Parameter estimation in numerical weather prediction

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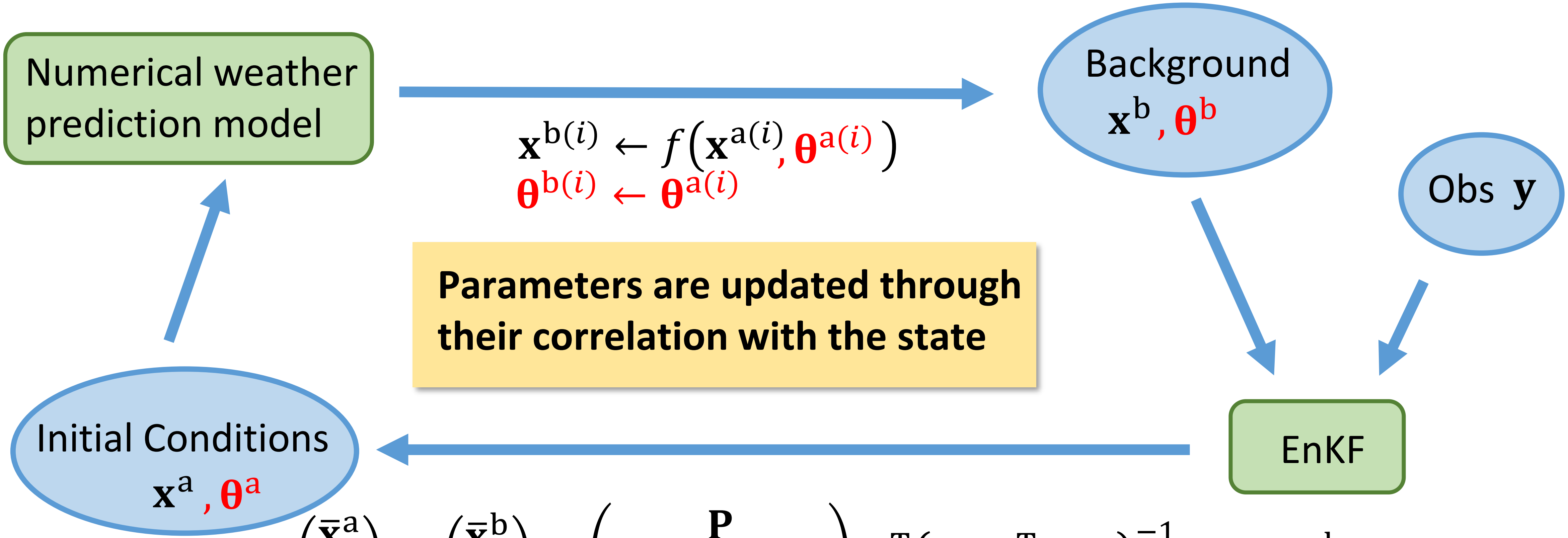
Outline

- Estimation of the roughness length in COSMO-KENDA
  Ruckstuhl and Janjic, (2020)
- How can we deal with non-Gaussianity?
  Quadratic Filter, Particle Filter
  Ruckstuhl and Janjic, (2018)
- How can we get accurate full error statistics of the background?
  Stochastic Galerkin
  Janjic, Lukacova, Ruckstuhl and Wiebe (under review)

Augmented state parameter estimation



$$\mathbf{x}^{\mathrm{b}(i)} \leftarrow f\left(\mathbf{x}^{\mathrm{a}(i)}, \boldsymbol{\theta}^{\mathrm{a}(i)}\right)$$

$$\boldsymbol{\theta}^{\mathrm{b}(i)} \leftarrow \boldsymbol{\theta}^{\mathrm{a}(i)}$$

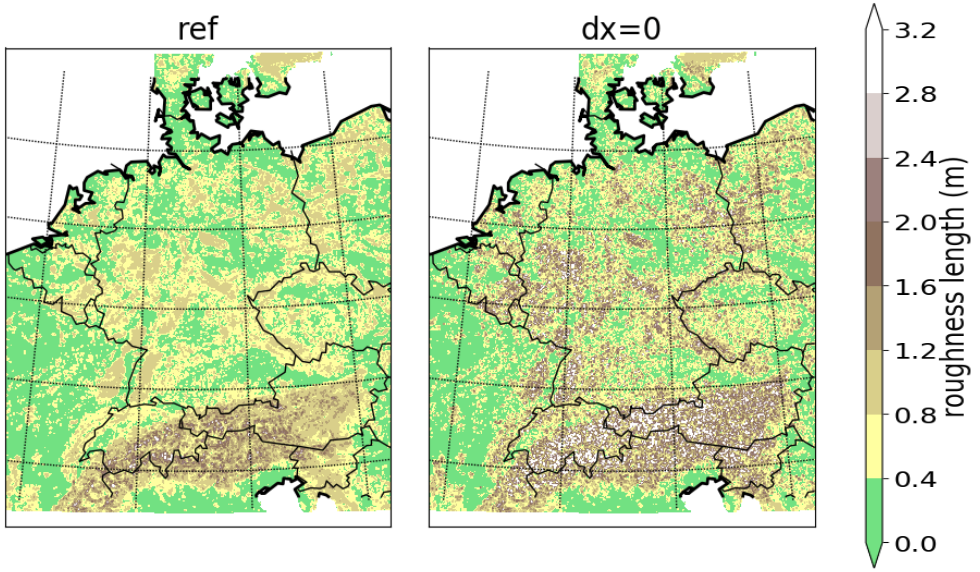
Parameters are updated through their correlation with the state

$$\begin{pmatrix} \overline{\mathbf{x}}^{\mathrm{a}} \\ \overline{\boldsymbol{\theta}}^{\mathrm{a}} \end{pmatrix} - \begin{pmatrix} \overline{\mathbf{x}}^{\mathrm{b}} \\ \overline{\boldsymbol{\theta}}^{\mathrm{b}} \end{pmatrix} = \begin{pmatrix} \mathbf{P} \\ \mathrm{Cov}(\boldsymbol{\theta}^{\mathrm{b}}, \mathbf{x}^{\mathrm{b}}) \end{pmatrix} \mathrm{H}^{\mathrm{T}} \left(\mathrm{HPH}^{\mathrm{T}} + \mathrm{R}\right)^{-1} (\mathbf{y} - \mathrm{H}\overline{\mathbf{x}}^{\mathrm{b}})$$

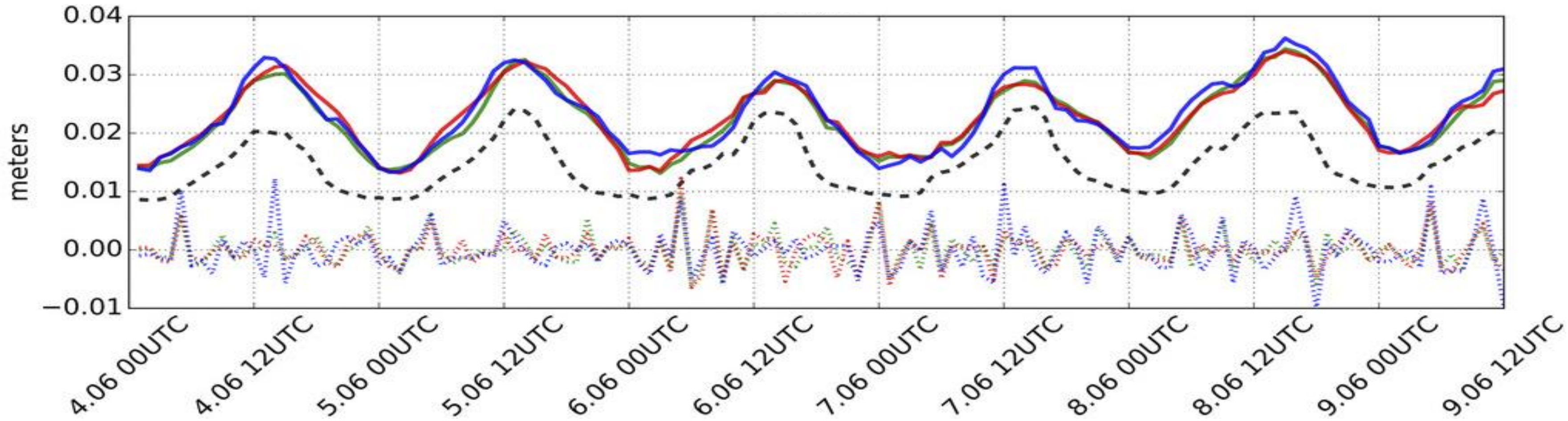
Application to roughness length in COSMO-KENDA

Ruckstuhl and Janjic (2020)

- Roughness length accounts for subgrid scale orography and land use
- Operational configuration
- Assimilate conventional observations and radar reflectivity



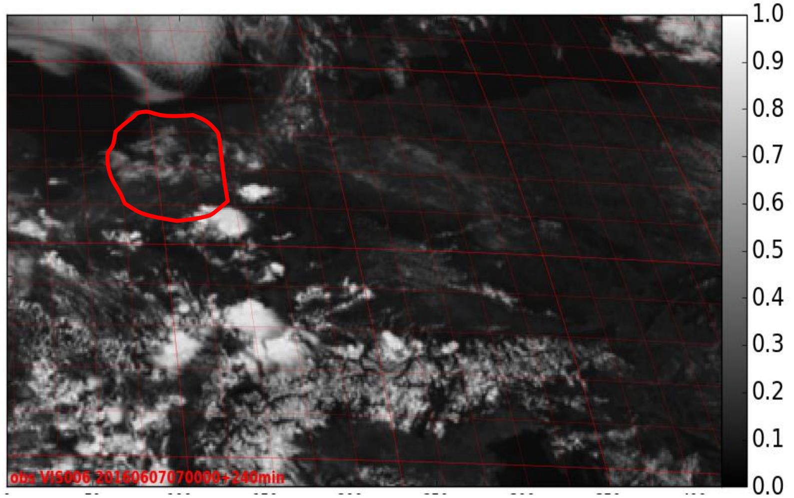
Spatially averaged parameter and momentum surface flux increments.

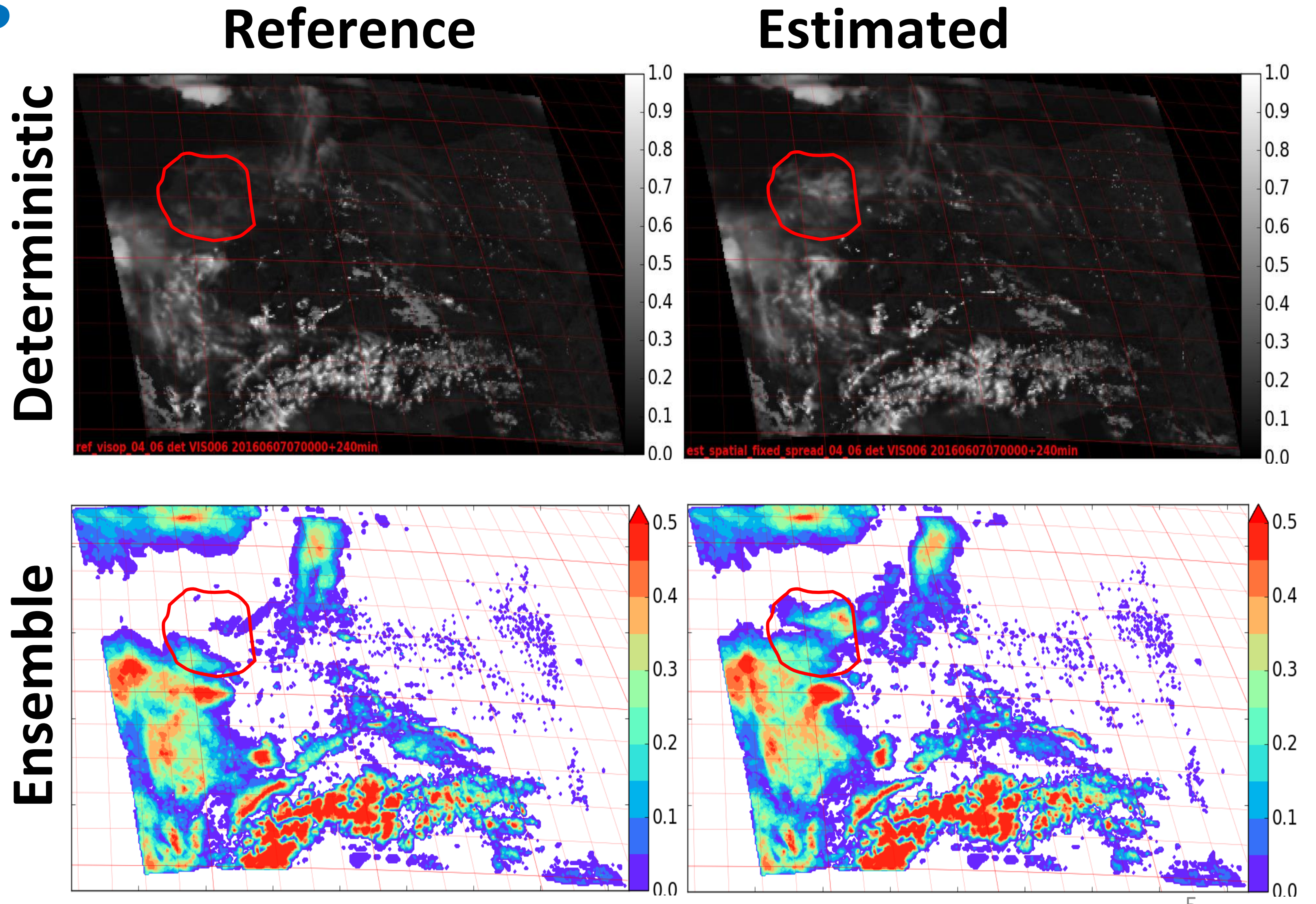


Model error related to surface fluxes is projected onto the roughness length

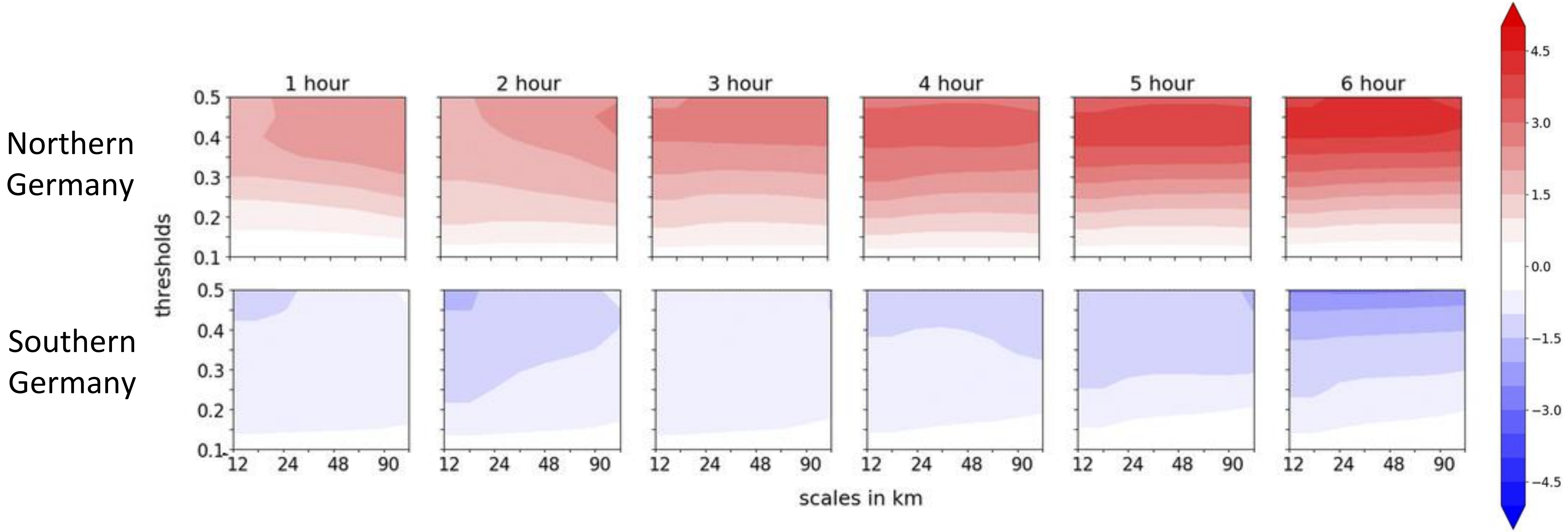
Verification against visible satellite images

Observations



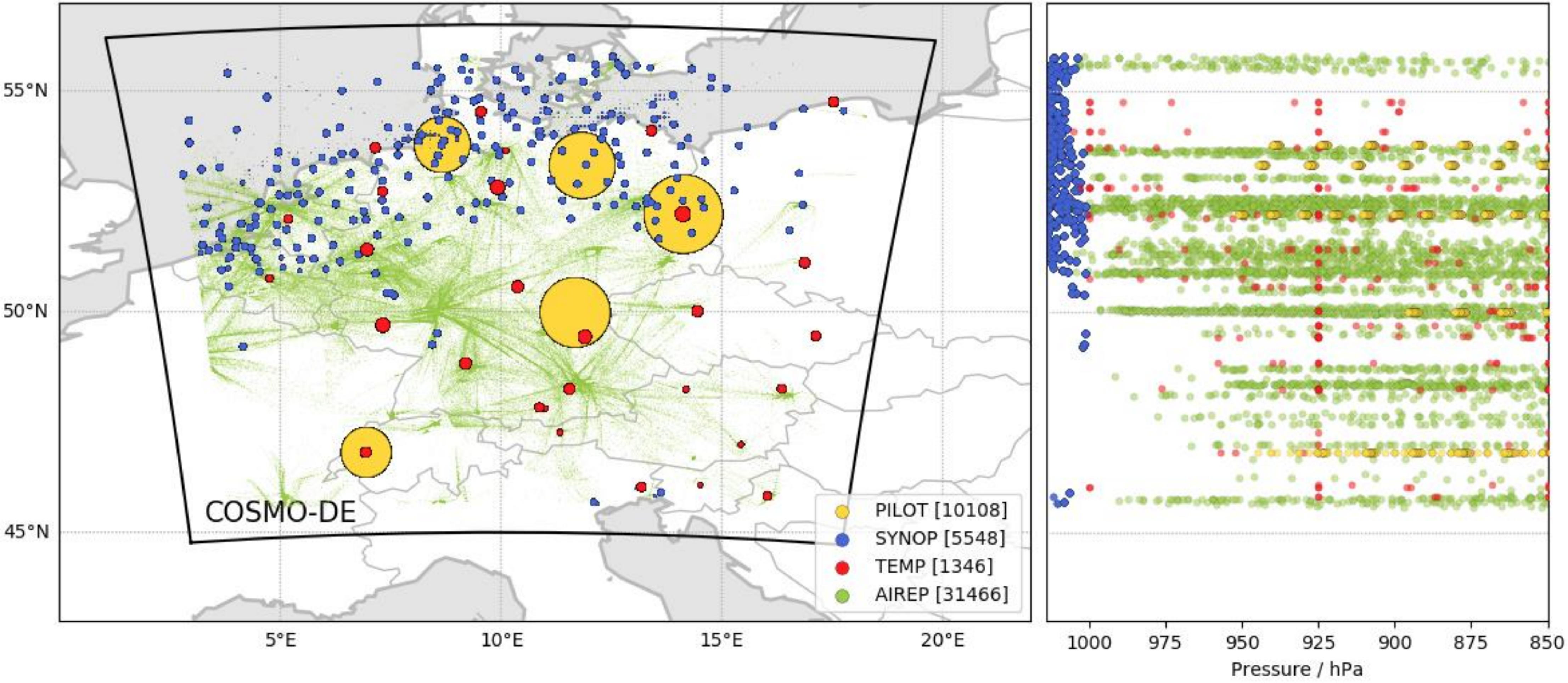


Verification against visible satellite images



Relative Fraction Skill Score of satellite reflectance averaged over 60 forecasts

Assimilated wind observations



What have we learned?

- Parameter compensates for other model errors (in this case surface fluxes)
- Estimating the roughness length significantly reduces short term forecast errors of clouds and precipitation where surface wind measurements are assimilated
- Sufficiently constraining the parameter is key

How can we better constrain the parameters?

- Increase observational coverage
- Use observations more effectively
  - reduce sampling errors
    (larger ensemble size/localization/reduce degrees of freedom)
  - choose DA algorithms that alleviate the Gaussian assumption

Quadratic Filter

EnKF

$$\hat{x} = \mathrm{K}v$$

$$x^a - x^b = \mathrm{P}^b\mathrm{H}^\mathrm{T}\left(\mathrm{H}\mathrm{P}^b\mathrm{H}^\mathrm{T} + \mathrm{R}\right)^{-1} v$$

$$v = y - \mathrm{H}x^b$$

Deriving EnKF as the
Best Linear Unbiased Estimate…

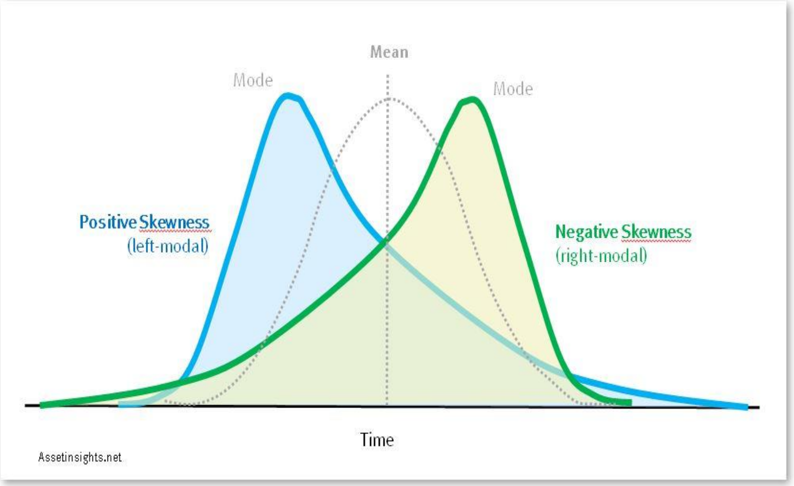
$$x^a - x^b = \min_{\hat{x}} \mathrm{E}[||x - \hat{x}||^2]$$

Quadratic Filter (QF), Hodyss (2012)

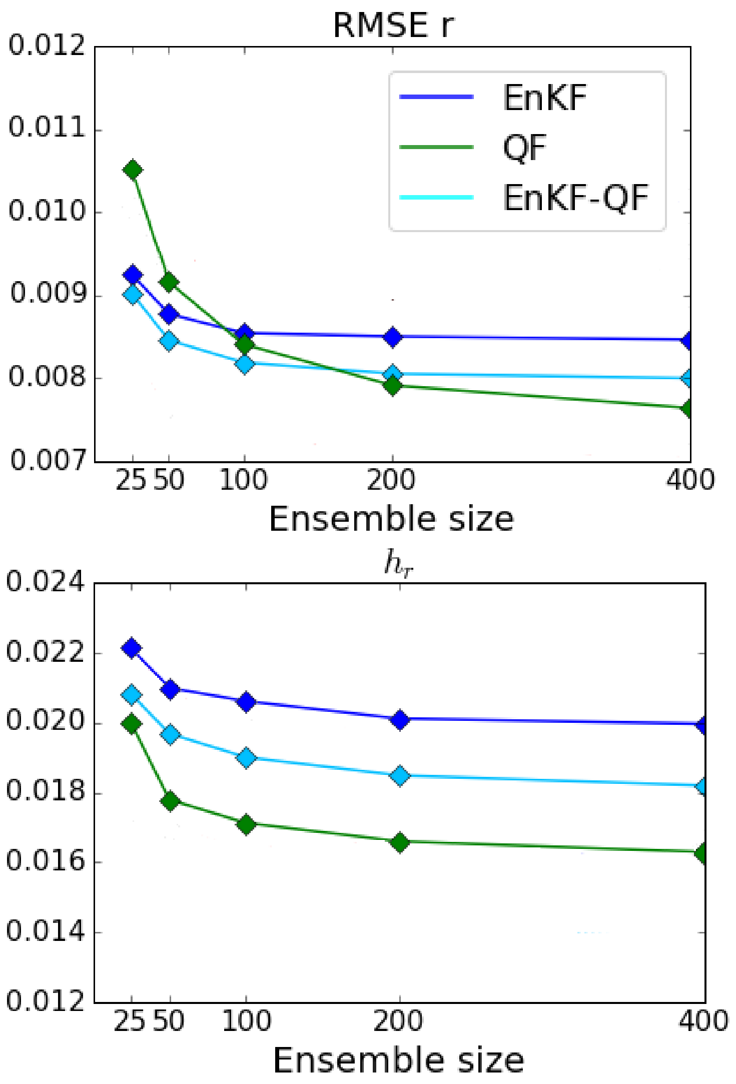
$$\hat{x} = \mathrm{K}v + \mathrm{G}(v \odot v)$$

$$x^a - x^b = \tilde{\mathrm{P}}\mathrm{H}^\mathrm{T}\left(\mathrm{H}\tilde{\mathrm{P}}\mathrm{H}^\mathrm{T} + \tilde{\mathrm{R}}\right)^{-1} \tilde{v}$$

$$\tilde{\mathrm{P}} = \begin{bmatrix} \mathrm{P}^f & \mathrm{P}_{\mathrm{skew}} \\ \mathrm{P}_{\mathrm{skew}} & \mathrm{P}_{\mathrm{kurt}} \end{bmatrix} \quad \tilde{\mathrm{R}} = \begin{bmatrix} \mathrm{R} & \mathrm{R}_{\mathrm{skew}} \\ \mathrm{R}_{\mathrm{skew}} & \mathrm{R}_{\mathrm{kurt}} \end{bmatrix} \quad \tilde{v} = \begin{bmatrix} v \\ v \odot v \end{bmatrix}$$



Results modified shallow water model



Ruckstuhl and Janjic (2018)

- QF is more sensitive to ensemble size than EnKF
- QF outperforms EnKF when ensemble size is sufficiently large
- EnKF-QF outperforms EnKF already for small ensemble sizes

EnKF-QF is feasible option!

But maybe we can do even better…

Stochastic Galerkin as alternative to ensemble for background error statistics

In collaboration with **Bettina Wiebe** and **Maria Lukacova**

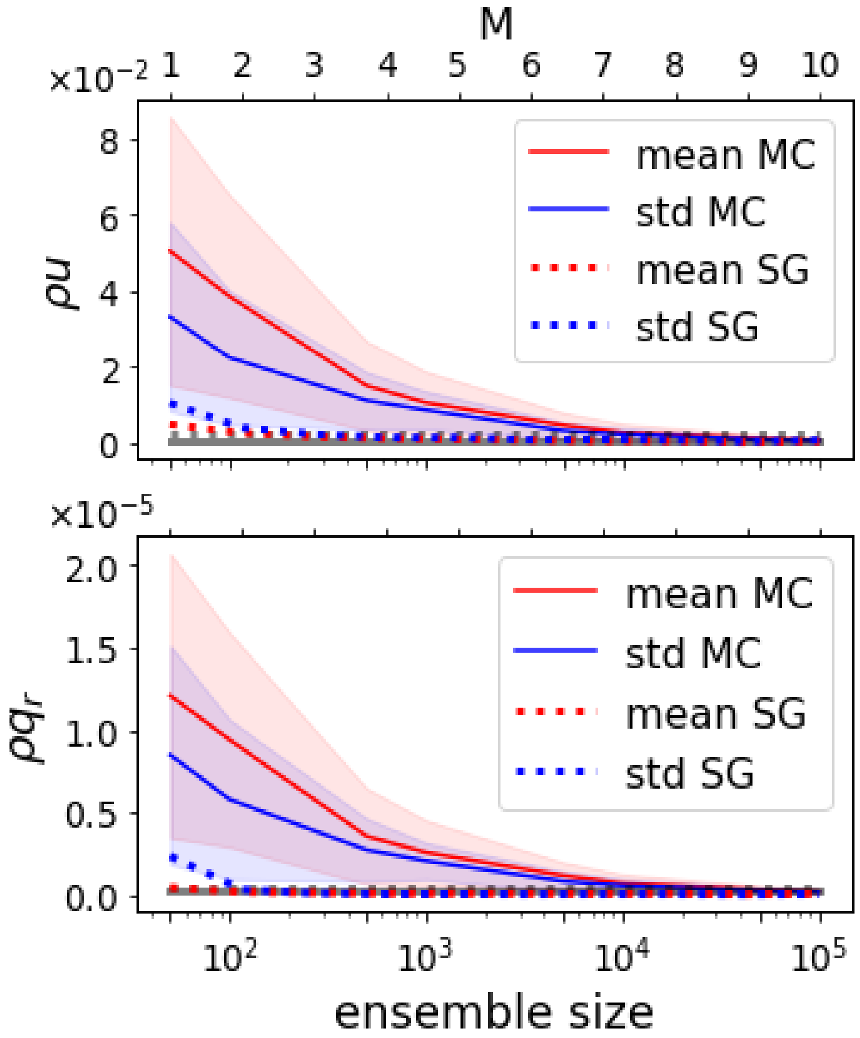
- Assume all variables are stochastic: $\theta(x,t) \leftarrow \theta(x,t,\omega)$, $\omega \sim N(0,1)$
- Approximate stochastic variables with a polynomial expansion

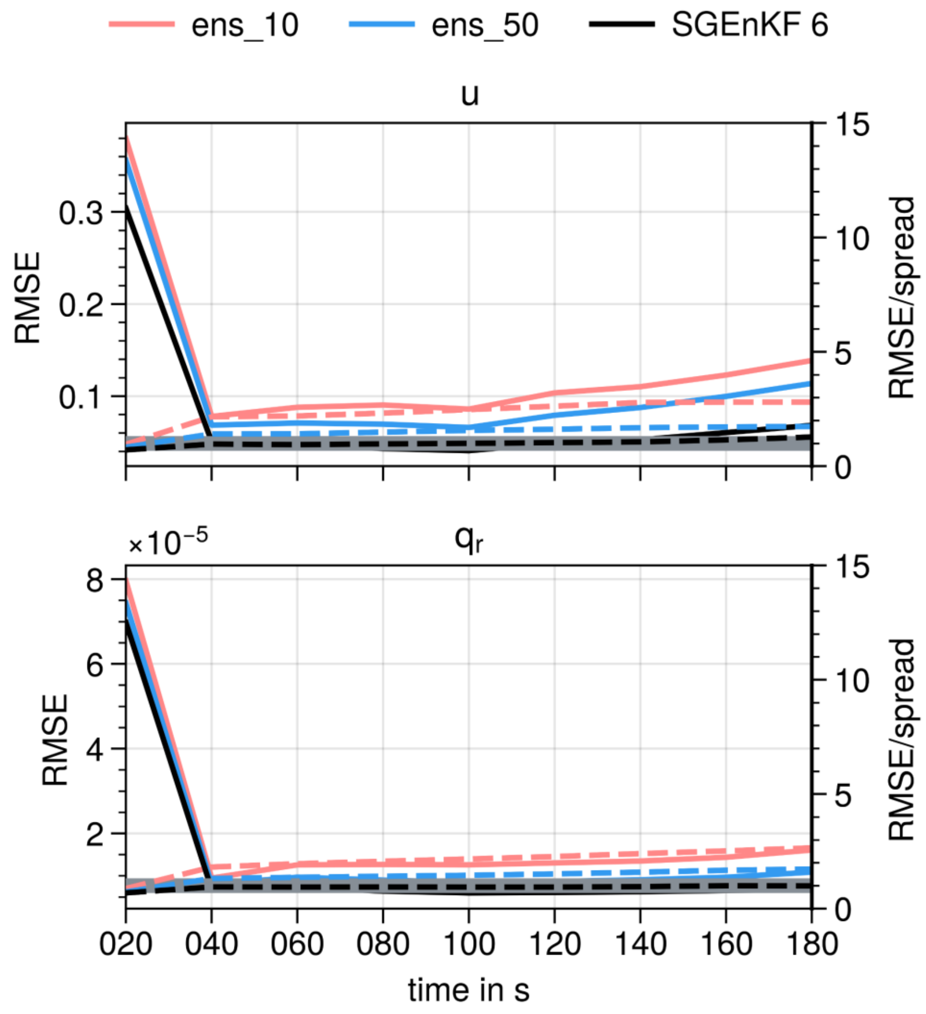
  $$\theta(x,t,\omega) \approx \sum_{k=0}^{M} \hat{\theta}_k(x,t)\varphi_k(\omega)$$

  where $\varphi_k(\omega)$ are Hermite polynomials $(1, \omega, \omega^2 - 1, \ldots)$ and substitute into model
- Apply weak formulation and use orthogonality of $\varphi_k(\omega)$ wrt to Gaussian pdf to get deterministic system of PDEs
- Solve numerically for $\hat{\theta}_k(x,t), k = 1,2,\ldots,M$

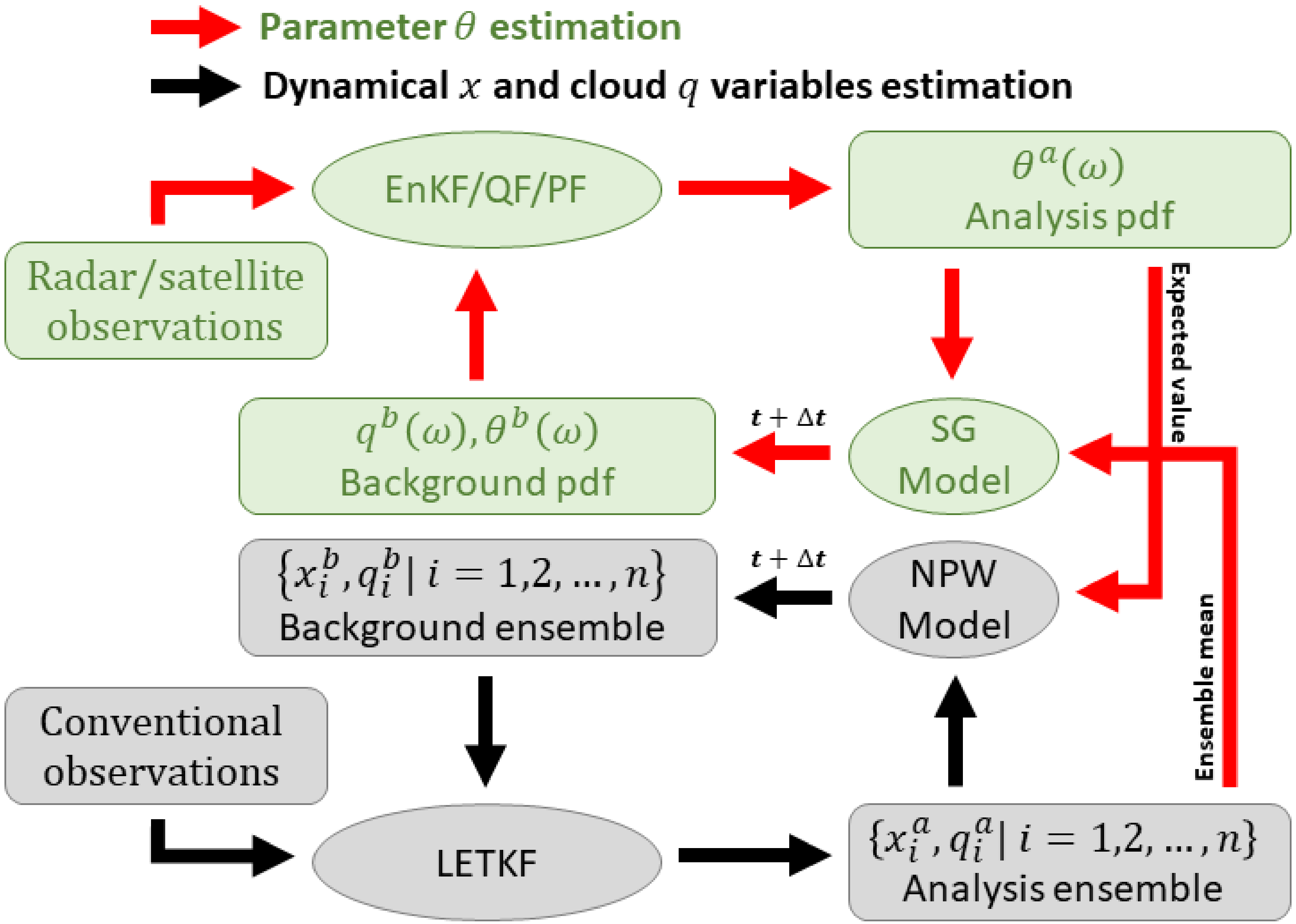
Ensemble versus Stochastic Galerkin

Janjic et al. (under review)





Parameter estimation with SG-DA hybrid



Goal:

Estimate cloud parameters in ICON

Summary

- Estimating the roughness length improves short term cloud and precipitation forecasts
- Parameters need to be sufficiently constrained for successful estimation
- Parameters are better constrained when reducing sampling errors and using higher order moments of background error statistics (QF)
- Using the stochastic Galerkin instead of an ensemble to obtain accurate full error statistics may open the door to DA algorithms like particle filters for parameter estimation

References

- Ruckstuhl, Y., and T. Janjić, 2020: Combined State-Parameter Estimation with the LETKF for Convective-Scale Weather Forecasting. Mon. Wea. Rev., 148, 1607–1628, https://doi.org/10.1175/MWR-D-19-0233.1
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