Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

ct.qmat
Complexity and Topology in Quantum Matter

Two new quantum information measures in AdS/CFT

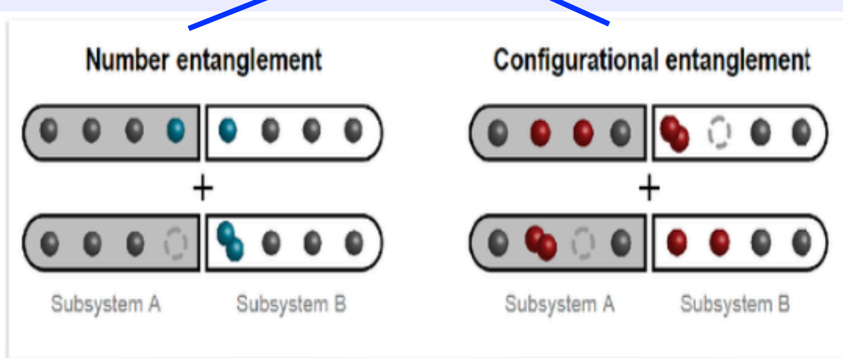
Priv. Doz. Dr. René Meyer
Lehrstuhl für Theoretische Physik III
Institut für Theoretische Physik & Astrophysik
Julius Maximilians Universität Würzburg

Topic 1

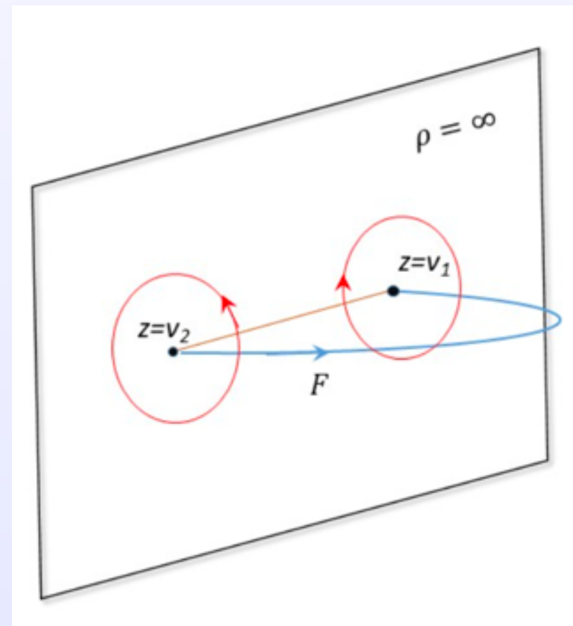
Symmetry resolved entanglement in AdS_3/CFT_2

Entanglement at fixed subregion charge

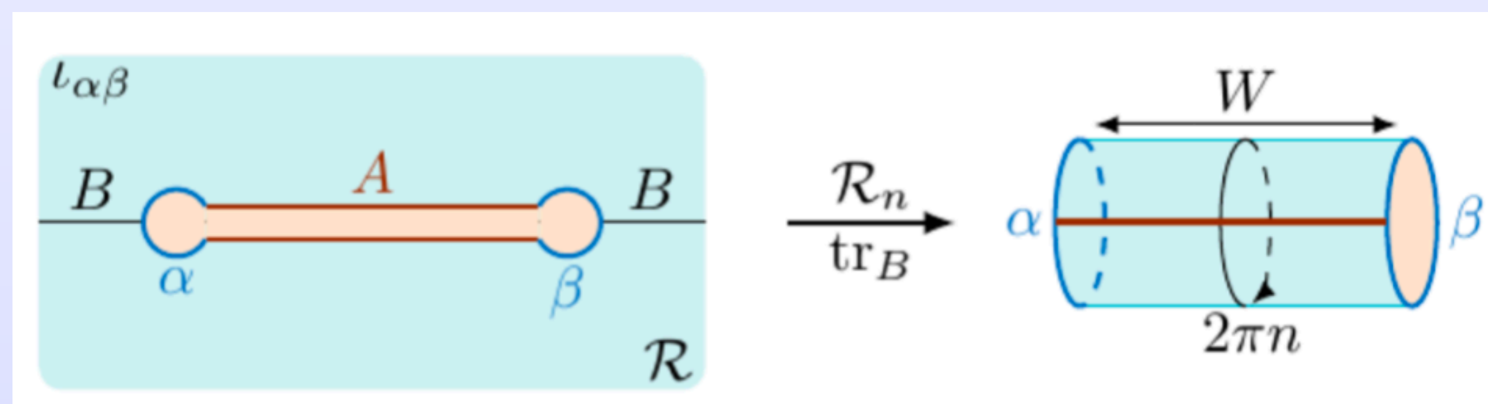
$$S_1 = \sum_q P_{\mathcal{A}}(q) S_1(q) - \sum_q P_{\mathcal{A}}(q) \log P_{\mathcal{A}}(q)$$



A. Lukin, M. Rispoli, R. Schittko, M. E. Tai, A. M. Kaufman,S. Choi, V. Khemani, J. Leonard, and M. Greirer, Probing entanglement in a many-body localized system, Science 364, 6437 (2019).



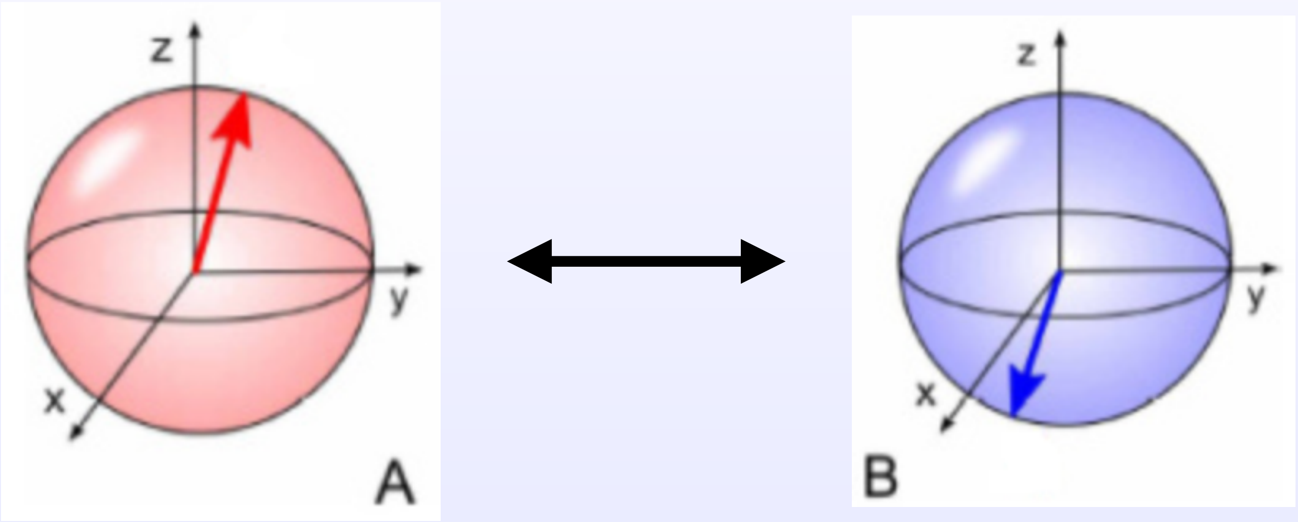
U(1) Wilson line

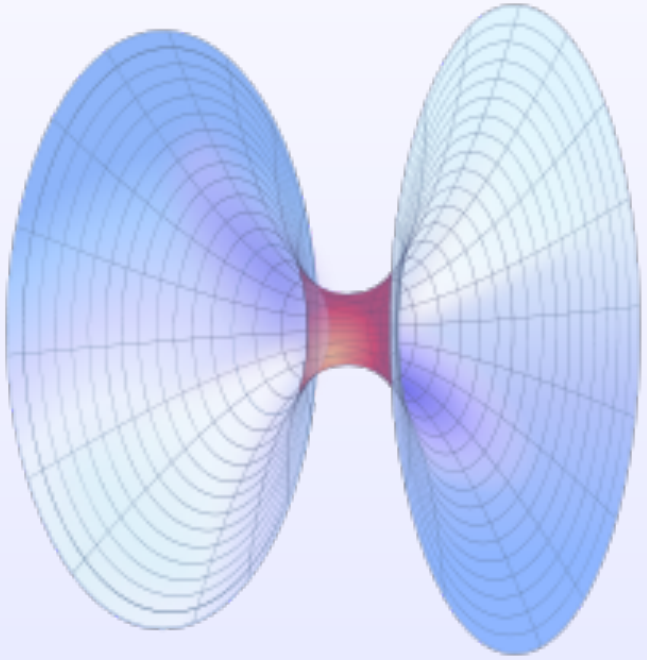


S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)
S. Zhao, C. Northe, K Weisenberger, RM, JHEP 2022 (2202.11111)
G. Di Giulio, C. Northe, R. Meyer, H. Scheppach, S. Zhao 2212.09767

Topic 2

Berry phases in AdS/CFT and beyond





EPR pair of spins

Wormhole (ER bridge)

Berry phases probe non-factorization of Hilbert space

Berry phase in QM and wormholes

F. S. Nogueira, S. Banerjee, M. Dorband, RM, J. v. d. Brink, J. Erdmenger, PRD 2022

Berry phases in AdS3/CFTs

S. Banerjee, M. Dorband, J. Erdmenger, RM, A.-L. Weigel, 2202.11717

Quantum Information and AdS/CFT

Emergence of holographic space-time from quantum information?

Building up spacetime with quantum entanglement

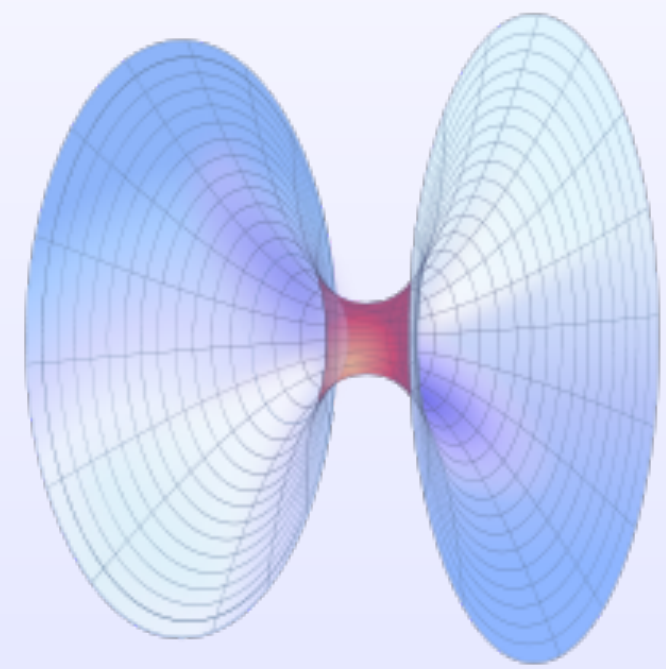
Mark Van Raamsdonk

Department of Physics and Astronomy, University of British Columbia
6224 Agricultural Road, Vancouver, B.C., V6T 1W9, Canada
mav@phas.ubc.ca

Abstract

In this essay, we argue that the emergence of classically connected spacetimes is intimately related to the quantum entanglement of degrees of freedom in a non-perturbative description of quantum gravity. Disentangling the degrees of freedom associated with two regions of spacetime results in these regions pulling apart and pinching off from each other in a way that can be quantified by standard measures of entanglement.

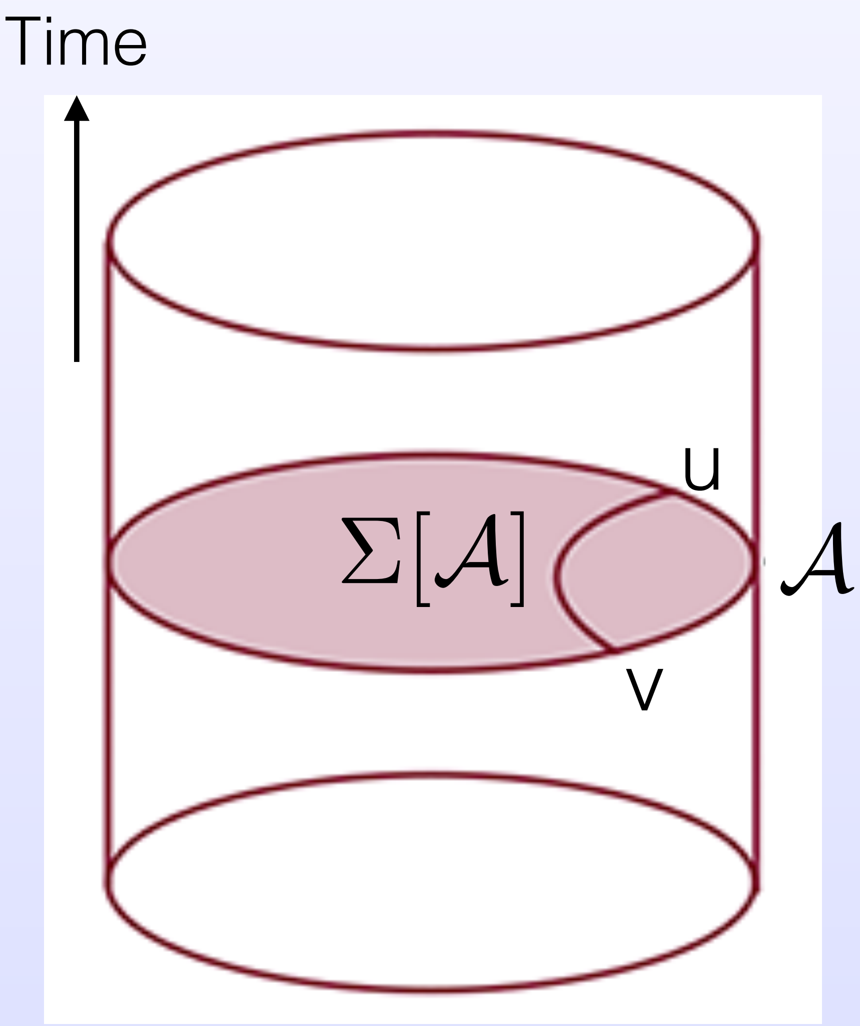
arXiv:1005.3035 [hep-th]



Wormhole (Einstein-Rosen bridge)

Entanglement Entropy in AdS3/CFT2

Minimal length curve (geodesic)
anchored at the ends of the entangling interval



$$S(\mathcal{A}) = \frac{\mathrm{Length}(\Sigma[\mathcal{A}])}{4G_3}$$

$$c = \frac{3L}{2G_3} \gg 1$$

L… Curvature Radius of AdS3 space-time

For ground state of 2D CFTs:

$$S(\mathcal{A}) = \frac{c}{3} \log \frac{|v-u|}{\epsilon}$$

ϵ … Short Distance Cutoff

Ryu, Takayanagi 2006
Hubeny-Rangamani-Takayanagi 2007

Holzhey-Larsen-Wilczek 1994,
Cardy-Calabrese 2004

Symmetry Resolved Entanglement

Entanglement entropy in each charge sector, e.g. U(1)

charge operator Q $Q = Q_{\mathcal{A}} \oplus Q_{\mathcal{B}}$.

eigenstate of Q, $[\rho, Q] = 0$. → $[\rho_{\mathcal{A}}, Q_{\mathcal{A}}] = 0$.

Block decomposition: $\rho_{\mathcal{A}} = \oplus_q \rho_{\mathcal{A}}(q)$

Symmetry Resolved Renyi and Entanglement Entropy:

$$S_n(q) = \frac{1}{1-n} \log \mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)^n$$

$$P_{\mathcal{A}}(q) = \frac{\mathrm{Tr}\rho_{\mathcal{A}}(q)}{\mathrm{Tr}\rho_{\mathcal{A}}} = \mathrm{Tr}\rho_{\mathcal{A}}(q)$$

$$S_1(q) = \lim_{n \to 1} S_n(q) = -\mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \log \frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)$$

Sela, Goldstein PRL 2018

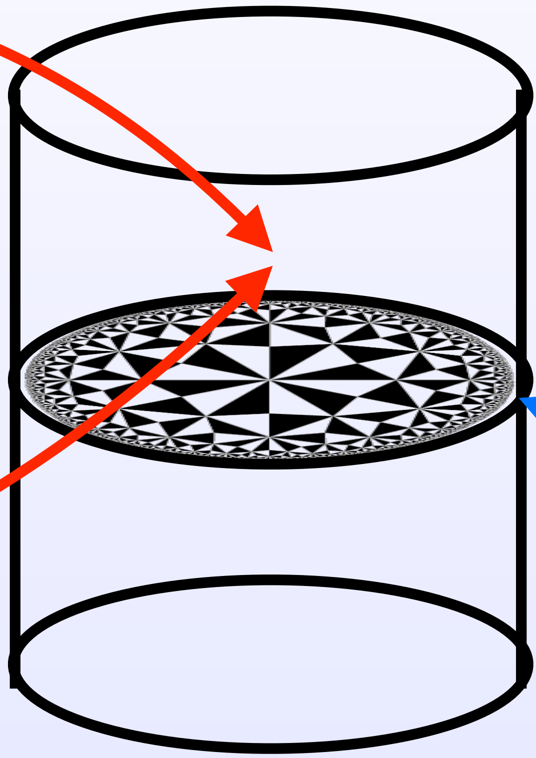
AdS$_3$ dual to U(1)$_k$ Kac-Moody CFT

3D Einstein-Hilbert gravity

$$S_g = \frac{1}{16\pi G_3} \int d^3x \sqrt{g} \left(R + \frac{2}{L^2}\right)$$

L… Curvature Radius of AdS3 space-time

To suppress quantum gravity effects: $c = \frac{3L}{2G_3} \gg 1$



U(1)$_k$ Chern-Simons theory

$$S_{CS} = \frac{ik}{8\pi} \int A \wedge dA$$

k… Chern-Simons level

Asymptotic symmetry analysis:

Boundary theory has U(1)$_k$ Kac-Moody symmetry

Hilbert space factorizes into gravitational and U(1) part

P. Kraus arXiv:hep-th/0609074

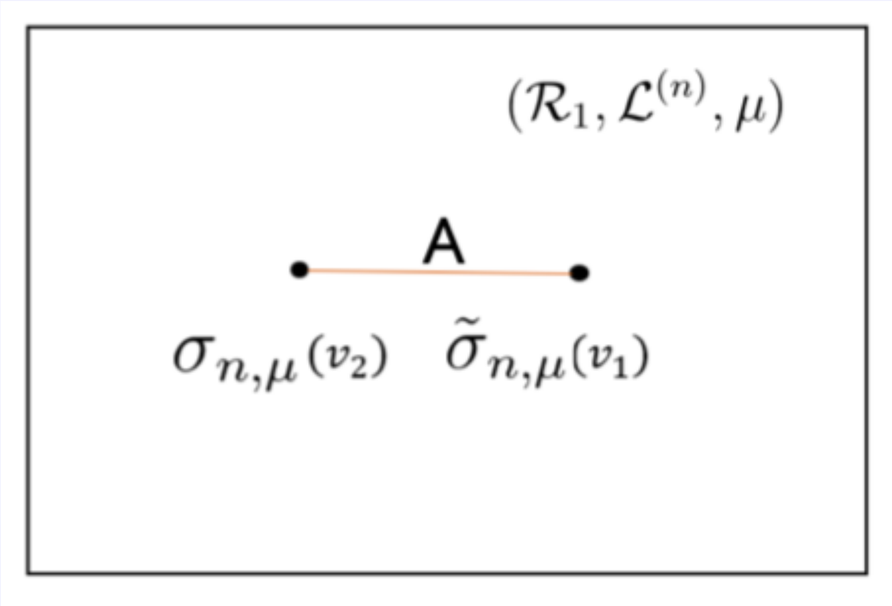
S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)

K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

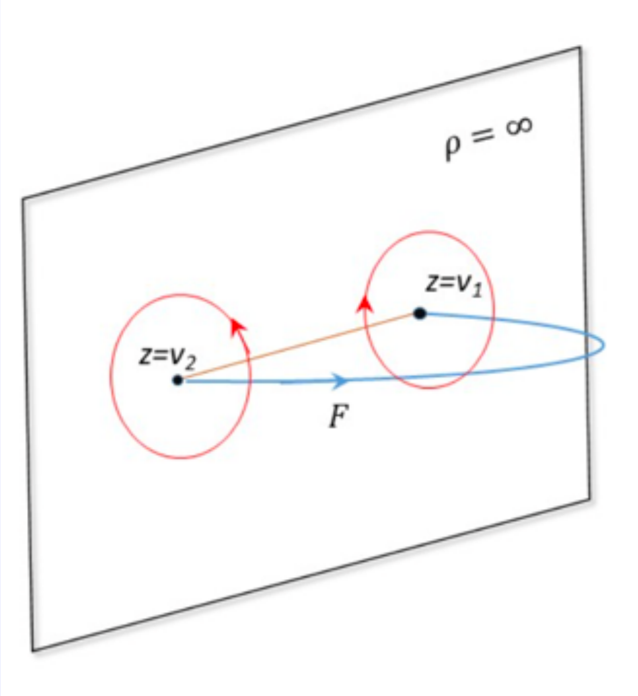
Single Interval SREE in AdS_3/CFT_2

Charged twist operator induces flux:

Sela, Goldstein PRL 2018



Wilson line following the Ryu-Takayanagi geodesic



CFT_2 result matches AdS_3 result for c>>1

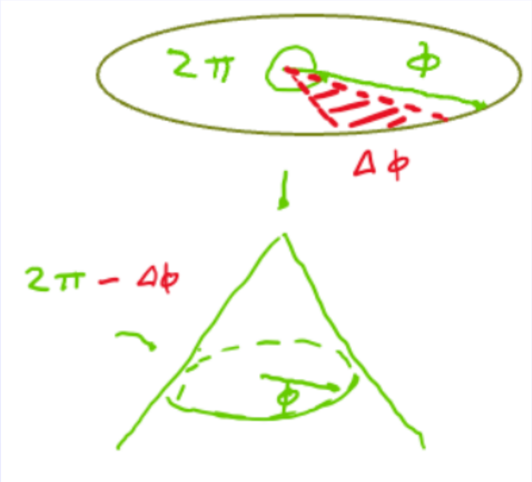
$$S_1(q) = \frac{c}{6}\ell - \frac{1}{2}\log\left(\frac{k\ell}{2\pi}\right) \quad \text{with} \quad \ell = 2\log\frac{|v_1 - v_2|}{\epsilon}$$

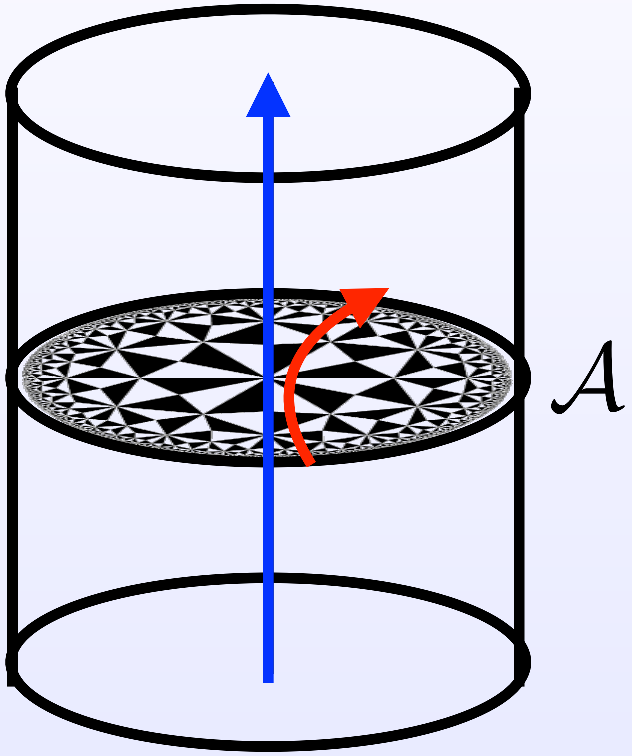
Equipartiton of entanglement!

Suting Zhao, Christian Northe, RM, JHEP 2021 (arXiv: 2012.11274)
Xavier et.al. 2018, Belin, Hung et.al. 2013

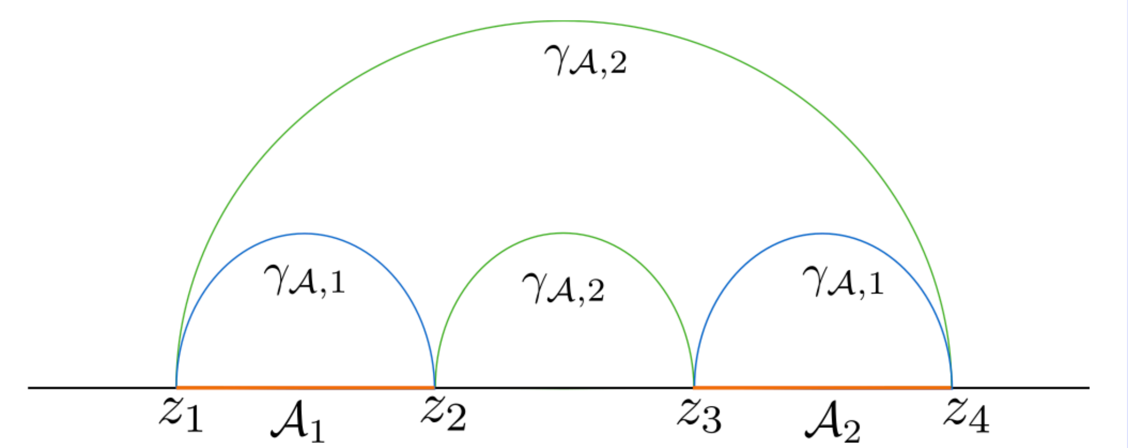
Further Checks

- Single interval with uncharged/charged heavy primary insertions





- Two intervals in the ground state

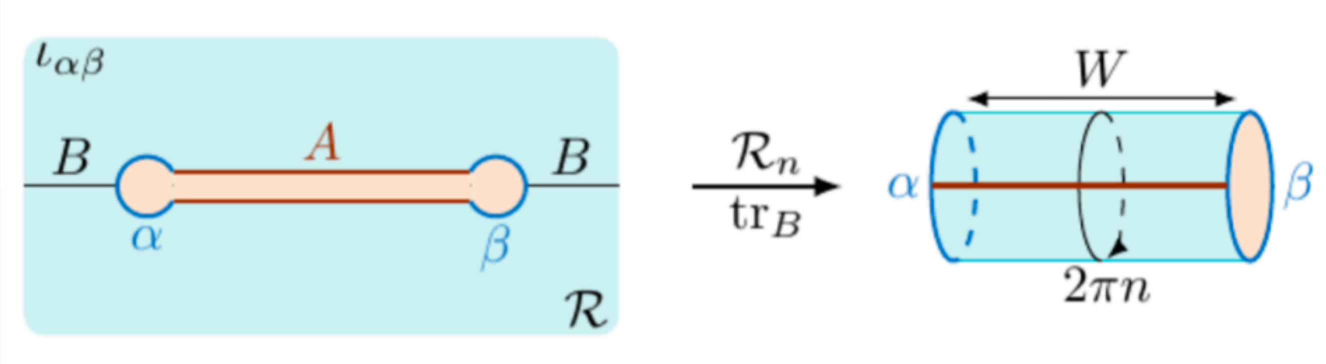


S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

BCFT corrections to SRE

Boundary states change the factorized Hilbert space

Ohmori & Tachikawa 2015, Cardy & Tonni 2016



Choose boundary conditions which preserve conformal symmetry and $U(1)$

$$T = \bar{T}|_{bdy} \qquad J = \pm\bar{J}|_{bdy}$$

For CFT vacuum:

$$\rho_A = \frac{q^{L_0 - c/24}}{Z_{\alpha\beta}(q)}$$

where $Z_{\alpha\beta}(q) = \mathrm{tr}_{\,A} q^{L_0 - c/24}$ and $q = e^{-2\pi^2/W}$

Partition function splits into representations $\mathcal{H}_i$ of chiral algebra with characters $\chi_i(q) = \mathrm{tr}_{\,\mathcal{H}_i} q^{L_0 - c/24}$

$$Z_{\alpha\beta}(q) = \sum_{i\in\sigma} \mathrm{n}^i_{\alpha\beta}\, \chi_i(q) \approx \chi_\Omega(\tilde{q}) \underbrace{\times \mathrm{g}_\alpha \mathrm{g}_\beta}_{S^{\alpha\beta}_{\text{bdy}} = \log[\mathrm{g}_\alpha \mathrm{g}_\beta]} ,$$

Free Boson: Exact equipartition to all orders in UV cutoff

G. Di Giulio, C. Northe, RM, H. Scheppach, S. Zhao, arXiv: 2212.09767

Breakdown of Equipartition

SL(3,R) Higher Spin Gravity in 3D: W_3 symmetric CFT

Energy-momentum tensor plus Spin 3 current

$$T(z)W(w) = \frac{3W(w)}{(z-w)^2} + \frac{\partial W(w)}{z-w} + \cdots$$

Charged moments for a single interval:

Topological black hole grand canonical partition function

Perturbative result in μ

Kraus+Perlmutter (2011)

Gaberdiel+Hartman+Jin (2012)

$$\log \mathrm{Tr}\left(e^{-2\pi n\mathcal{H}+2\pi i\mu Q_{\mathcal{A}}}\right) = \frac{c\ell}{6n}\left(-\frac{1}{3}\frac{\mu^2}{n^4} + \frac{10}{27}\frac{\mu^4}{n^8} + \cdots\right)$$

Fourier transformation and taking the replica limit yields breakdown of equipartition in SREE at large c.

Suting Zhao, Christian Northe, Konstantin Weisenberger, RM, JHEP 2022, 2202.11111

Berry Phases and Wormholes

- Wormholes relate entanglement and geometry in AdS/CFT (EPR=ER) **van Raamsdonk; Maldacena, Susskind**
- Wormhole-like structures also in QM systems **Verlinde**
- Mathematical structure: Non-exact symplectic form

- Berry phases for entangled two-qubit system and for wormhole geometries share similar structure
- States with same entanglement differ in Berry phase
- Berry phases related to non-exact symplectic form
- Indicate wormhole-like structures & non-factorization
- May be realized experimentally

F. S. Nogueira, S. Banerjee, M. Dorband, RM, J. v. d. Brink, J. Erdmenger, PRD 2022
S. Banerjee, M. Dorband, J. Erdmenger, RM, A.-L. Weigel, 2202.11717

Nonexact Symplectic Structures & Wormholes

H. Verlinde 2021

$$Z(\beta) = \mathrm{tr}(e^{-\beta H})$$

$$Z(D) = \int [dX]\, e^{\int_D \Omega - \oint_{\partial D} H dt}$$

$D =$ 

generalized coordinates and momenta X^a, symplectic form $\Omega = \frac{1}{2}\omega_{ab} dX^a \wedge dX^b$

Exact symplectic structure: $\Omega = d\alpha,$ $\int_D \Omega = \oint_{\partial D} \alpha$

$$Z(\beta) = Z(D)$$

If symplectic structure is non-exact:

$$\langle Z(\beta)^n \rangle = Z(\Sigma_n)$$

$\Sigma_n =$  .

Berry Phases in Quantum Mechanics

Time-dependent Schrödinger eq.:

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H(\lambda(t))|\psi\rangle$$

$\lambda(t)$ adiabatic parameter

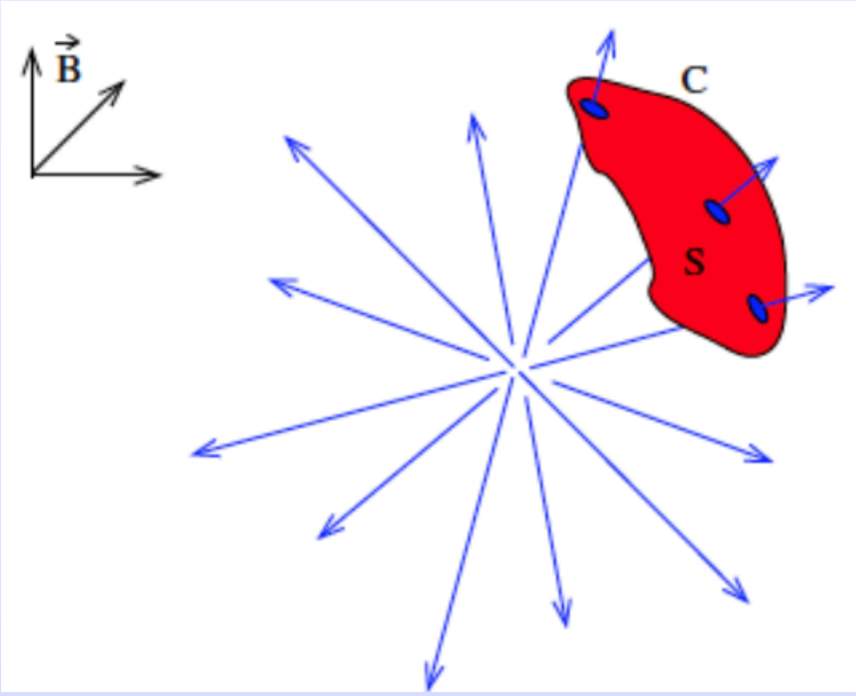
Adiabatic theorem:

$$|\psi(t)\rangle = U(t)\,|n(\lambda(t))\rangle$$

$|n(\lambda(t))\rangle$ instantaneous energy Eigenstates

Berry connection:

$$\mathcal{A}_i(\lambda) = -i\langle n|\frac{\partial}{\partial\lambda^i}|n\rangle$$

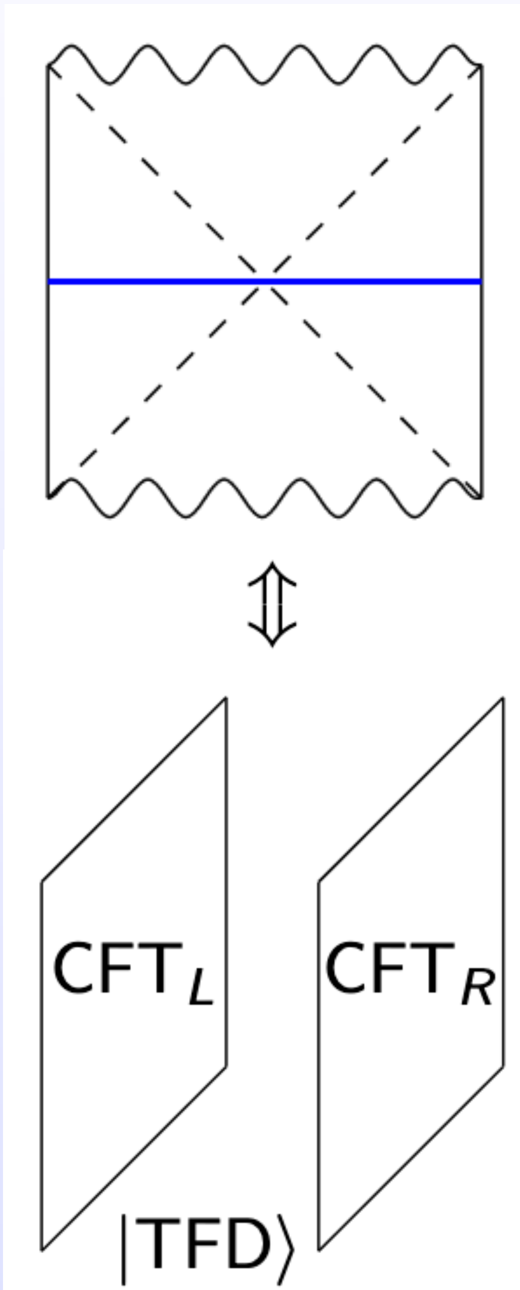


Berry phase:

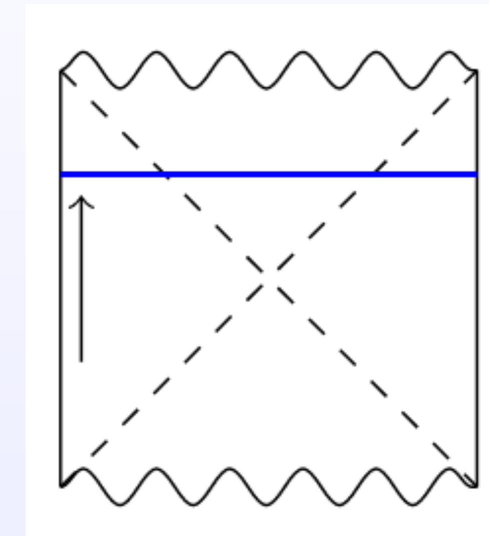
$$e^{i\gamma} = \exp\left(-i\oint_C \mathcal{A}_i(\lambda)\,d\lambda^i\right)$$

Review: Lectures by David Tong

Gauge Berry Phase in Bulk Wormhole



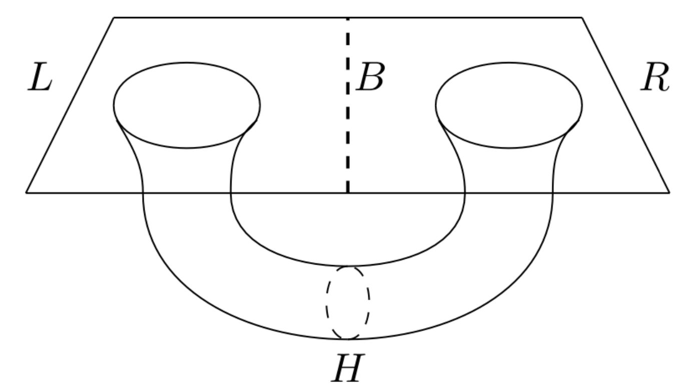
$$U_+|\mathrm{TFD}\rangle \equiv e^{-\mathrm{i}(H_L+H_R)\delta}|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}}\sum_n e^{-2\mathrm{i}E_n\delta}e^{-\frac{\beta}{2}E_n}|n\rangle_L|n\rangle_R^*$$



- Boundary times t_L and t_R naturally identified at the boundary
- Further identification at the horizon necessary [H. Verlinde '20]

$$\begin{cases} \text{at } B: & t_L = t_R \\ \text{at } H: & t_L = 2\delta - t_R \end{cases}$$

- δ can be understood as topological phase



$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}}\sum_n e^{-\frac{\beta}{2}E_n}|n\rangle_L|n\rangle_R^*$$

F. S. Nogueira, S. Banerjee, M. Dorband, RM, J. v. d. Brink, J. Erdmenger, PRD 2022

Gauge Berry Phase in Bulk Wormhole

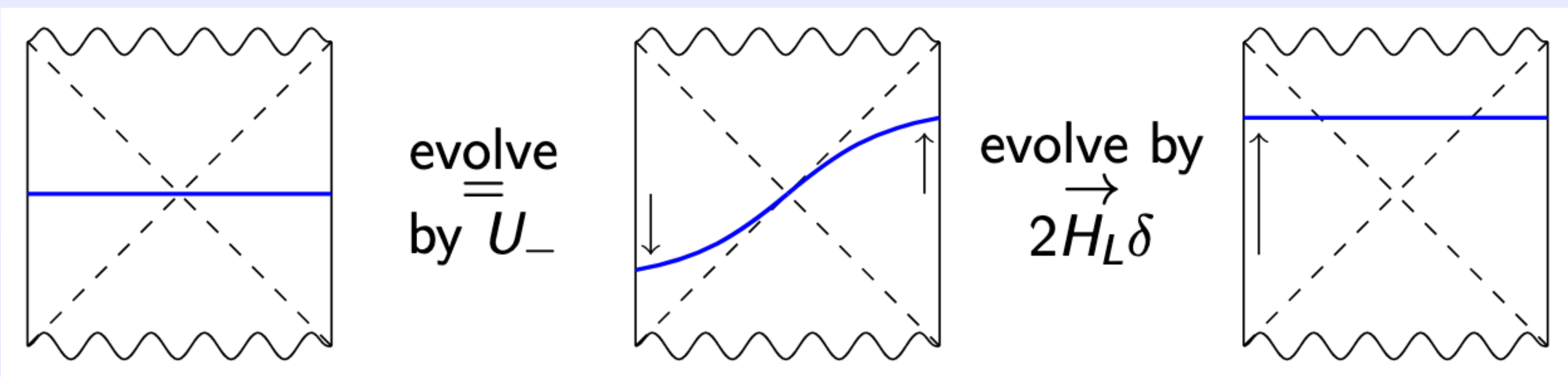
CFT side

$$U_+|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}}\sum_n e^{-2\mathrm{i}E_n\delta}e^{-\frac{\beta}{2}E_n}|n\rangle_L|n\rangle_R^*$$

$$A_\delta = i\langle\mathrm{TFD}|U_+^\dagger\partial_\delta U_+|\mathrm{TFD}\rangle$$
$$= \frac{2}{Z}\sum_n E_n e^{-\beta E_n}$$

$$U_-|\mathrm{TFD}\rangle \equiv e^{\mathrm{i}(H_L-H_R)\delta}|\mathrm{TFD}\rangle = |\mathrm{TFD}\rangle$$

(JT) Gravity side



Phases generated by proper bulk diffeomorphisms

Phase space: $(\delta, H_L + H_R)$ **Non-exact symplectic form**

Holonomy measure non-factorization

Harlow, Jafferis 2019

Geometric Phase and Operator Algebras

- In bipartite quantum systems, geometric phases can be used to describe the entanglement between the subsystems.
- In this case, the geometric phase provides information about the geometry of state space inaccessible to a local observer.
- In the context of vN algebras, the existence of a trace functional for von Neumann algebras are related to the absence of a geometric phase (2306.00055).
- In particular, the non-vanishing geometric phase of the cyclic and separating vector for a type III algebra provides a geometric interpretation for the absence of the trace.
- In type II algebras, the trace is defined since the cyclic and separating vector has a vanishing geometric phase.
- This was realised in holography for the eternal black hole in terms of the geometric phase of the TFD state [Nogueira et al '21]

S. Banerjee, M. Dorband, J. Erdmenger, A.-L. Weigel, 2306.00055

Conclusions & Outlook

- Symmetry resolved entanglement provides new tests of “extended” AdS_3/CFT_2 correspondence
- First example of breakdown of equipartition at large c
- BCFT corrections to SRE, exact equipartition proof

- Outlook:
  Improved understanding of AdS Quantum Gravity
  as well as strongly interacting physics by symmetry resolution
  Applications to condensed matter systems **Oblak et.al. (2022)**
  BCFT corrections on the gravity side

- Non-Factorization due to non-exact symplectic form
- Results in non-zero Berry phase
- Same mathematical structure in QM models
- Berry phase is experimentally measurable
- Route to a better understanding of factorization puzzle (Type III vs. type II operator algebras)