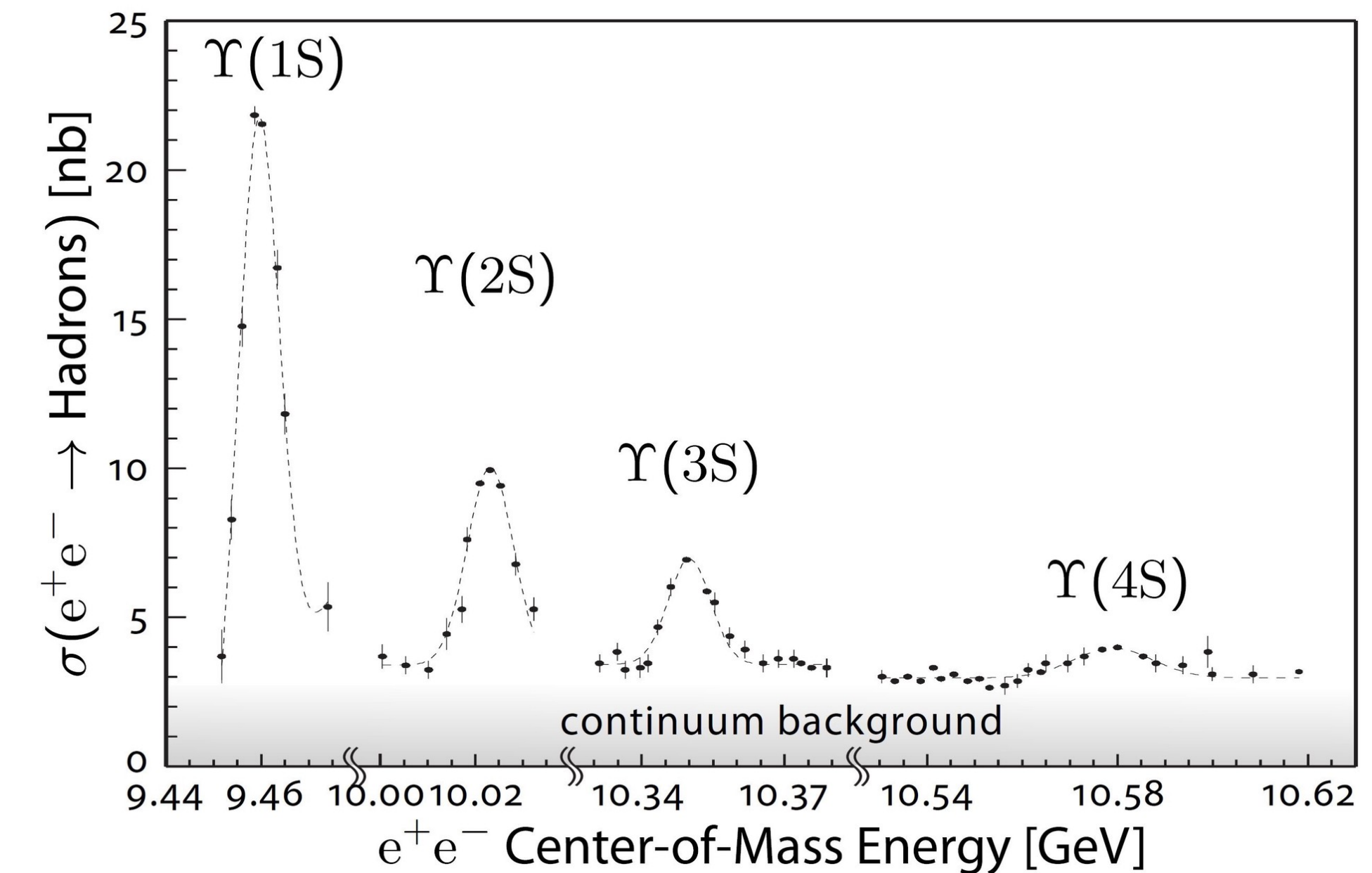


# Backgrounds at $e^+e^-$ Collider

Four types of backgrounds at  $e^+e^-$  colliders:

- Continuum Backgrounds  $e^+e^- \rightarrow q\bar{q}$ , where  $q \in (s, c, d, u)$  and  $e^+e^- \rightarrow \tau\bar{\tau}$
- $B$ -backgrounds
  - misidentified
  - mis-reconstructed
  - combinatorial

Continuum backgrounds



$\sqrt{s} = 10.52$  GeV  $\rightarrow$  control sample to constrain continuum backgrounds

# Backgrounds at $e^+e^-$ Collider

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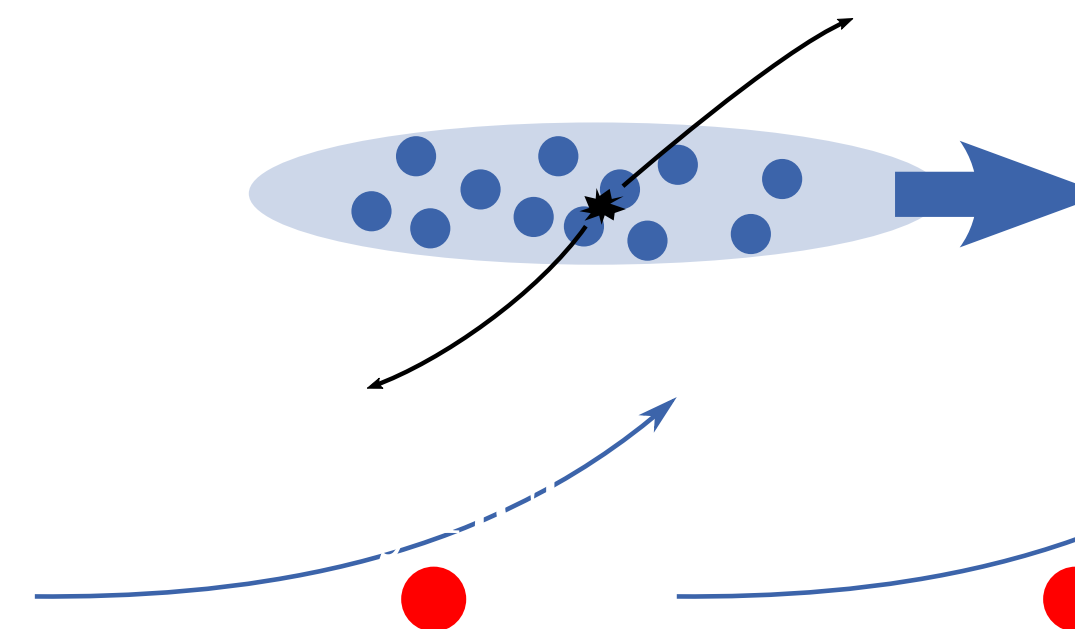
- Beam-backgrounds:

- Touschek scattering, Coulomb scattering, synchrotron radiation, injection background, ...

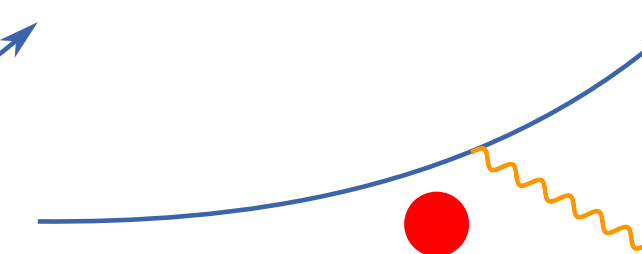
- Luminosity Backgrounds:

- $e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-e^+e^-$

Increase with  $\mathcal{L}_{inst}$

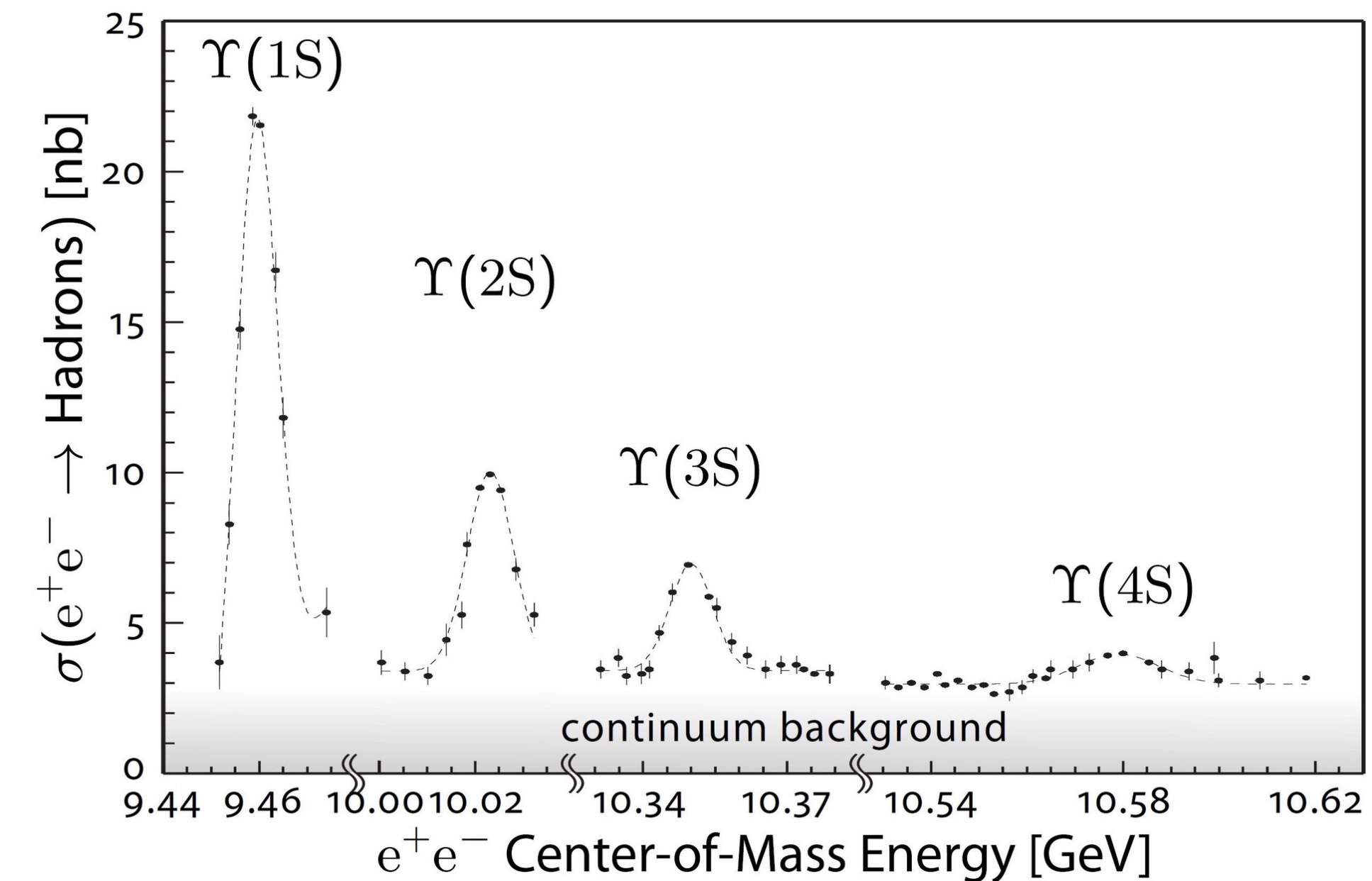


Touschek scattering



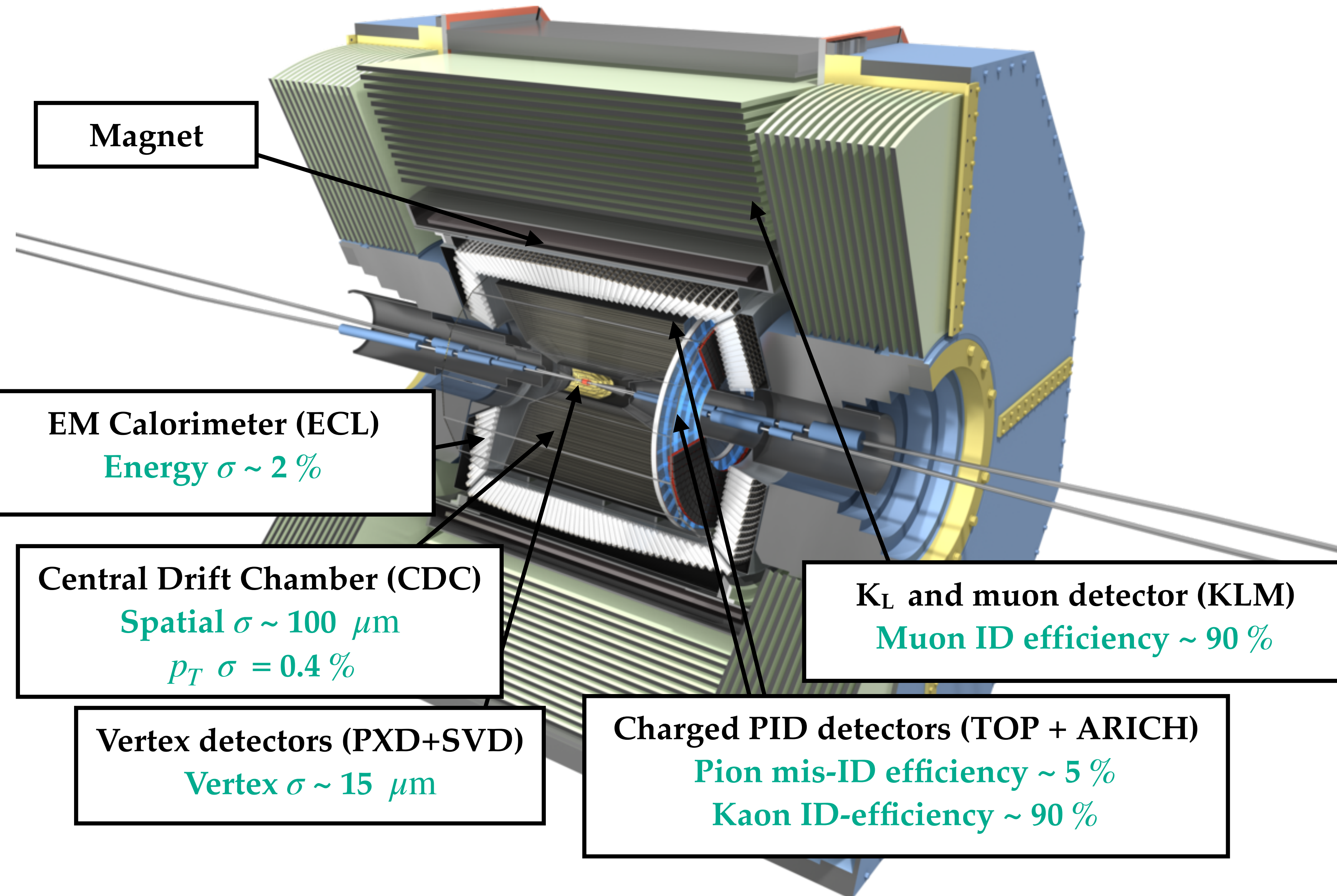
Coulomb scattering

## Continuum backgrounds



$\sqrt{s} = 10.52 \text{ GeV} \rightarrow$  control sample to constrain continuum backgrounds

# Belle II Detector



**Magnet**

**EM Calorimeter (ECL)**  
Energy  $\sigma \sim 2\%$

**Central Drift Chamber (CDC)**  
Spatial  $\sigma \sim 100 \mu\text{m}$   
 $p_T \sigma = 0.4\%$

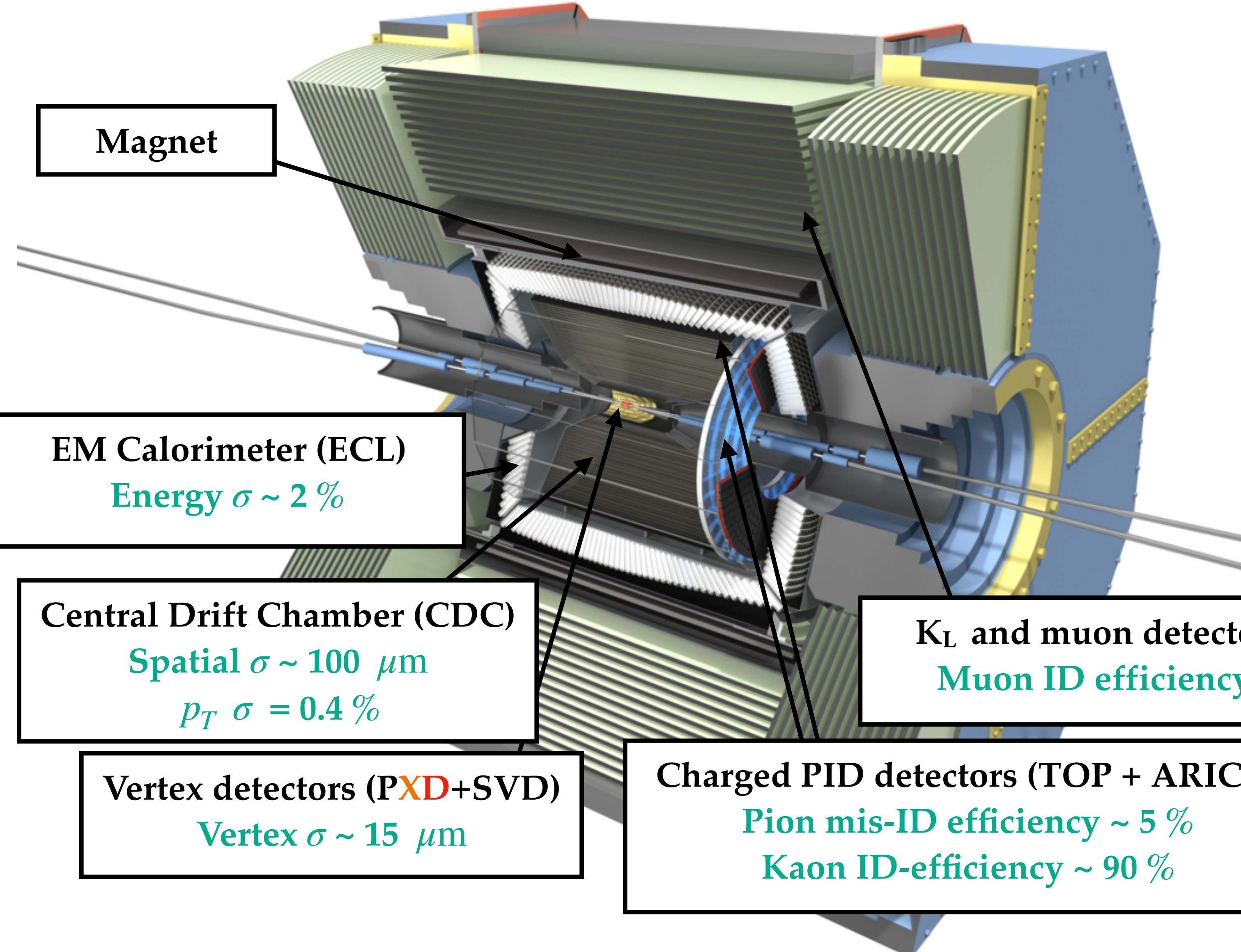
**Vertex detectors (PXD+SVD)**  
Vertex  $\sigma \sim 15 \mu\text{m}$

**Charged PID detectors (TOP + ARICH)**  
Pion mis-ID efficiency  $\sim 5\%$   
Kaon ID-efficiency  $\sim 90\%$

**K<sub>L</sub> and muon detector (KLM)**  
Muon ID efficiency  $\sim 90\%$

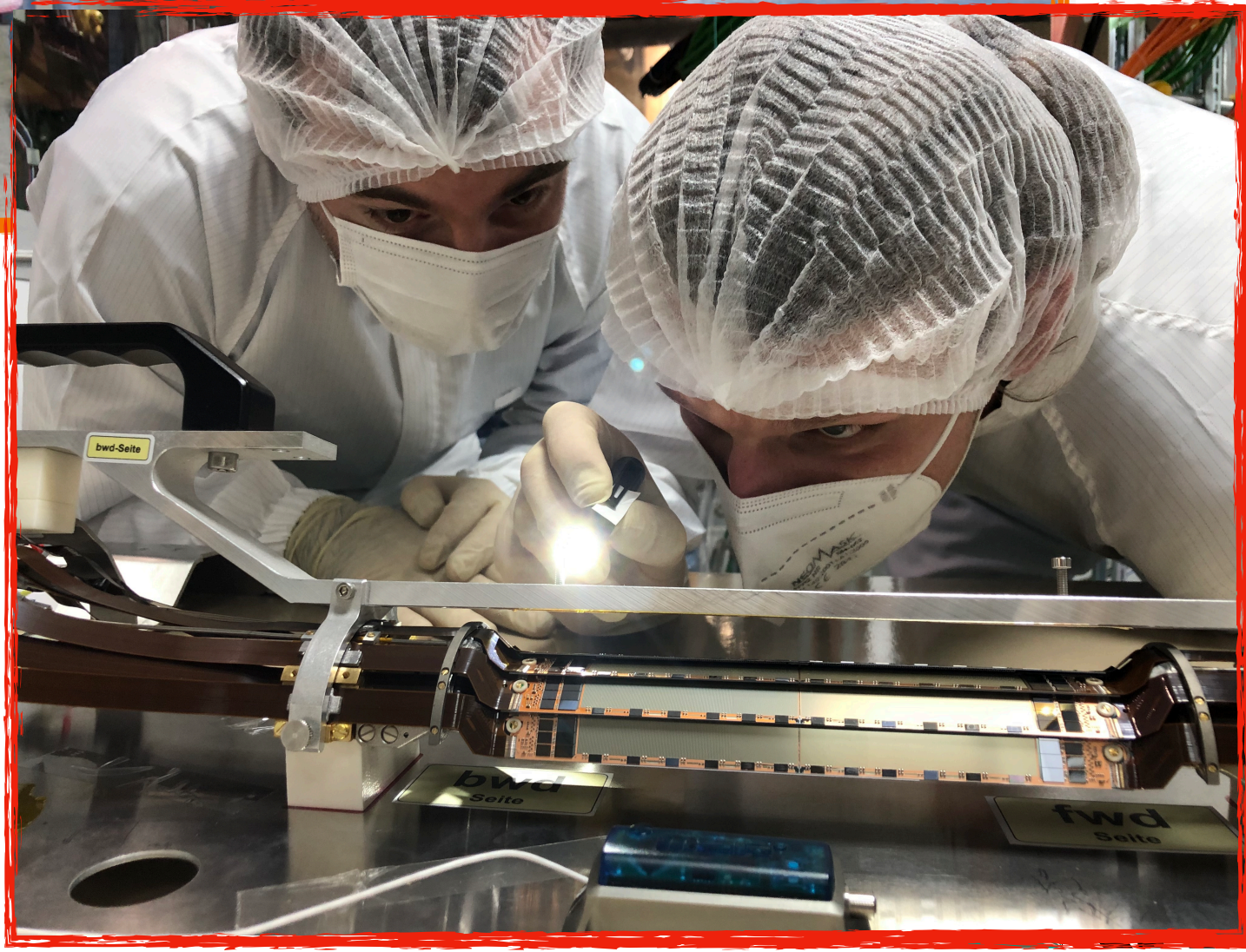
# Belle II Detector

New **PXD** arrived safely to KEK!



**Belle II Deutschland** @belle2germany · Mar 18

Auch Detektoren haben weite Wege hinter sich! Unser neuer Pixeldetektor PXD2 hat sich am Mittwoch morgen auf den Weg vom @desy in Hamburg nach Japan gemacht. Damit nichts kaputt geht, bekam der empfindliche Detektor einen eigenen Sitzplatz. Mehr dazu: [belle2.de/detail/belle-i-...](http://belle2.de/detail/belle-i-...)



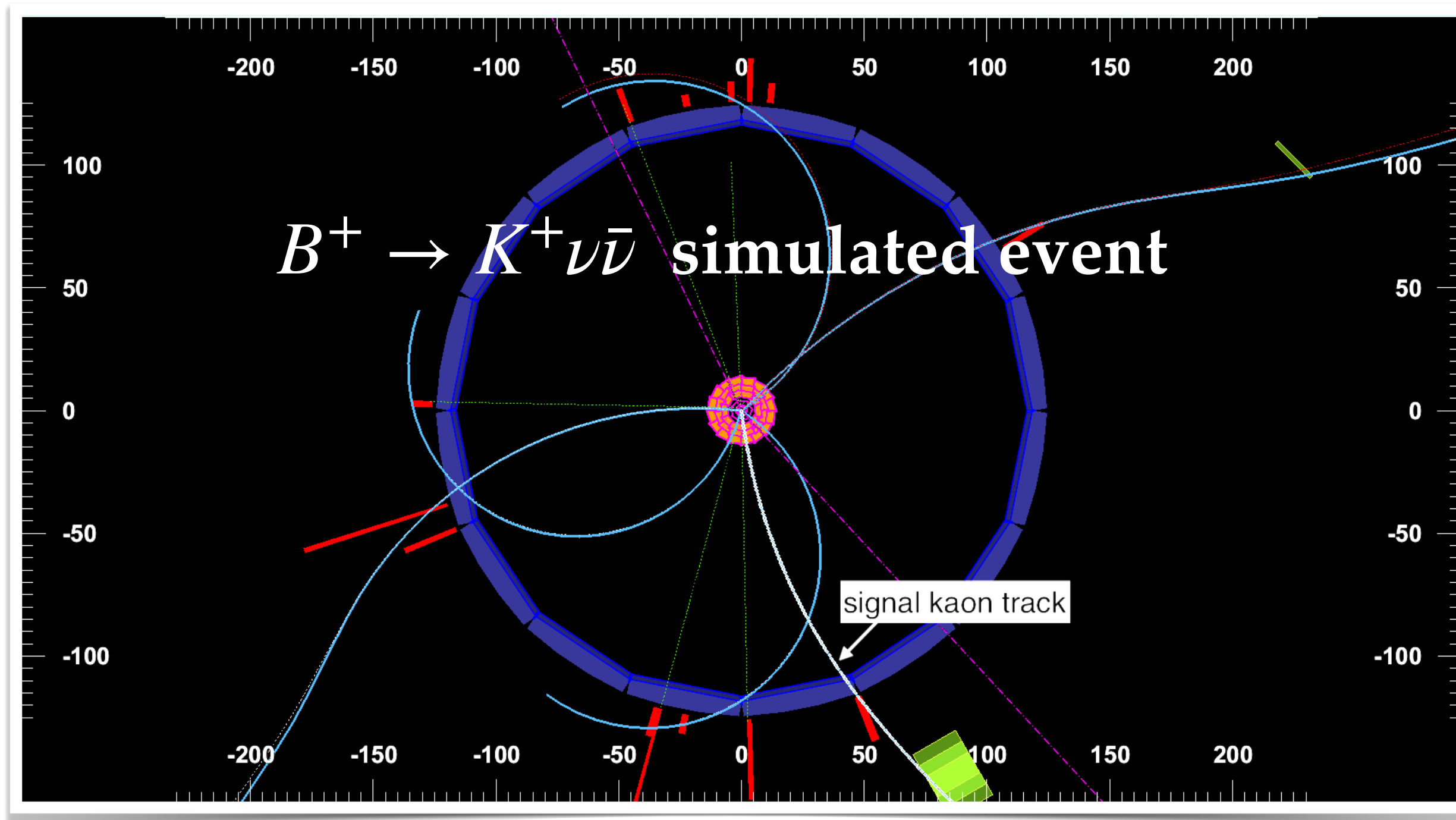
# $B \rightarrow K^{(*)} \nu \bar{\nu}$ Event in Belle II

Belle II is best-suited to measure  $B$ -decays with significant missing energy



Typical  $B \rightarrow K^{(*)} \nu \bar{\nu}$  event benefits from

- Detector with nearly full  $4\pi$  coverage with excellent sensitivity to low deposits
- Cleaner environment compared to LHCb
- Constraints from well-known initial state kinematics



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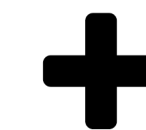


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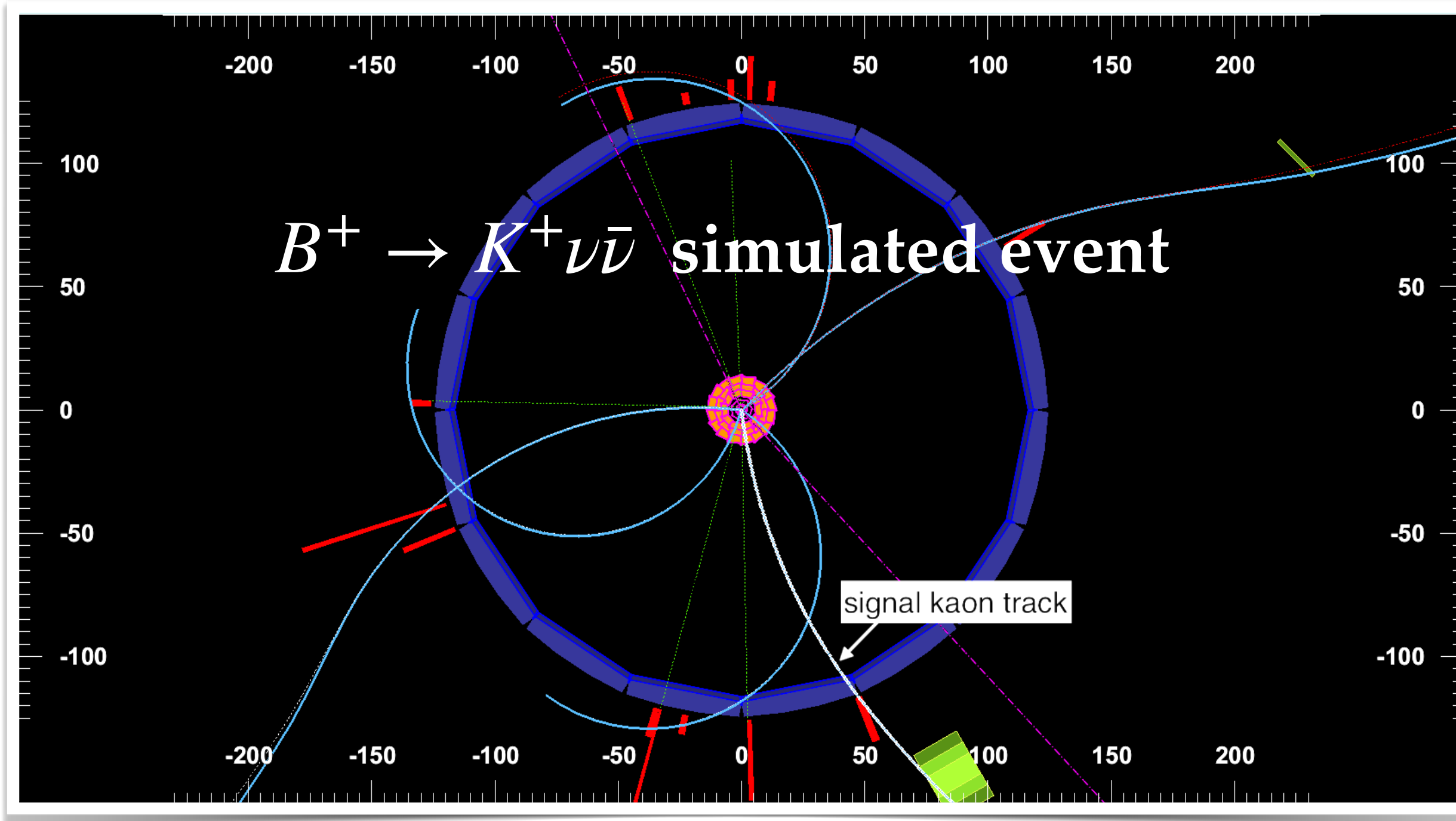
Challenges of rare  $B$ -decays

- High reconstruction efficiency for visible particles
- Excellent MC modelling



Challenges of channels with neutrinos

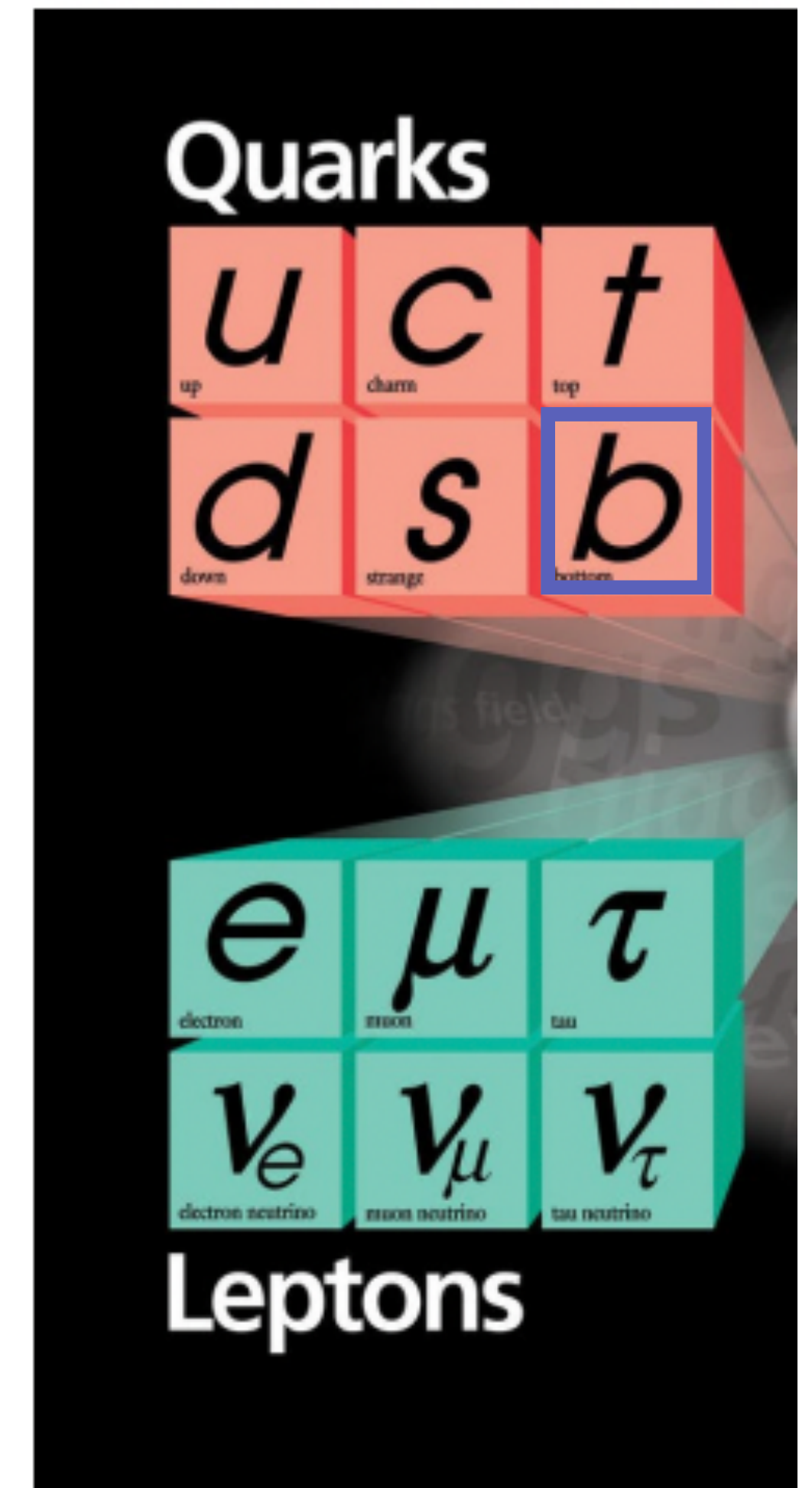
- Excellent understanding of the neutral objects  
( $\pi^0, K_L^0, K_S^0, n, \gamma, \dots$ )



# Why Flavour Physics?

- Quarks, leptons and interactions within the SM are the main protagonists of flavour physics
- $B$ -decays are especially good probes since
  - $B$ -hadrons are light enough to be produced abundantly and heavy enough to have many decays
  - Predictions for SM observables are well-known
- With  $B$ -decays we perform
  - Precision measurements of CP violation
  - (In-)direct searches for NP in rare decays

Focus of this talk



# Beam-backgrounds

Single-beam backgrounds:

- ▶ **Touschek scattering** → scattering of particles within a bunch →

$$\text{Touschek rate} \propto N_{\text{particles}} \times \rho \rightarrow I \times \frac{I}{\sigma_y n_b}$$

- ▶ **beam-gas scattering** → Coulomb scattering and Bremsstrahlung (scattering off gas molecules) → **Beam-gas rate**  $\propto N_{\text{gas molecules}} \times$

$$N_{\text{particles}} \rightarrow P \times I \times Z_{\text{eff}}^2$$

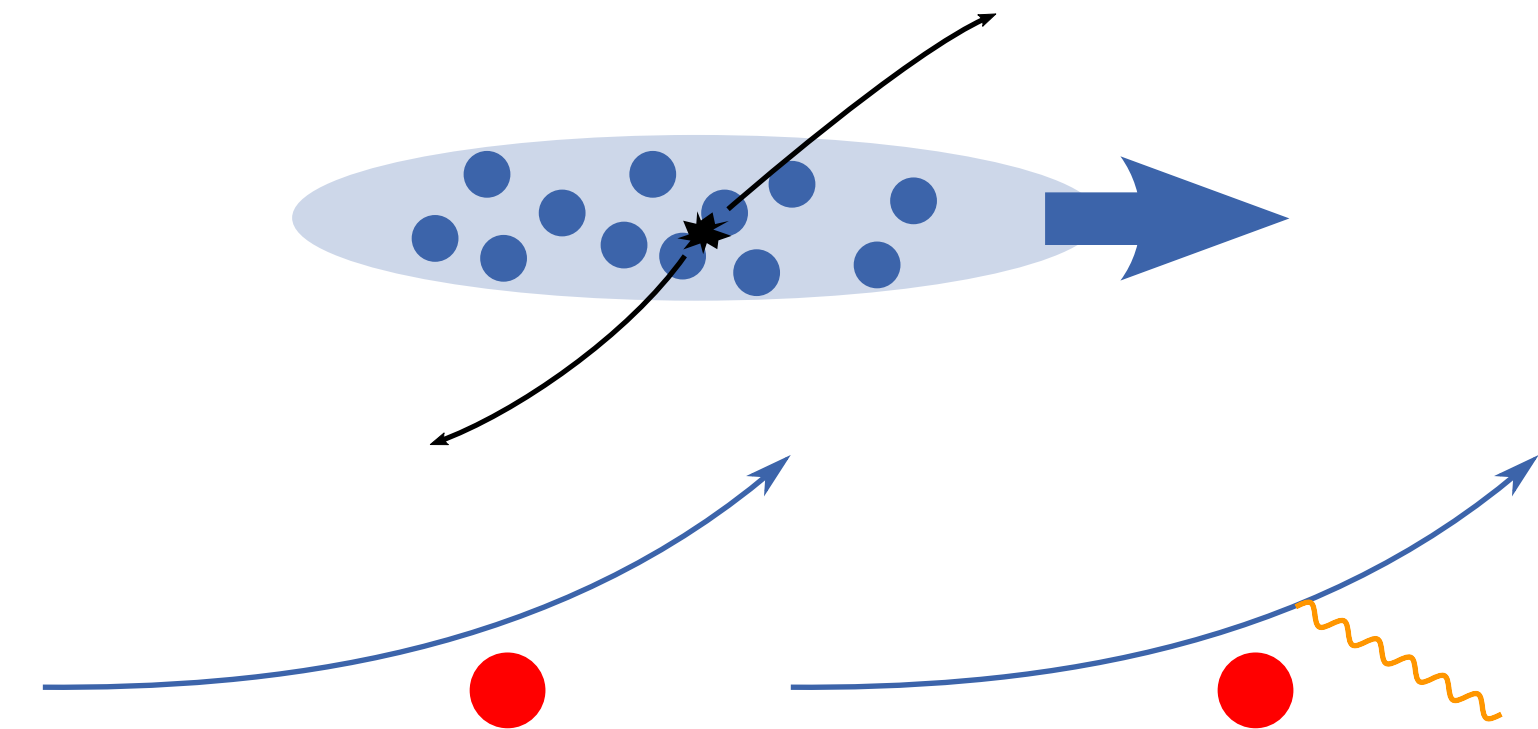
- ▶ **synchrotron radiation background** → consequence of a radial acceleration of the beam's particles achieved in bending magnets and quadrupoles

- ▶ **injection background** → continuous injection of charge into beam bunch modifying the beam bunch

Single-beam backgrounds can be mitigated with beam-steering, collimators, and vacuum-scrubbing

Luminosity backgrounds:

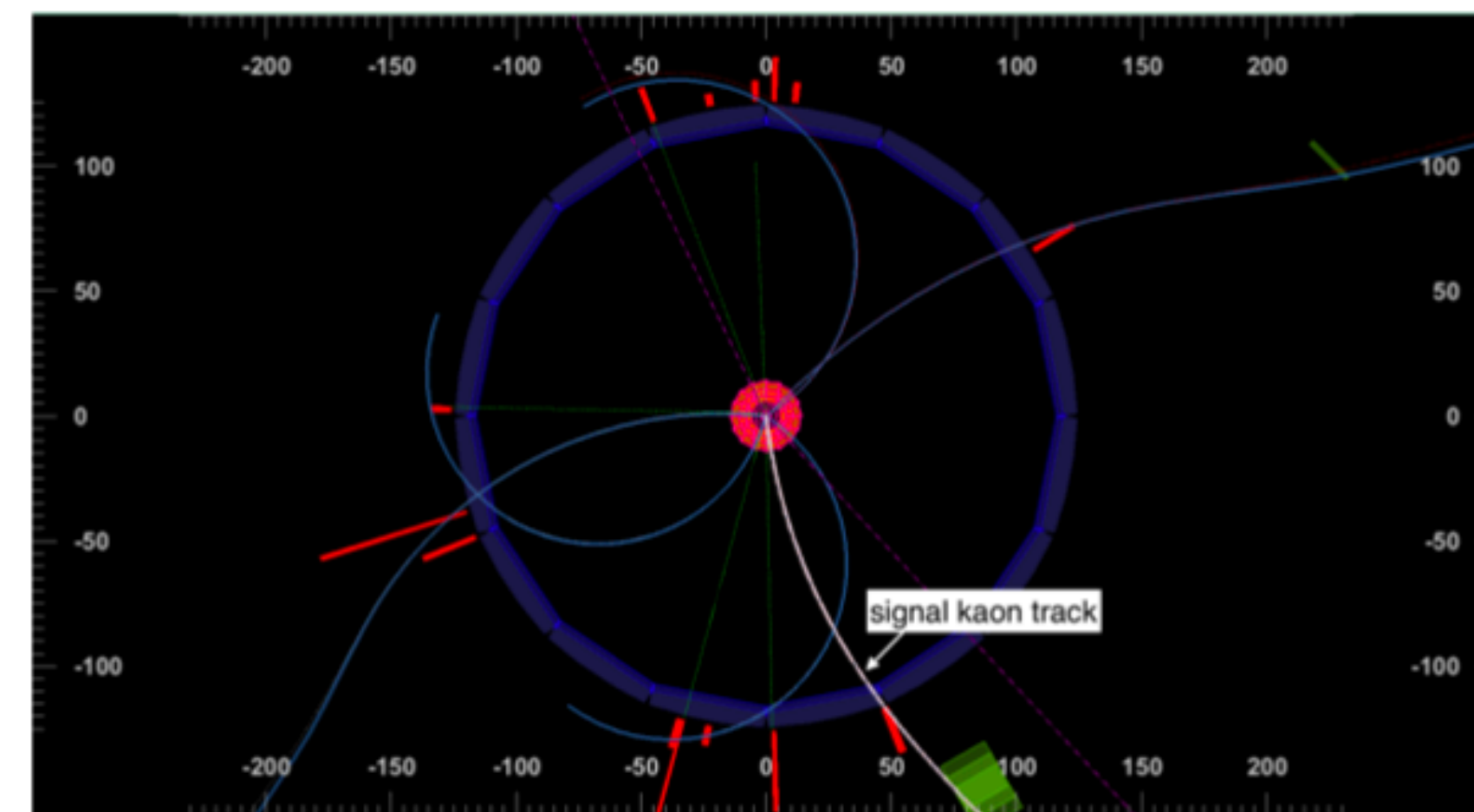
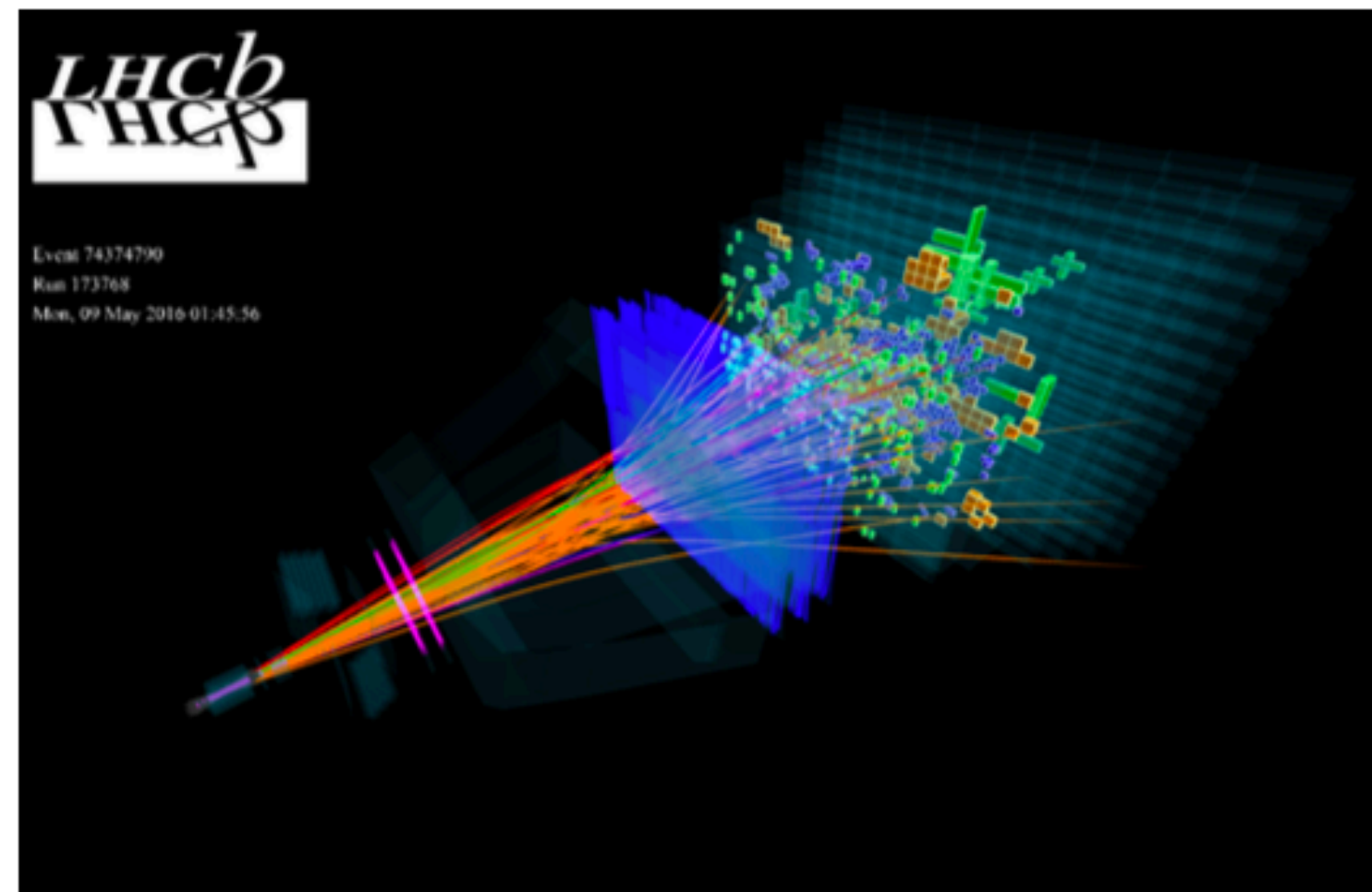
- ▶ **two-photon background** → leading luminosity background ( $e^+e^- \rightarrow e^+e^- \gamma\gamma \rightarrow e^+e^-e^+e^-$ ), unlike any of the backgrounds above cannot be reduced!



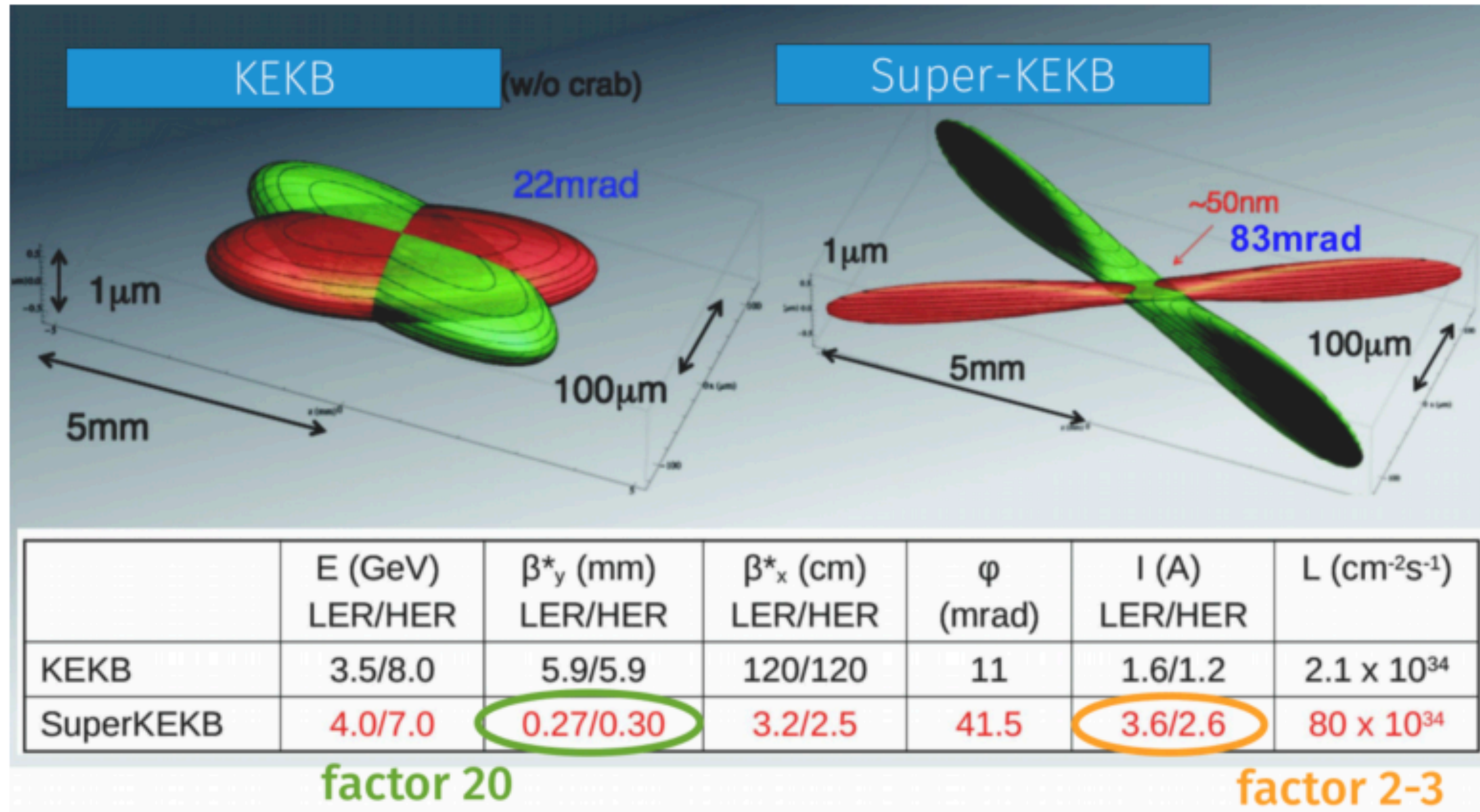


# Belle II vs LHCb

LHCb	Belle II
single-arm detector longitudinal momentum of $B$ not known	hermetic detector known initial state kinematics pro @ neutral object reconstruction (photon, $K_L$ )



# SuperKEKB vs KEKB

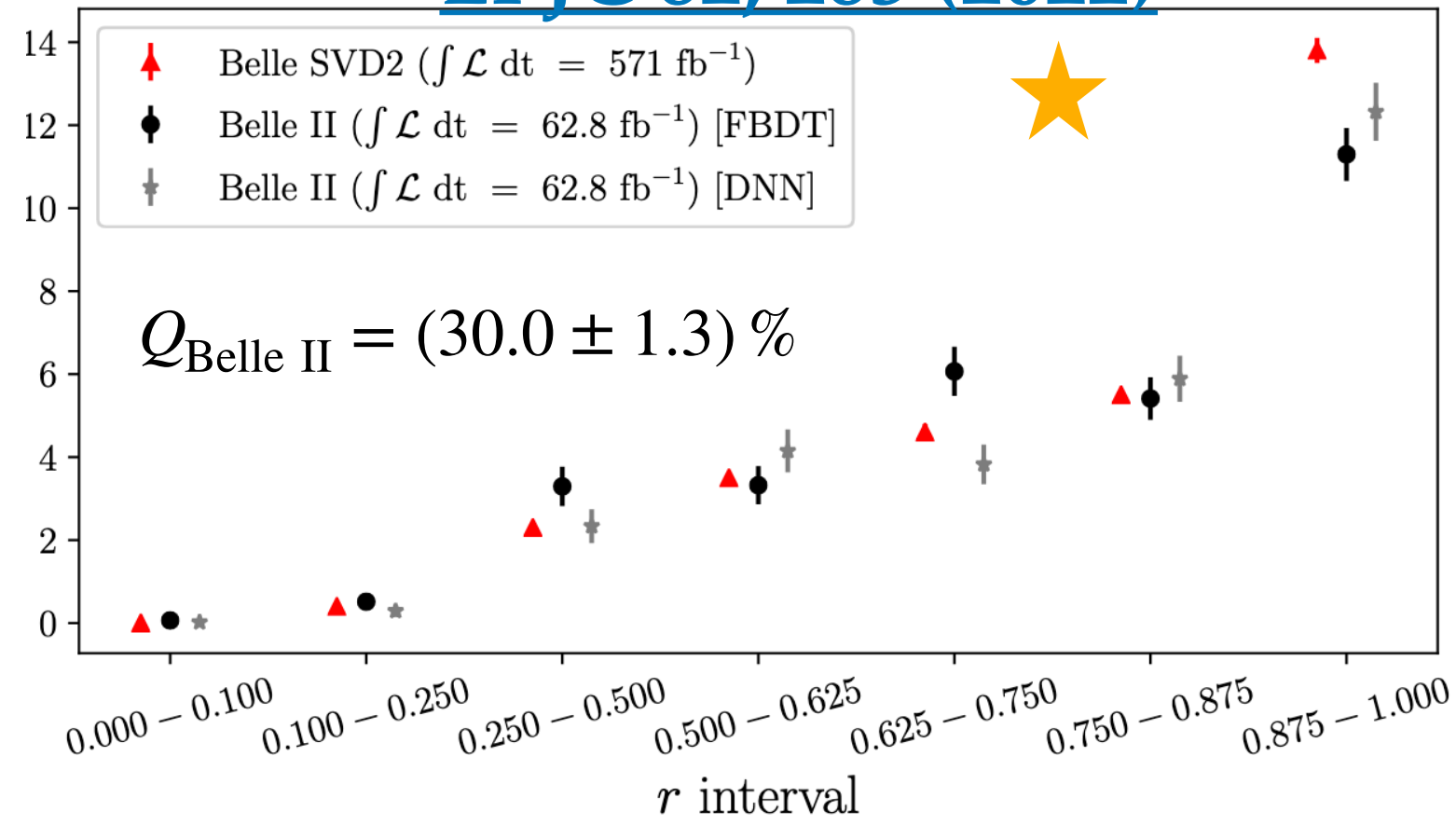


	KEKB		SuperKEKB (Juni 2022)		SuperKEKB Ziel	
	LER	HER	LER	HER	LER	HER
Energie [GeV]	3.5	8	4	7	4	7
#Bunches	1584		2249		1800	
$\beta^*_x/\beta^*_y$ [mm]	1200/5.9	1200/5.9	80/1.0	60/1.0	32/0.27	25/0.3
I [A]	1.64	1.19	1.46	1.15	2.8	2.0
Luminosität [ $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ ]	2.1		4.65 (Rekord!)		60	
Int. Luminosität [ $\text{ab}^{-1}$ ]	1		0.43		50	

# Performance

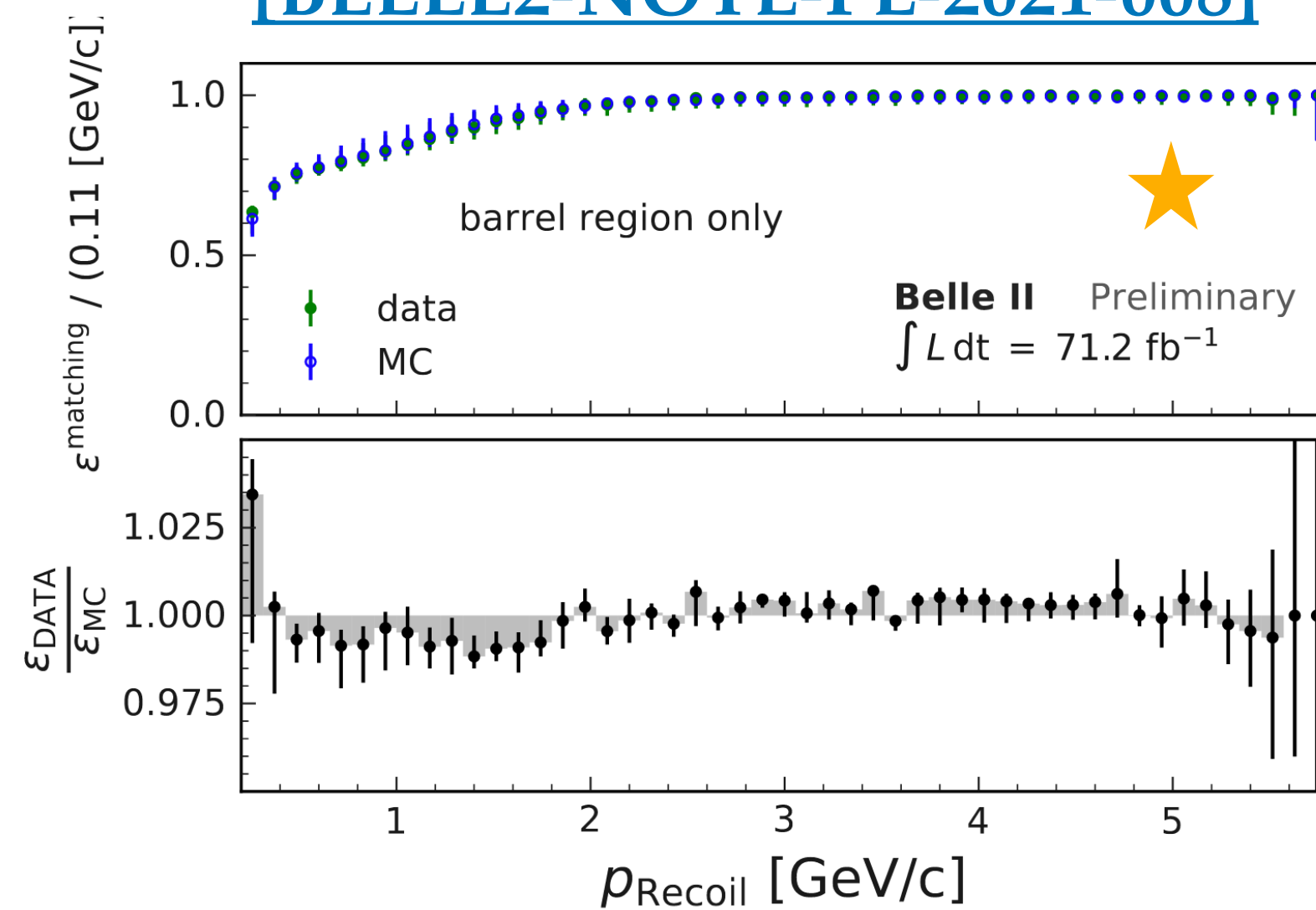
## Good flavour tagger performance

[EPJC 82, 283 \(2022\)](#)



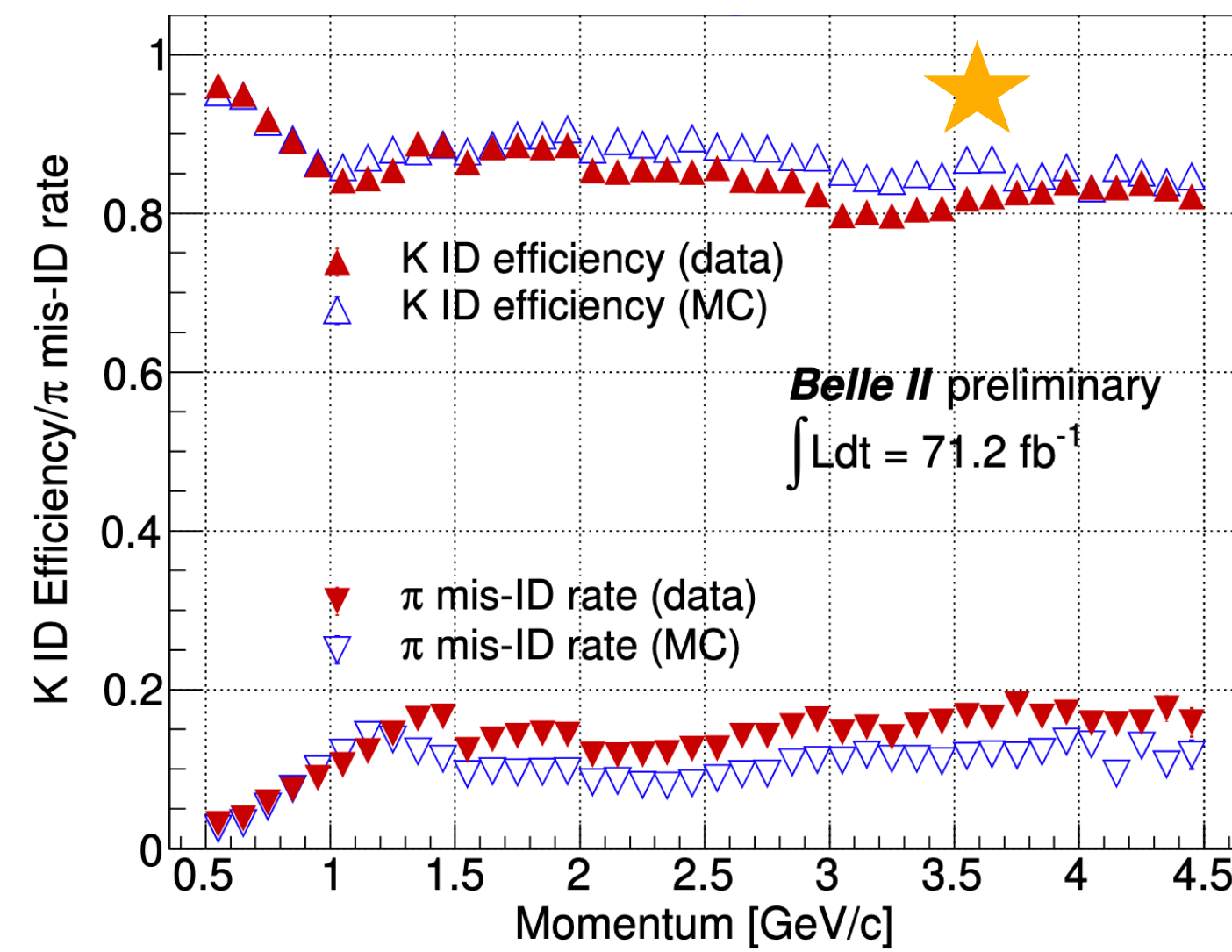
## High photon matching efficiency

[\[BELLE2-NOTE-PL-2021-008\]](#)

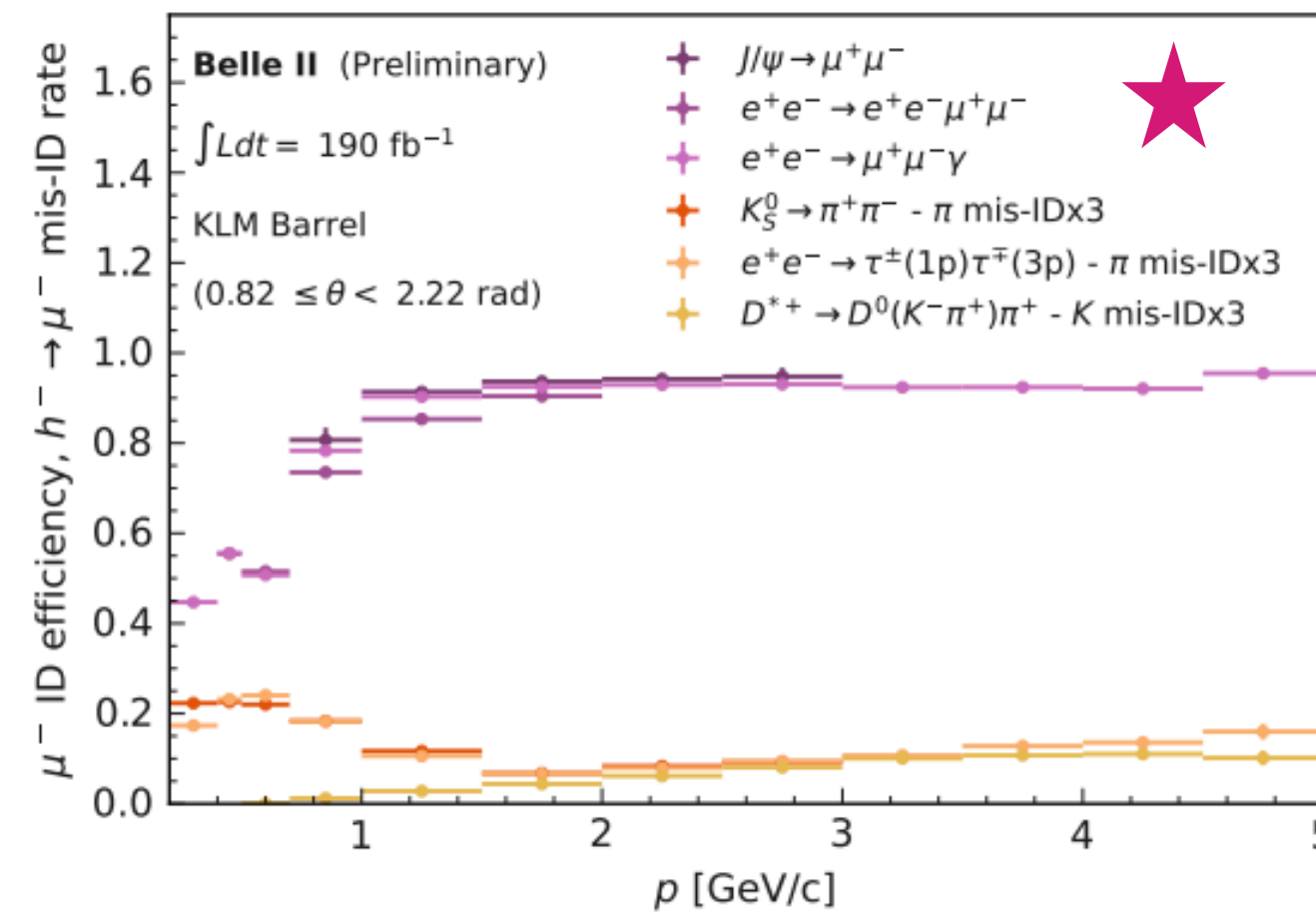


## Good particle identification

[\[BELLE2-NOTE-PL-2020-024\]](#)



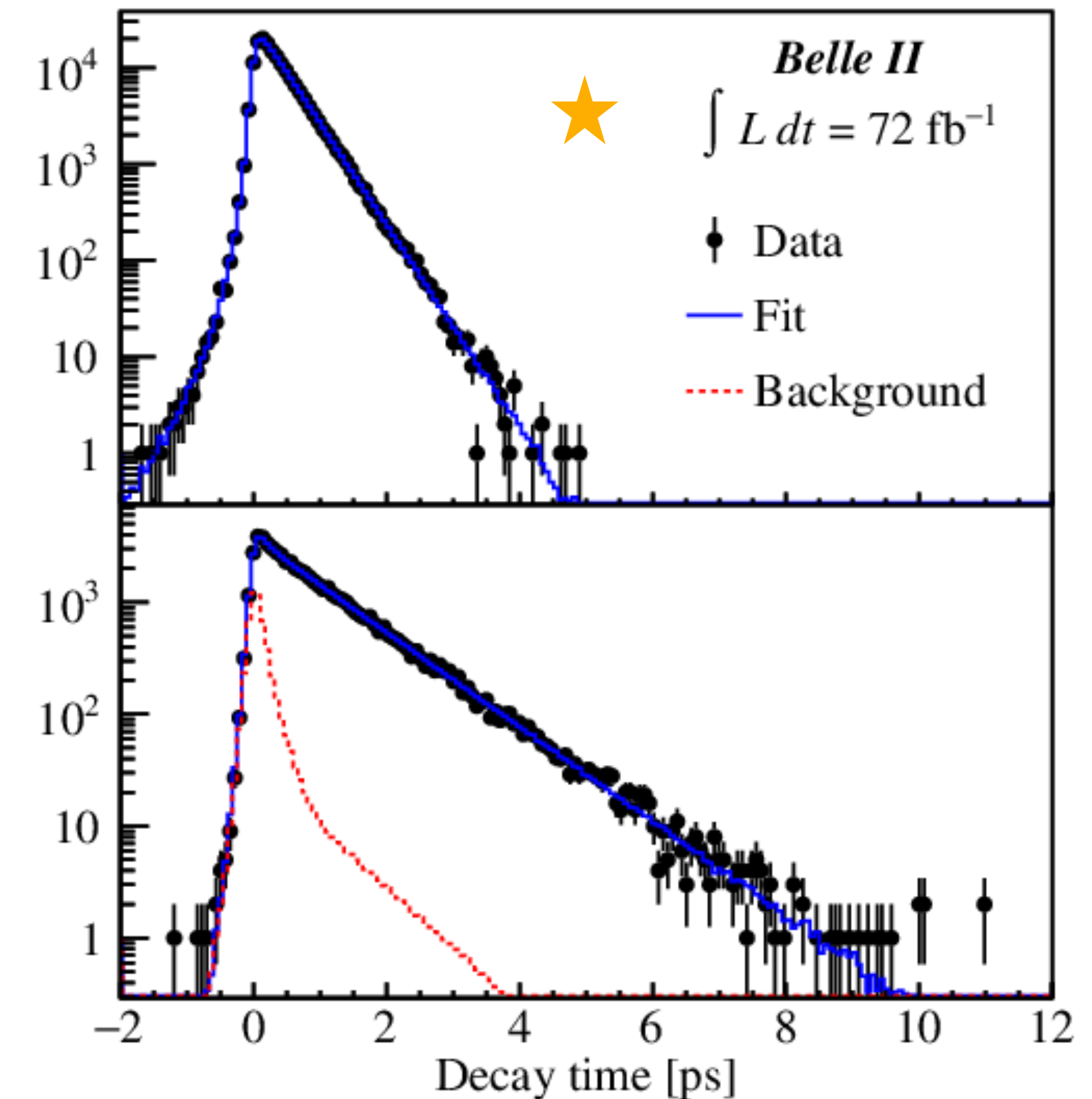
[\[BELLE2-NOTE-PL-2022-003\]](#)



## Most precise measurement of

### $D$ lifetimes

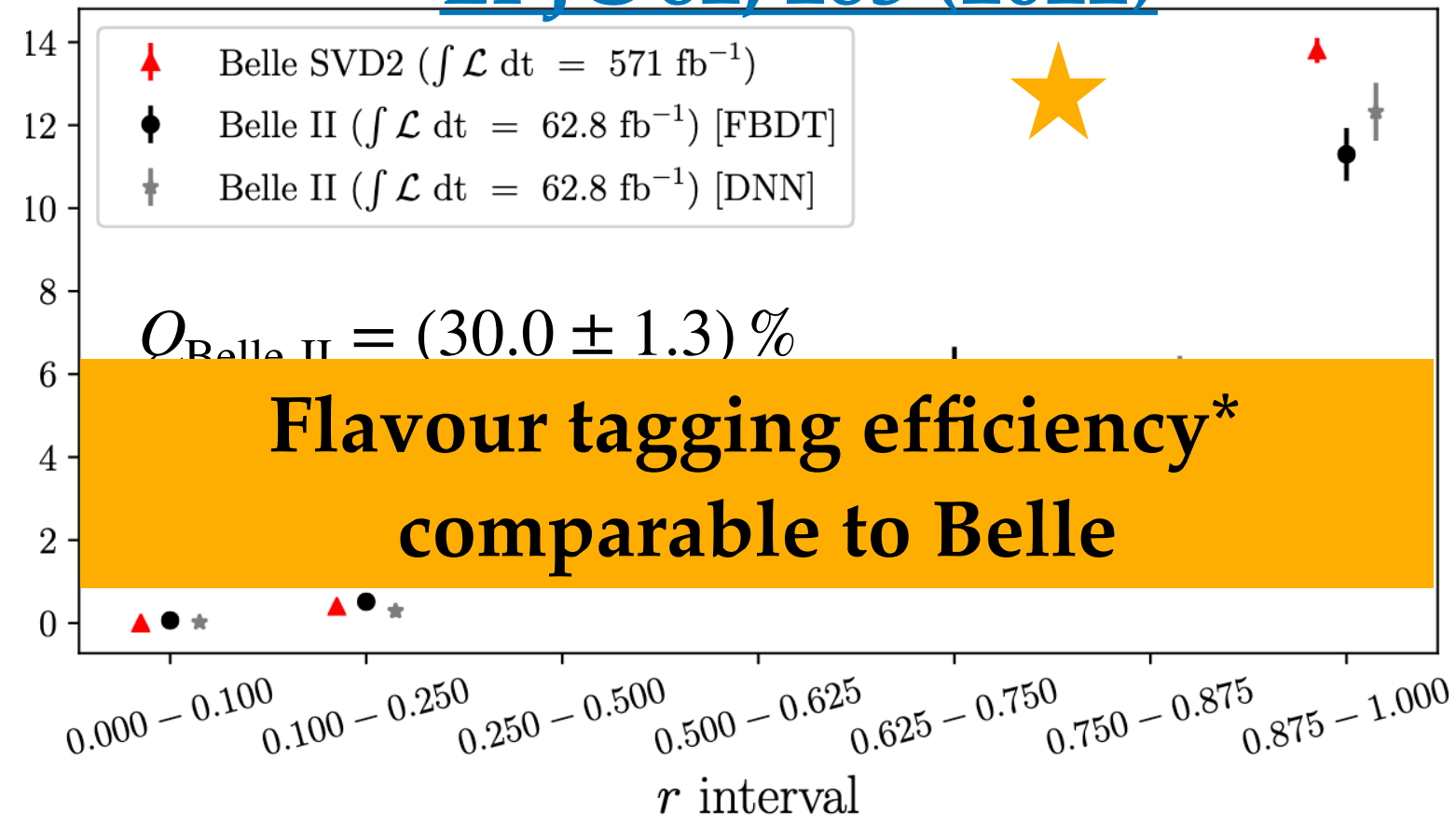
[PRL 127, 211801 \(2021\)](#)



# Performance

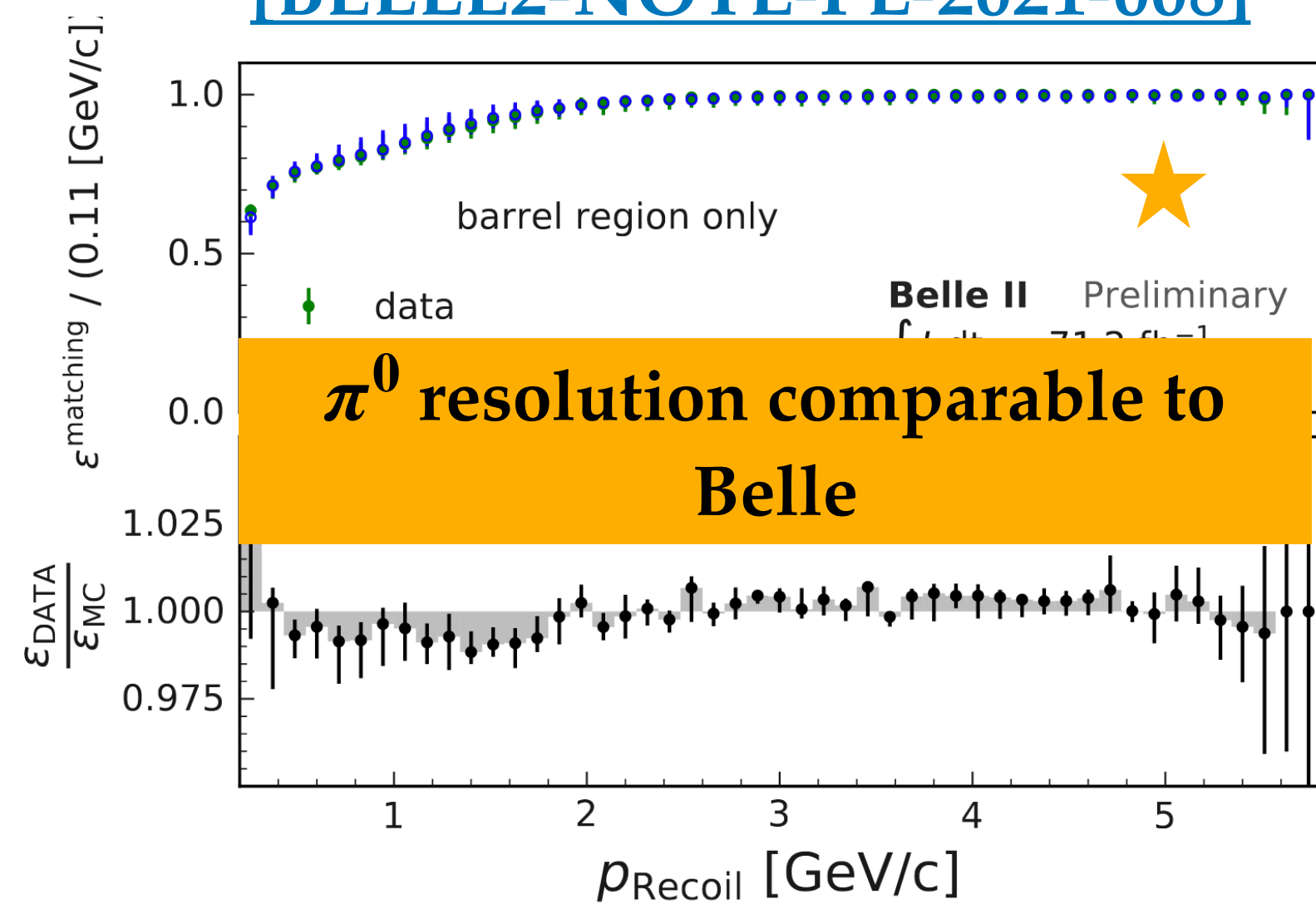
Good flavour tagger performance

[EPJC 82, 283 \(2022\)](#)



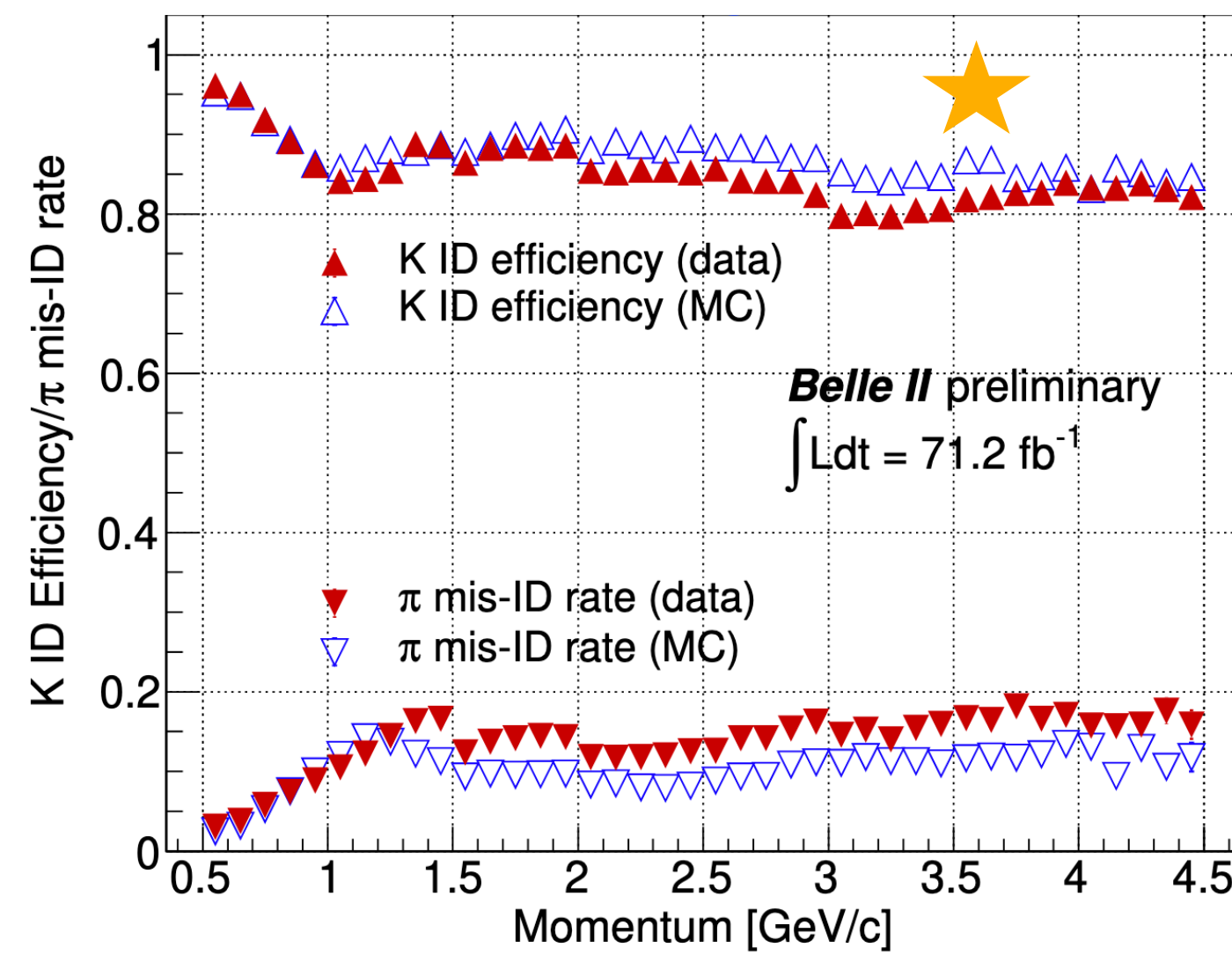
High photon matching efficiency

[\[BELLE2-NOTE-PL-2021-008\]](#)

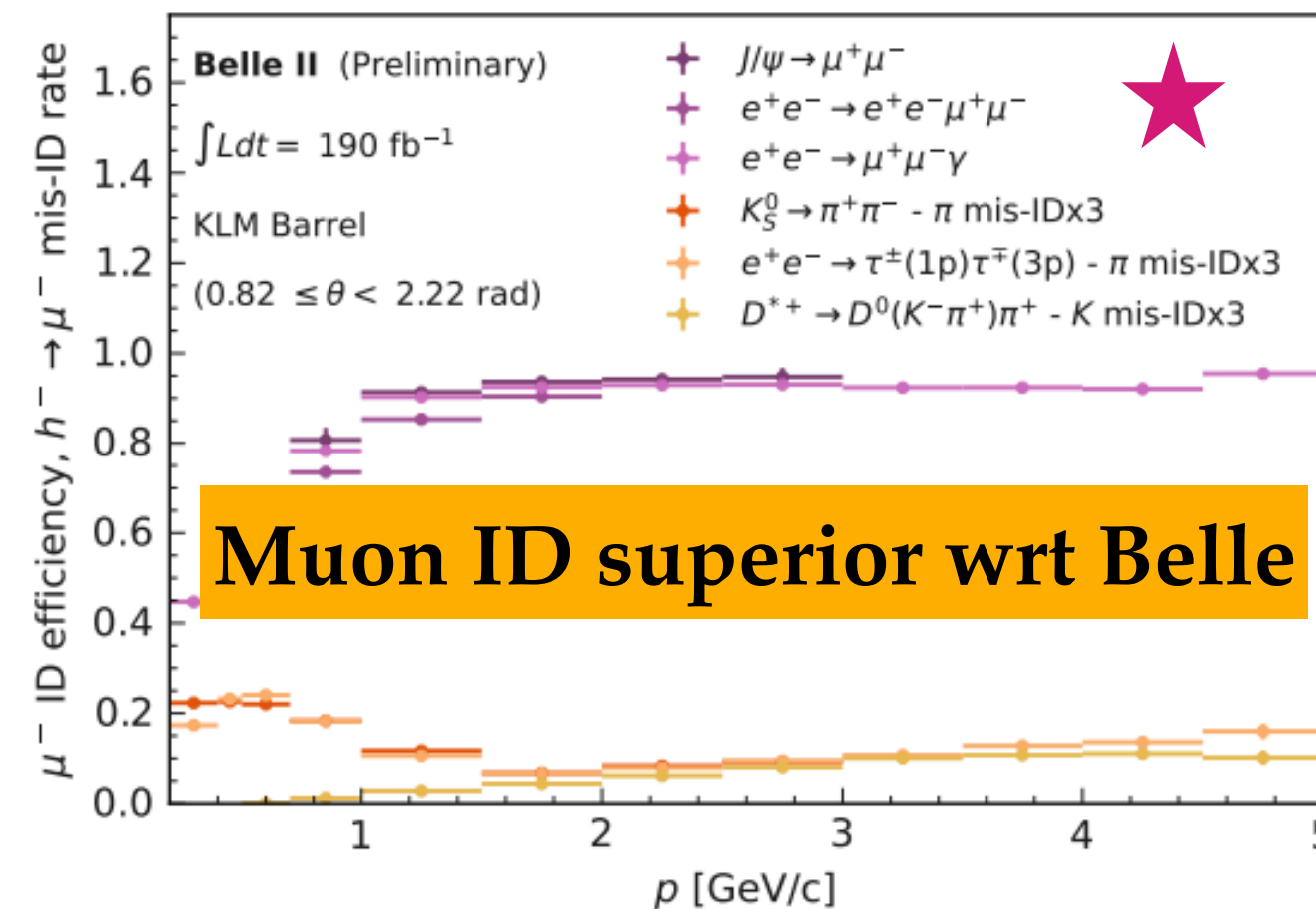


Good particle identification

[\[BELLE2-NOTE-PL-2020-024\]](#)



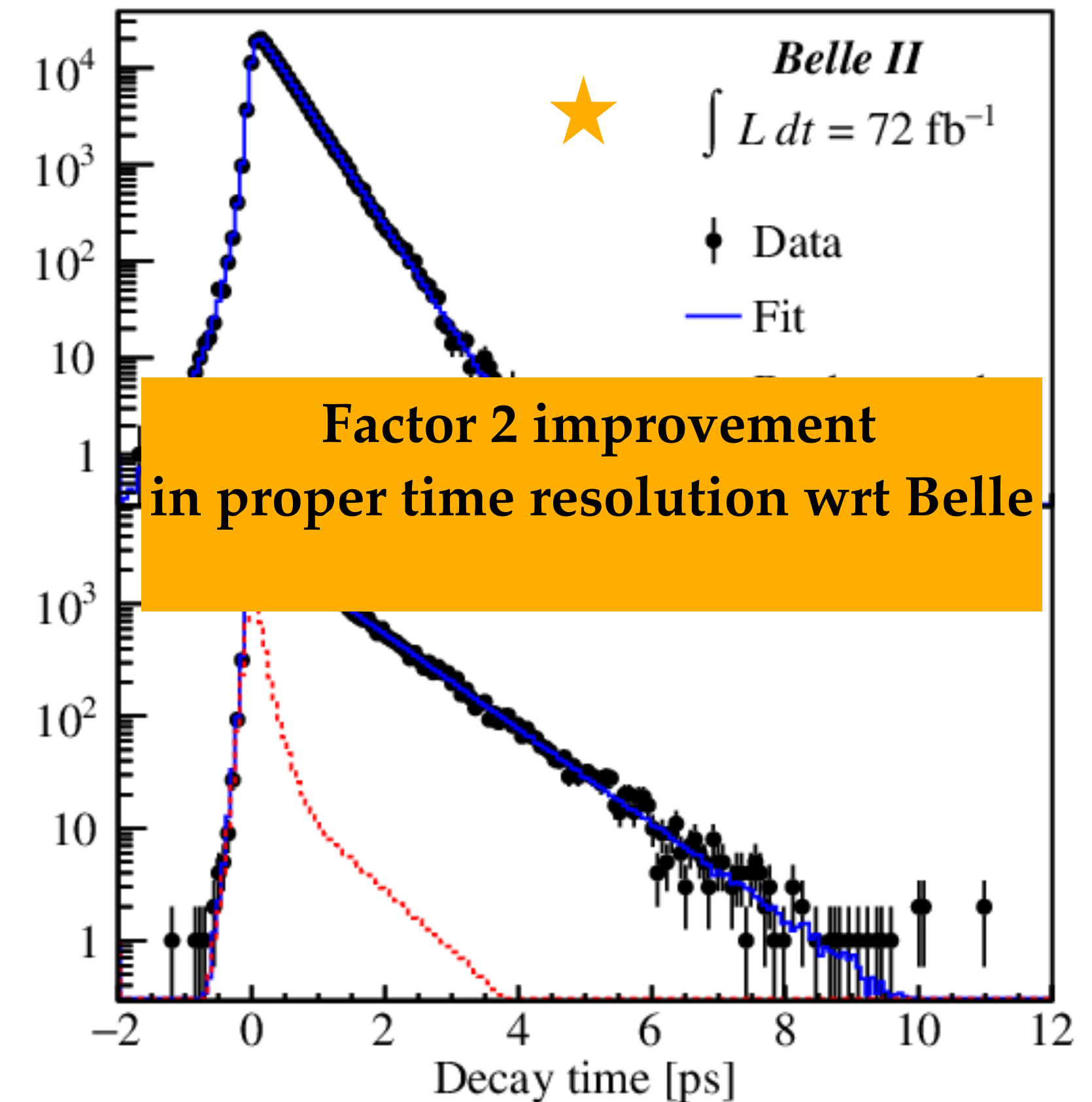
[\[BELLE2-NOTE-PL-2022-003\]](#)



Most precise measurement of

$D$  lifetimes

[PRL 127, 211801 \(2021\)](#)



# Statistical model

[PRL 127, 181802 (2021)]



## Set-up binned fit using HistFactory statistical model

- Likelihood based on [HistFactory](#) formalism implemented with [pyhf](#) + cross-check with sghf: simplified Gaussian model
- Signal and background templates from MC
- Separate templates for all backgrounds: mixed  $B$ , charged  $B$ ,  $c\bar{c}$ ,  $u\bar{u}$ ,  $s\bar{s}$ ,  $d\bar{d}$ ,  $\tau^-\tau^+$
- All systematics included via nuisance parameters:
  - background normalisation uncertainty
  - tracking inefficiency
  - neutral energy mis-calibration for photons
  - neutral energy mis-calibration for unmatched photons
  - uncertainty on PID correction due to limited statistics
  - uncertainty on branching fractions of leading background processes
  - uncertainty on SM form factor
- **Total number of fit parameters:**
  - 175 nuisance parameters  $\phi$
  - 1 parameter of interest (signal strength= $\mu$ )
  - $1 \mu = \text{SM } \mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu}) = (4.6 \pm 0.5) \times 10^{-6}$



$$f(n, a | \eta, \chi) = \prod_{r \in \text{regions}} \prod_{b \in \text{bins}} \text{Pois}(n_{rb} | \nu_{rb}(\eta, \chi)) \prod_{\chi} c_{\chi}(a_{\chi} | \chi)$$

$\eta$  = parameter of interest  
 $\chi$  = nuisance parameters

Simultaneous measurements of multiple regions

Constraints

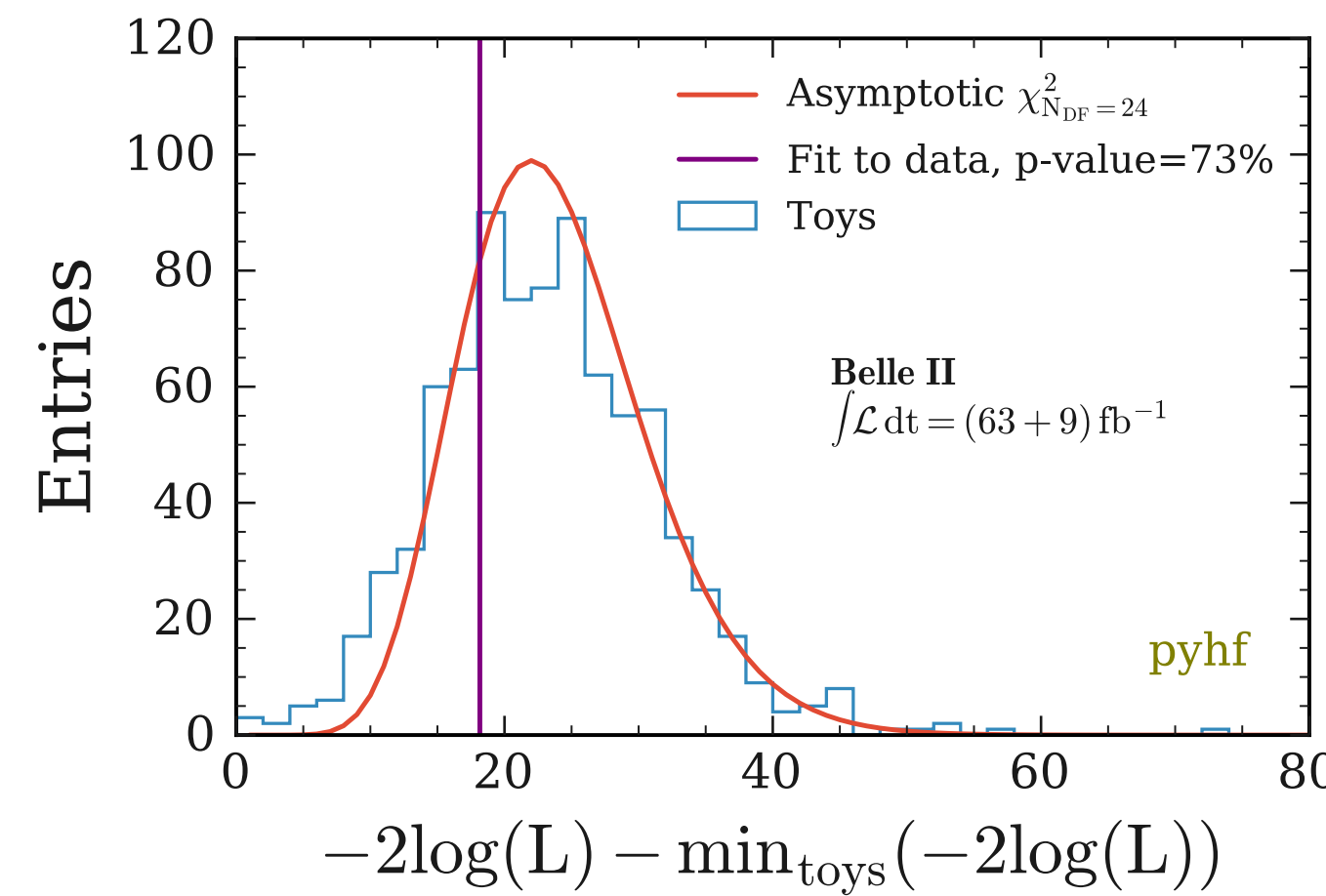
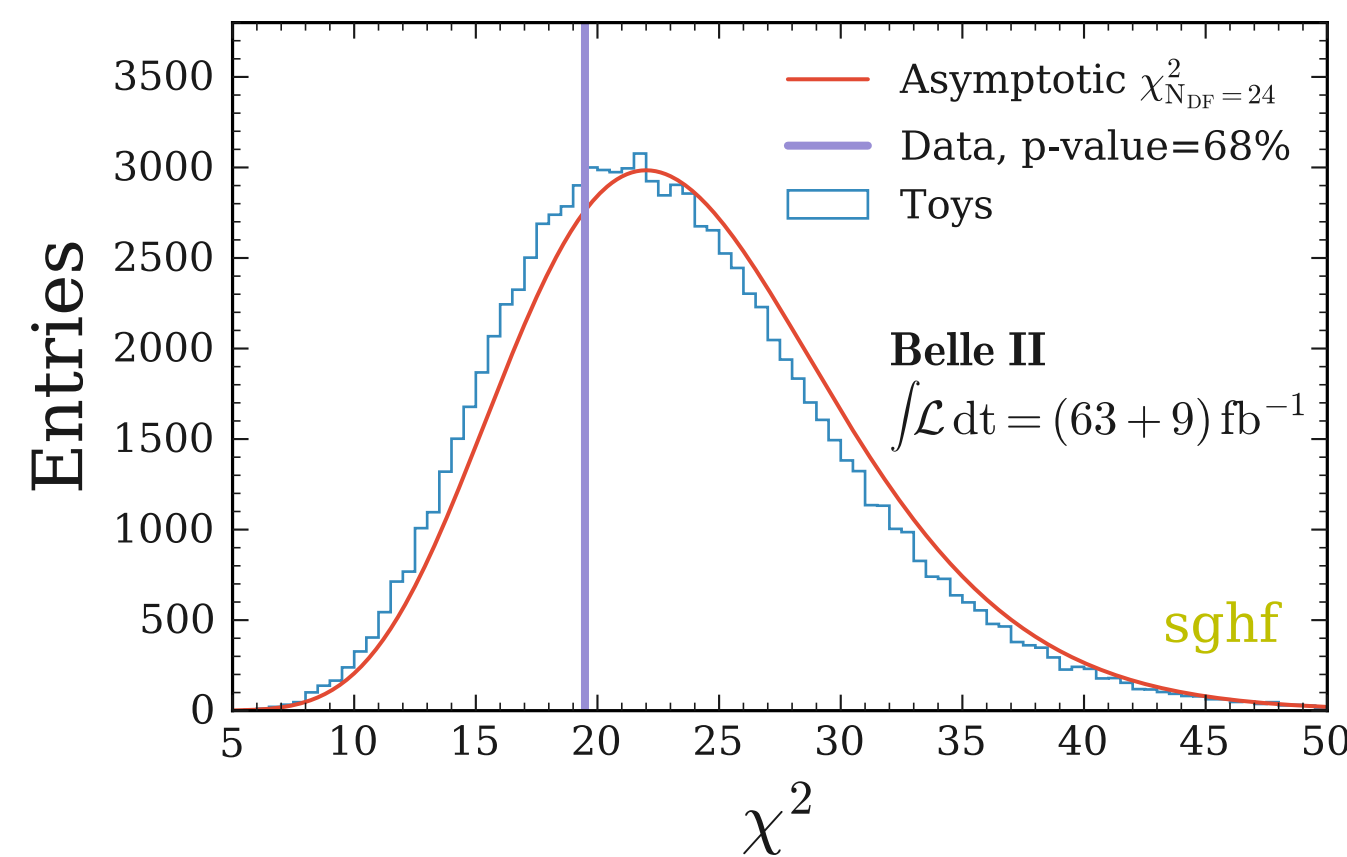
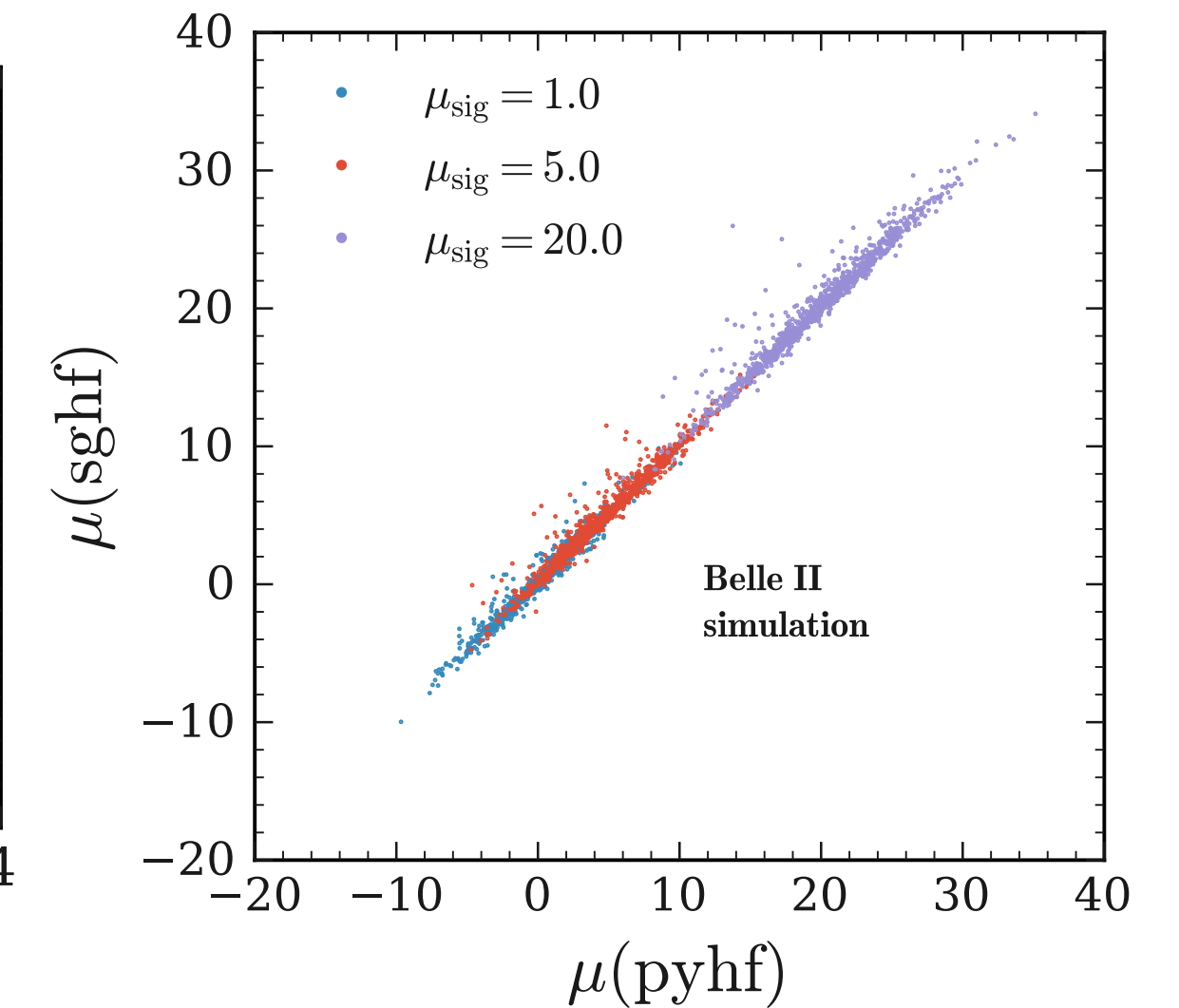
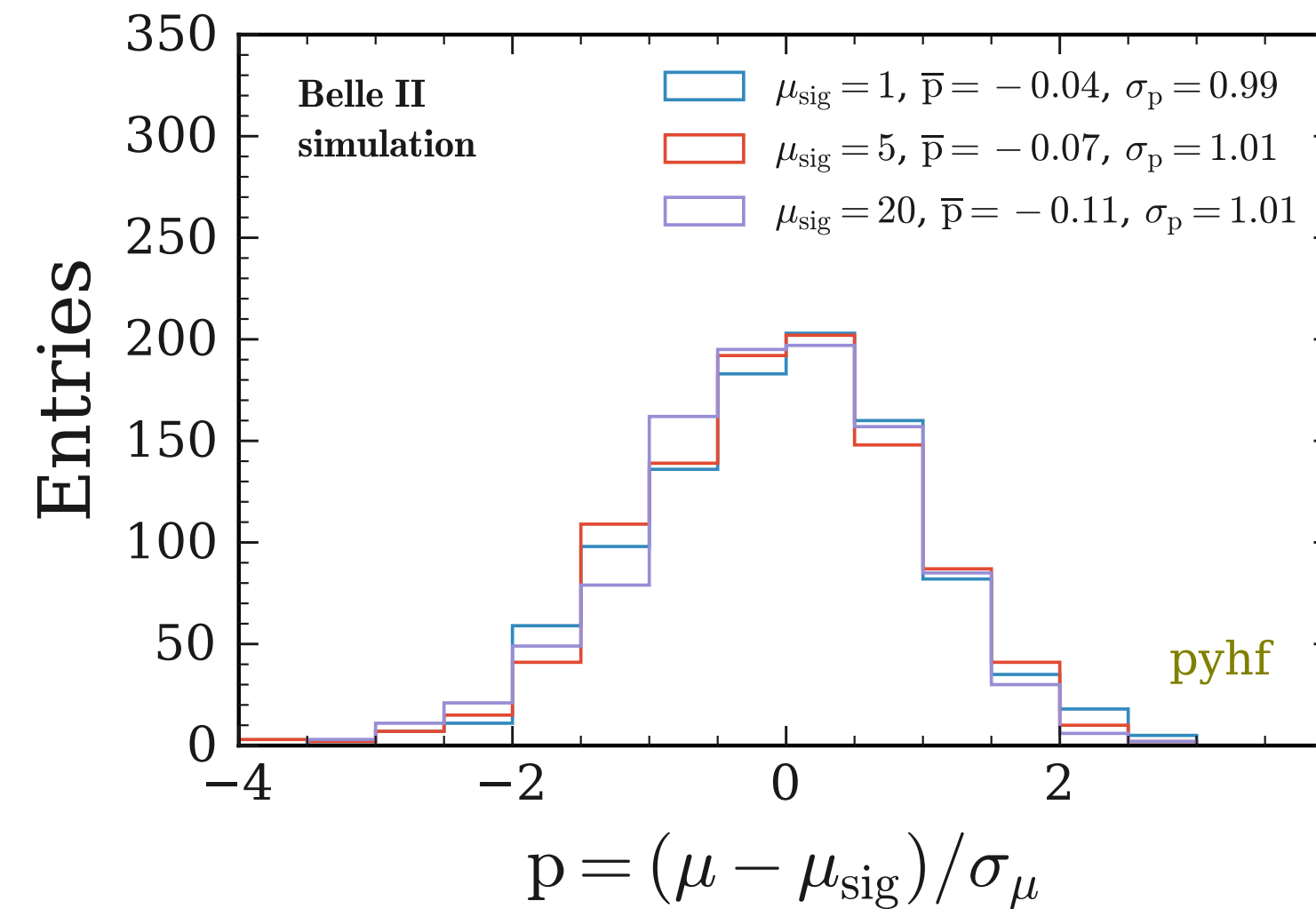
# Fit validation

[PRL 127, 181802 (2021)]



## Perform Fit Bias Check

- Used because of high  $\mathcal{B}$  and clean signature
- Generate toys with signal strength  $\mu = 1, 5, 20$  and check pulls  $= \frac{\mu_{fit} - \mu_{inj}}{\sigma_\mu}$
- Results: 0 bias, expected  $\mu$  recovered, very good agreement between **pyhf** and **sghf**



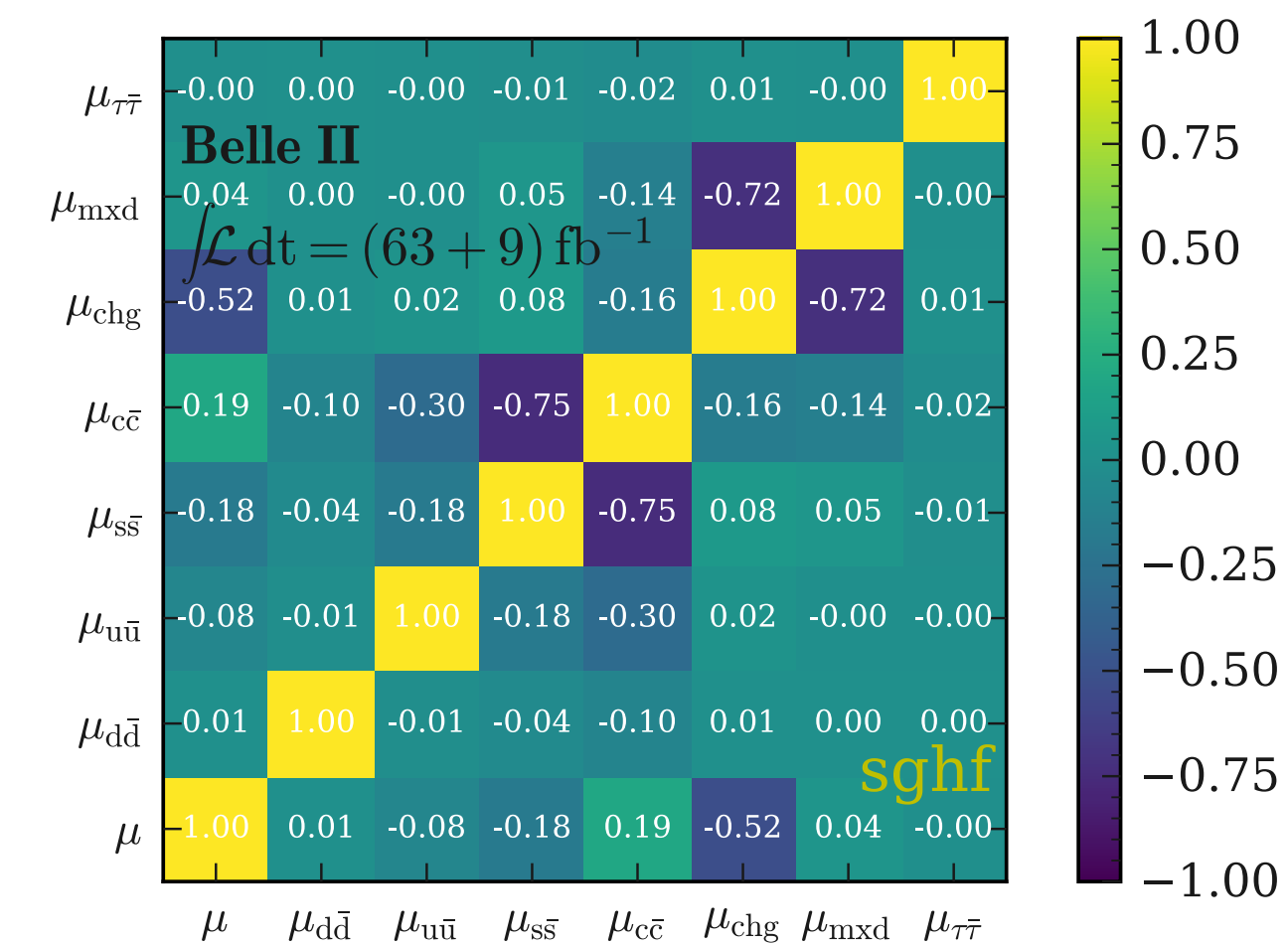
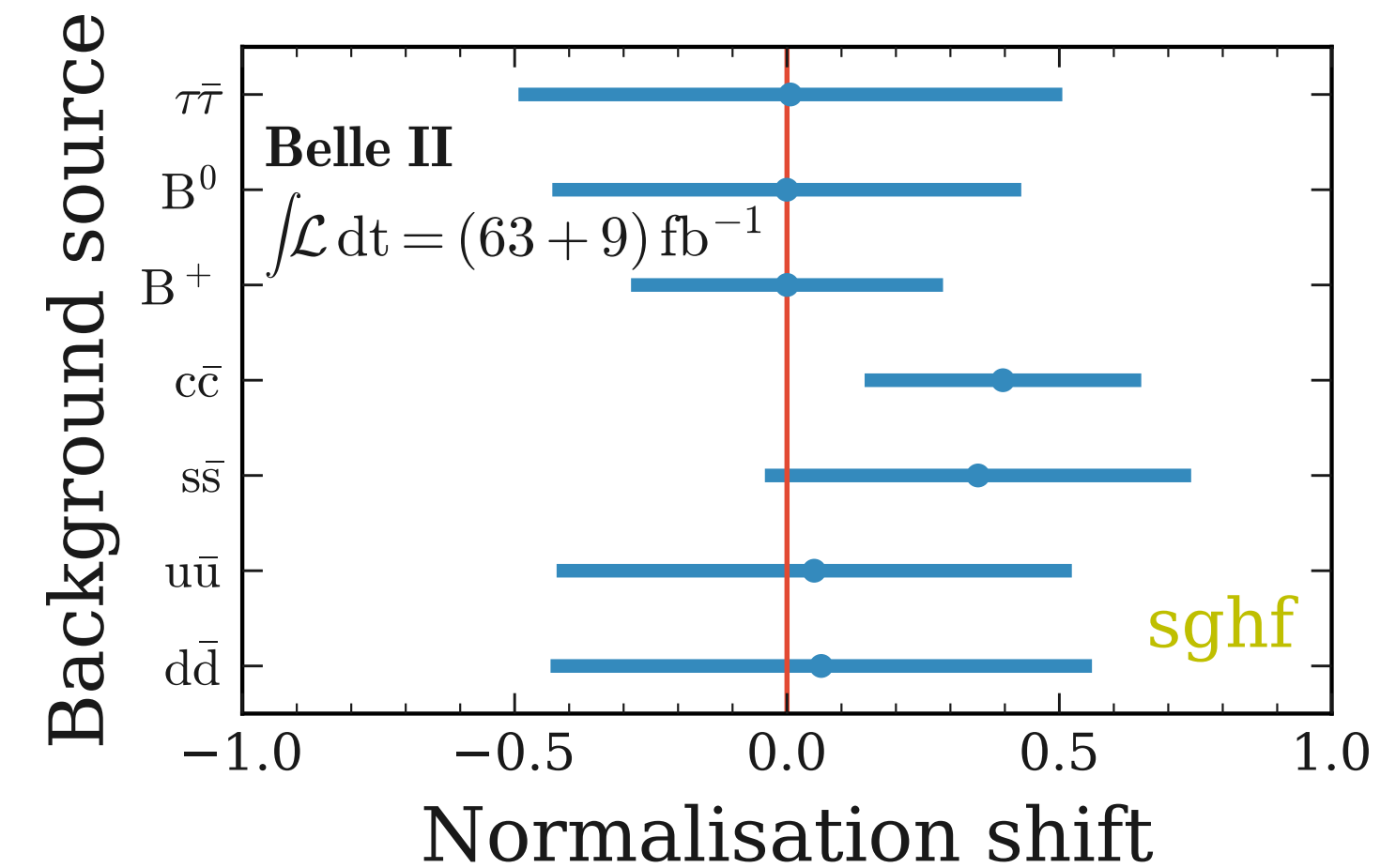
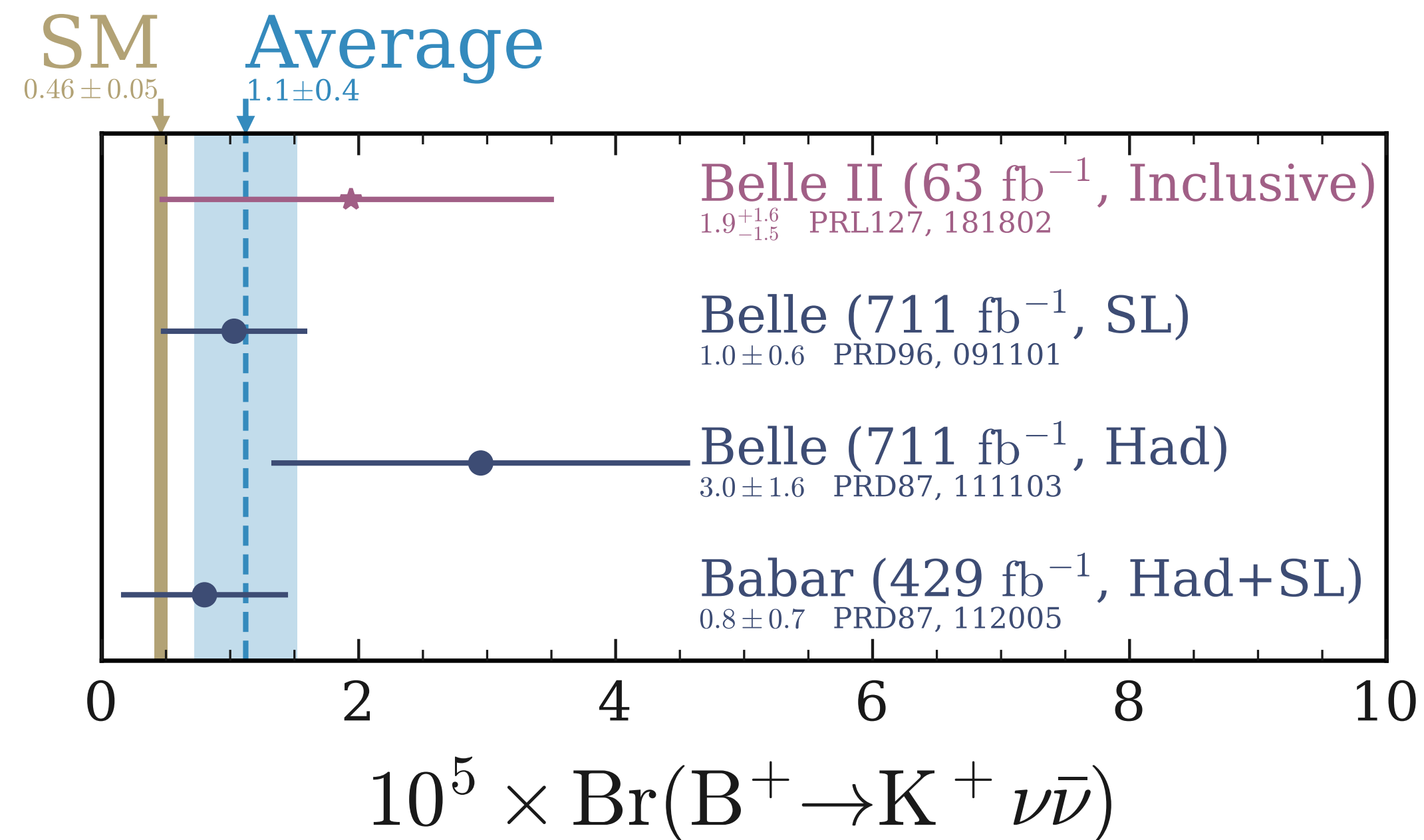
## Check Data-Model Compatibility

- Generate toys and check fit quality
- Results:  $p$ -value shows good data model compatibility for both **pyhf** and **sghf**

# What we learnt from fit? [PRL 127, 181802 (2021)]



1.  $c\bar{c}$ ,  $s\bar{s}$  continuum backgrounds are pulled up by 40%
2. Inclusive tag approach shows the best performance
  1. 3.5 better than HAD tag
  2. 20% better than SL Belle tag
  3. 10% better than HAD and SL tag
3. BSM  $B^+ \rightarrow K^+ \nu \bar{\nu}$  already with  $1 \text{ ab}^{-1}$



# Re-(interpretations)

[PRL 127, 181802 (2021)]



Partial reinterpretation can be done as Belle II publishes  $\epsilon_{sig}$  as a function of  $q^2$ :

- Reminder: default signal model  $\rightarrow$  PHSP model with SM form factor reweighting [arXiv:1409.4557]
- At low  $q^2$  maximum signal efficiency of 13%
- No sensitivity for  $q^2 > 16 \text{ GeV}^2/c^2$
- All public plots at [HEPData](#)

For full re-(interpretation):

- Provide full likelihood

