Likelihood Free Frequentist Inference (LF2I) of atmospheric cosmic-ray showers

Alex Shen[1] Ann Lee[1] Luca Masserano[1] Tommaso Dorigo [2]
Michele Doro [2] [3] Federico Nardi [3]

[1]Department of Statistics and Data Science
Carnegie Mellon University

[2]Istituto Nazionale Fisica Nucleare

[3]Department of Physics and Astronomy
Università di Padova

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Study of high energy cosmic rays remains challenging

[1]Source: swgo.org (Richard White, MPIK)

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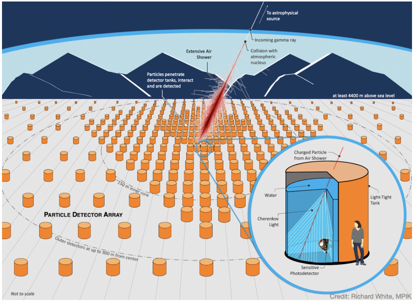

Proposed SWGO[1]

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Goals of the analysis

Given our observed detections associated with a secondary shower, we hope to

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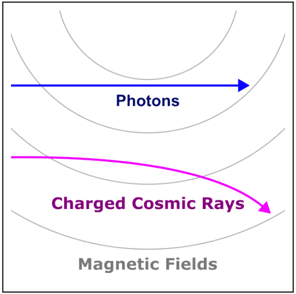

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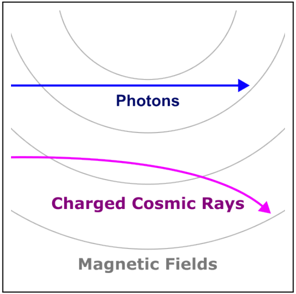

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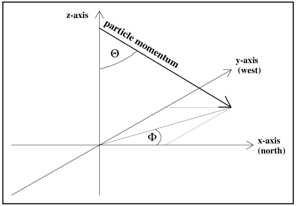
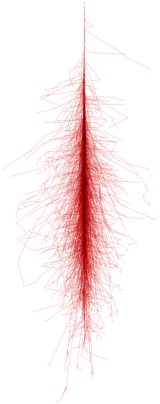


2. Estimate their orientation/energy

Data: CORSIKA Cosmic Ray Simulation Software

Simulation of secondary showers induced by cosmic ray interactions with atmosphere.

- Input: primary cosmic ray parameters
  - μ : particle identity
  - E : energy
  - Z, A : zenith, azimuthal angles
- Output X: identity, momenta, location, and timing of secondary particles observed at ground level



Example Simulation[1]

[1] Source: https://www.iap.kit.edu/corsika/

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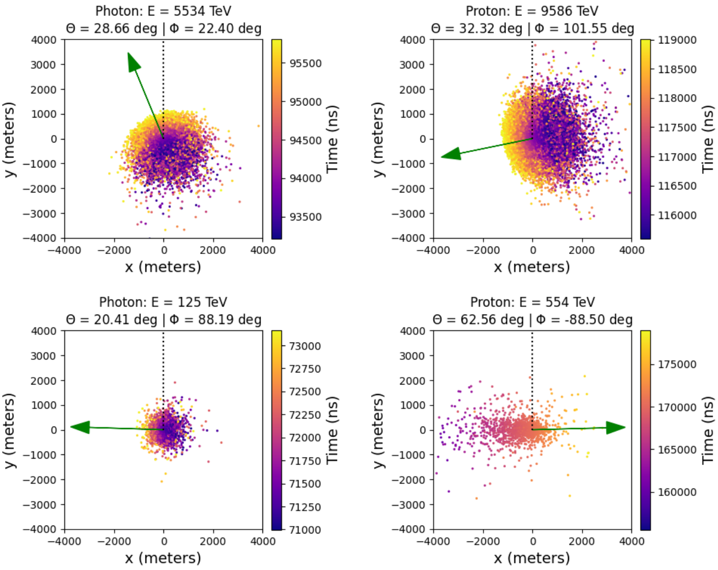

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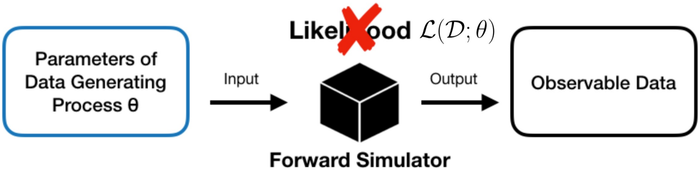
Likelihood $\mathcal{L}(\mathcal{D};\theta)$
Parameters of Data Generating Process θ
Input
Output
Observable Data
Forward Simulator

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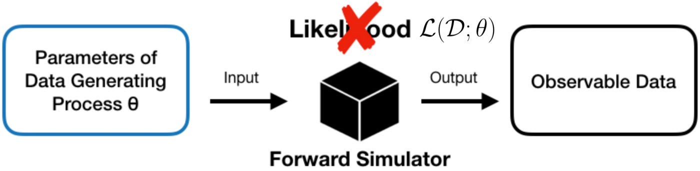

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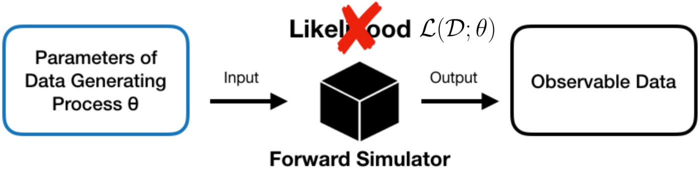

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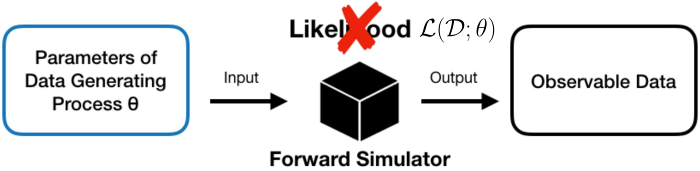

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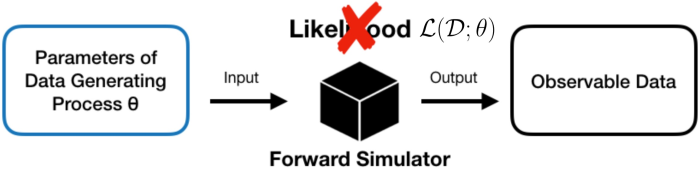

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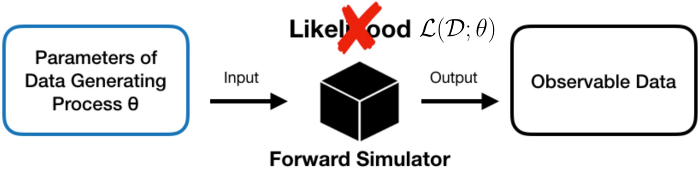

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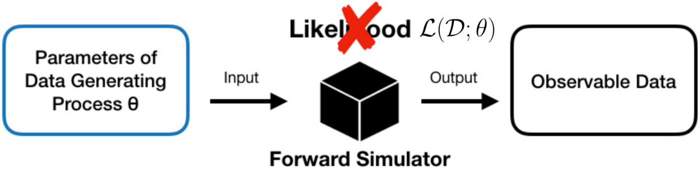

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4. Directly predicting μ from X is insufficient

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Given an observed footprint X, we test

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3. Reject if $\tau(X) \leq \inf_{\nu \in \mathcal{N}} C_\alpha(\nu)$

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2. Use quantile regression to estimate cutoffs $C_\alpha(\nu)$
3. Confidence Set: $\{\nu_0 : \tau(X) \geq C_\alpha(\nu)\}$

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Thanks! Any questions?