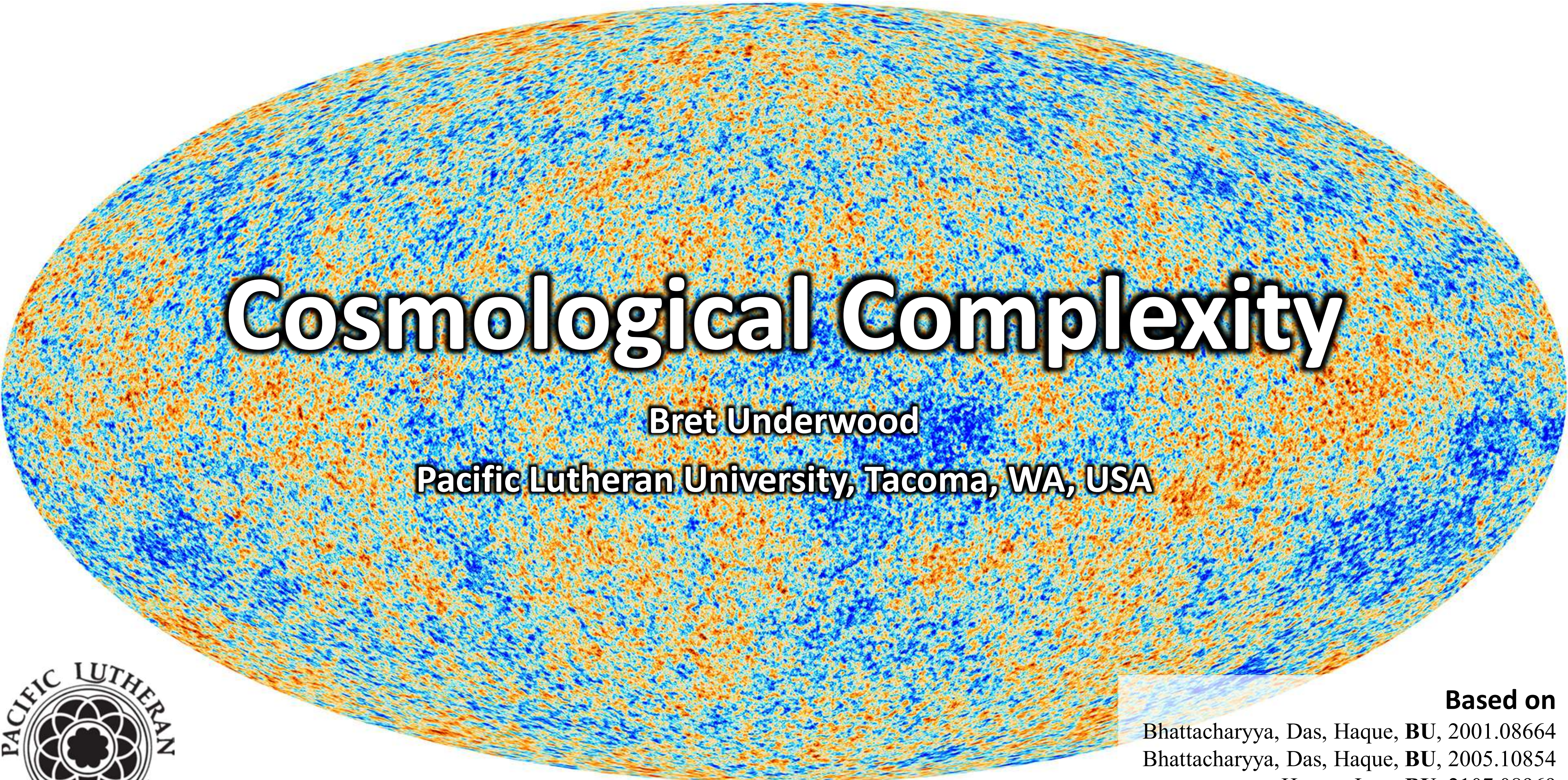


Cosmological Complexity

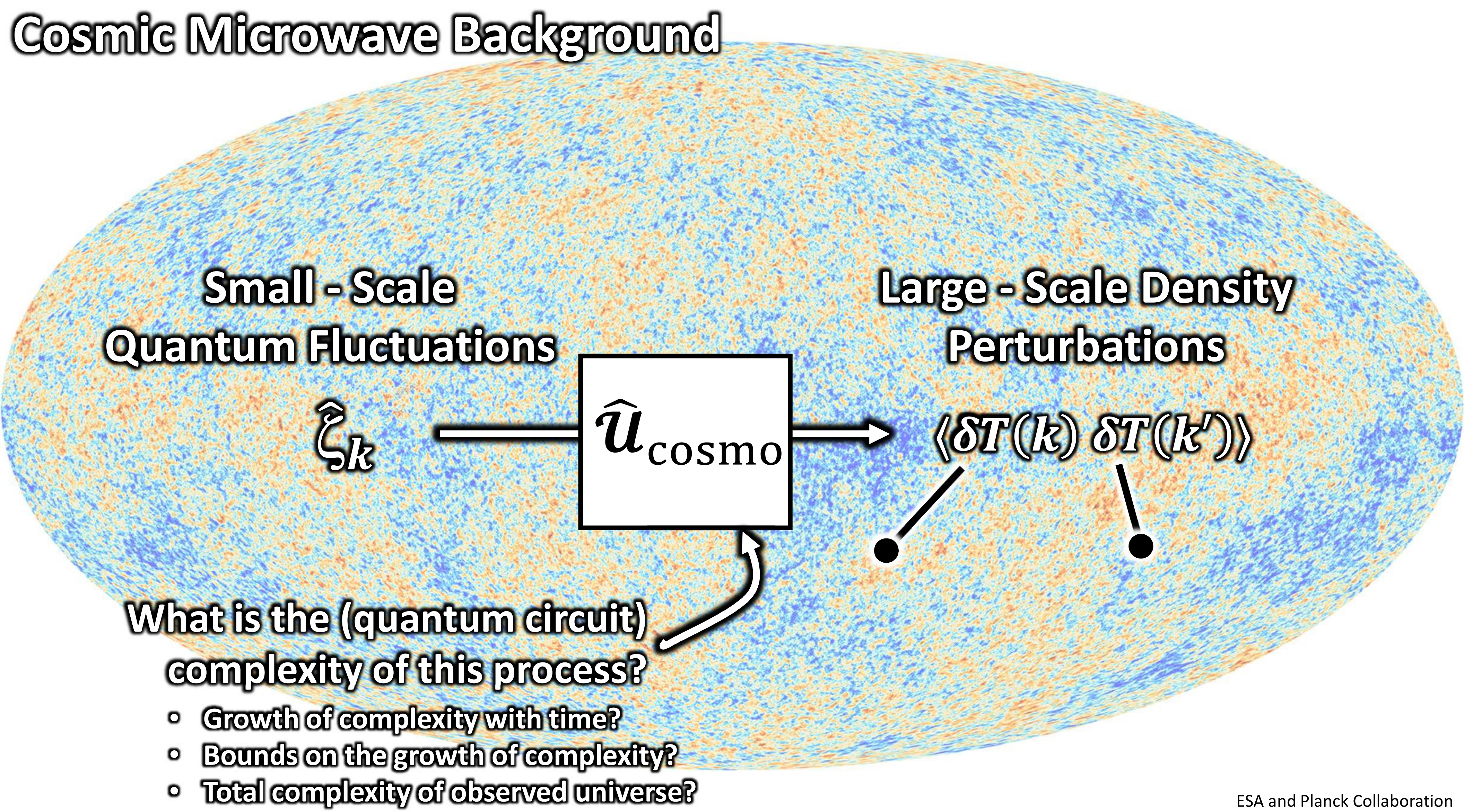
Bret Underwood

Pacific Lutheran University, Tacoma, WA, USA

PACIFIC LUTHERAN UNIVERSITY

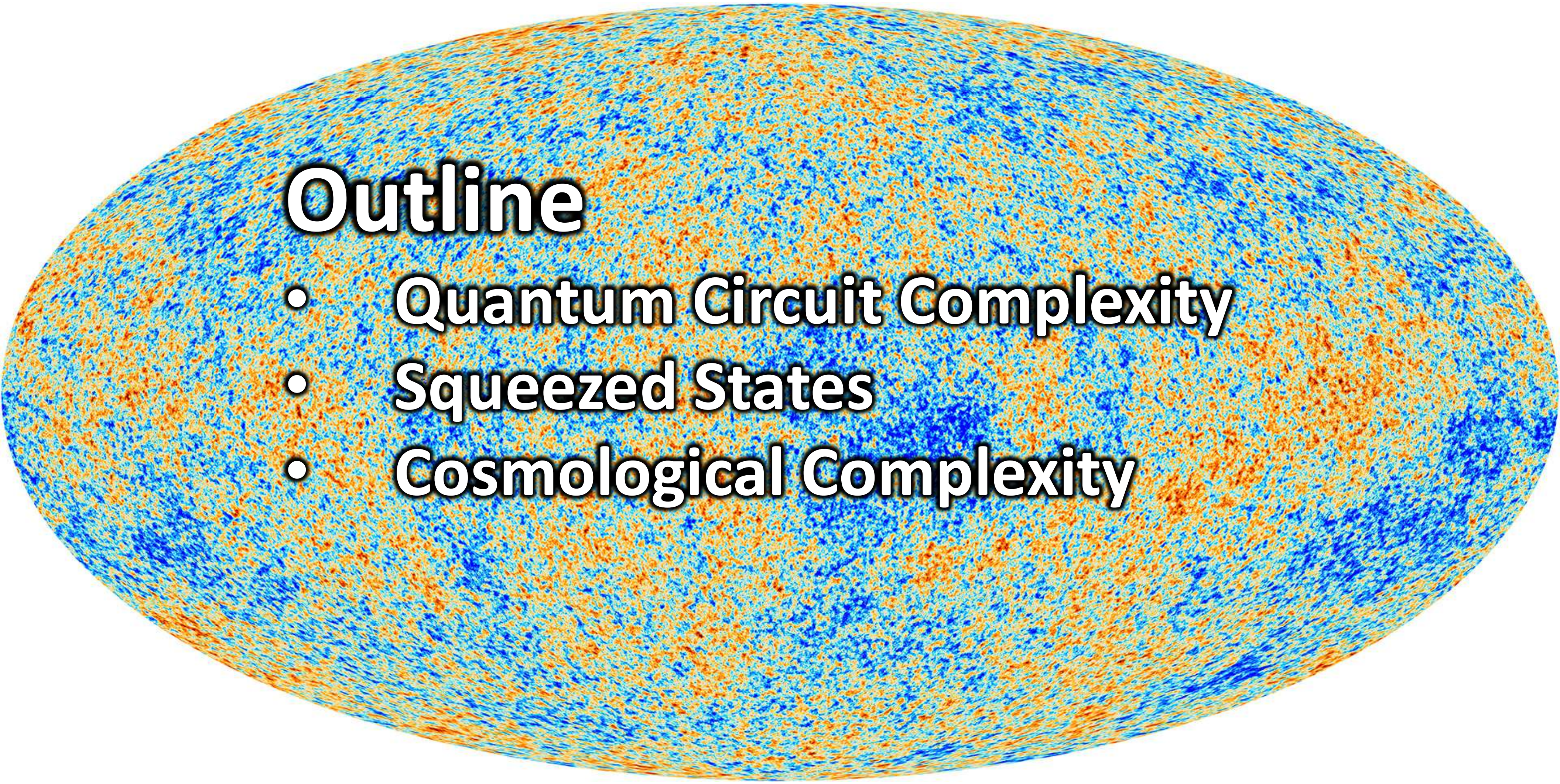
Based on

Bhattacharyya, Das, Haque, **BU**, 2001.08664
Bhattacharyya, Das, Haque, **BU**, 2005.10854
Haque, Jana, **BU**, 2107.08969
Haque, Jana, **BU**, 2110.08356


Cosmic Microwave Background
Small - Scale
Quantum Fluctuations
$\hat{\zeta}_k$
$\hat{U}_{\text{cosmo}}$
Large - Scale Density
Perturbations
$\langle \delta T(k)\, \delta T(k') \rangle$
What is the (quantum circuit)
complexity of this process?
• Growth of complexity with time?
• Bounds on the growth of complexity?
• Total complexity of observed universe?
ESA and Planck Collaboration

Outline

- Quantum Circuit Complexity
- Squeezed States
- Cosmological Complexity



ESA and Planck Collaboration

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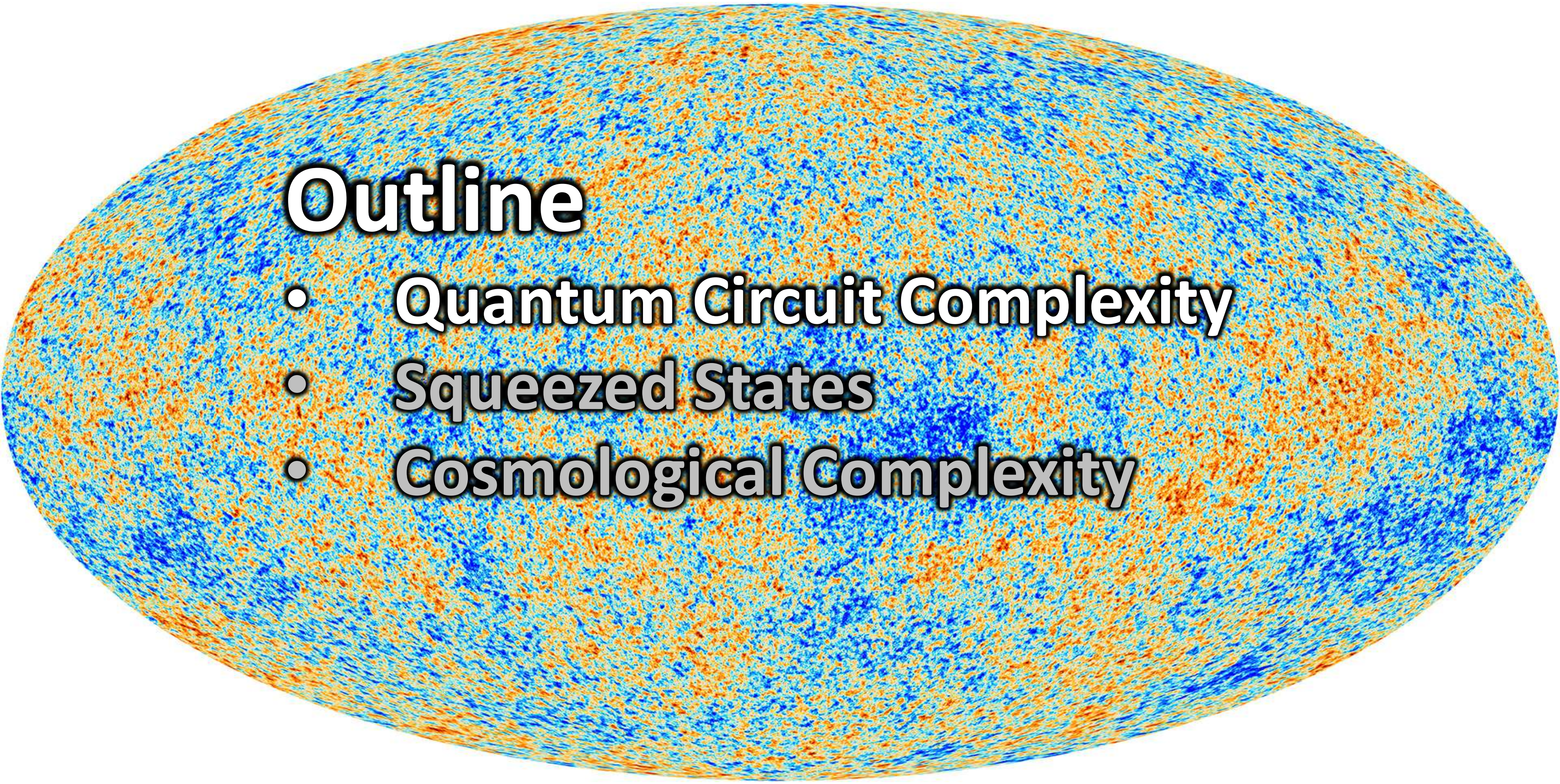


ESA and Planck Collaboration

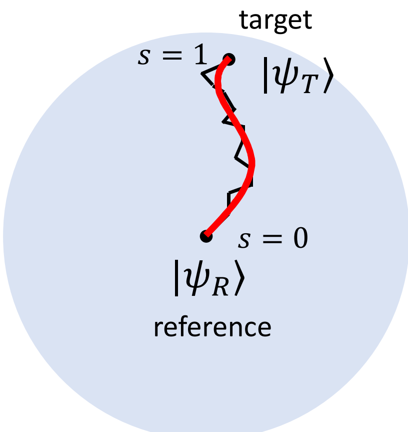
Quantum Circuit Complexity

Unitary evolution from reference state $|\psi_R\rangle$ to target state $|\psi_T\rangle$

$$|\psi_R\rangle \rightarrow \boxed{\hat{\mathcal{U}}_{\text{target}}} \rightarrow |\psi_T\rangle$$

e.g. $e^{-i\hat{H}t}$

$$|\psi_T\rangle = \hat{\mathcal{U}}_{\text{target}}\, |\psi_R\rangle$$



- Model as continuous application of operators Nielsen et al

$$\hat{\mathcal{U}}_{\text{target}} = \overleftarrow{P} \exp\left[\int_0^1 V^I(s)\, \hat{\mathcal{O}}_I\, ds\right]$$

$\{\hat{\mathcal{O}}_I\}$: basis of gates
$V^I(s)$: tangent vectors

- Assign a circuit depth to path

$$\mathcal{D} = \mathcal{D}[V^I]$$

$$\mathcal{D}[V^I] = \int_0^1 \sqrt{G_{IJ}\, V^I V^J}\, ds \quad \text{with } G_{IJ} = \delta_{IJ}$$

"gate cost"

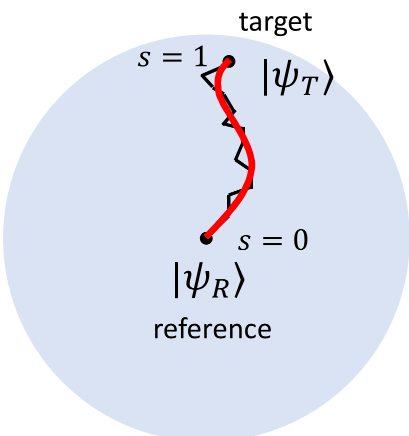
- Circuit Complexity is depth minimized over paths

$$\mathcal{C} = \min_{\{V^I\}} \mathcal{D}[V^I]$$

Quantum Circuit Complexity

Unitary evolution from reference state $|\psi_R\rangle$ to target state $|\psi_T\rangle$

$$|\psi_T\rangle = \hat{\mathcal{U}}_{\text{target}}\, |\psi_R\rangle$$



- Model as continuous application of operators Nielsen et al

$$\hat{\mathcal{U}}_{\text{target}} = \overleftarrow{P} \exp\left[\int_0^1 V^I(s)\, \hat{\mathcal{O}}_I\, ds\right]$$

$\{\hat{\mathcal{O}}_I\}$: basis of gates
$V^I(s)$: tangent vectors

Operator Circuit Complexity

- Characterize gates by structure constants
$$[\hat{\mathcal{O}}_I, \hat{\mathcal{O}}_J] = i\, f_{IJ}^K \hat{\mathcal{O}}_K$$
- Minimization:
$\Rightarrow$ Euler-Arnold eq on group manifold
$$G_{IJ} \frac{dV^J}{ds} = f_{IJ}^K\, G_{KL}\, V^J\, V^L$$

❑ Advantage: Not restricted to subset of states
❑ Disadvantage: Euler-Arnold eq can be difficult to solve

Balasubramanian, Decross, Kar, Parrikar
Basteiro, Erdmenger, Fries, Goth, Matthaiakakis, Meyer

(Gaussian) State Circuit Complexity

- Characterize target operator by its action on Gaussian states

$$\langle x|\psi_R\rangle \sim e^{-\frac{1}{2}\,\omega_0 \sum_k x_k^2}$$

$$\hat{\mathcal{O}}_k \sim e^{-i\hat{x}_k\hat{p}_k} \longrightarrow \langle x|\psi_T\rangle \sim e^{-\frac{1}{2}\sum_k \Omega_k\, x_k^2}$$

$\{\hat{\mathcal{O}}_I\}$: basis for $\mathrm{GL}(N,\mathbb{R})$ or $\mathrm{GL}(N,\mathbb{C})$

❑ Advantage: Simple to set up and find optimal path
❑ Disadvantage: Restricted to Gaussian states

Jefferson, Myers
Ali, Bhattacharyya, Haque, Kim, Moynihan, Murugan

also K-Complexity/Spread Complexity

Dixit, Magan, Kim, Dymarsky, Watanabe

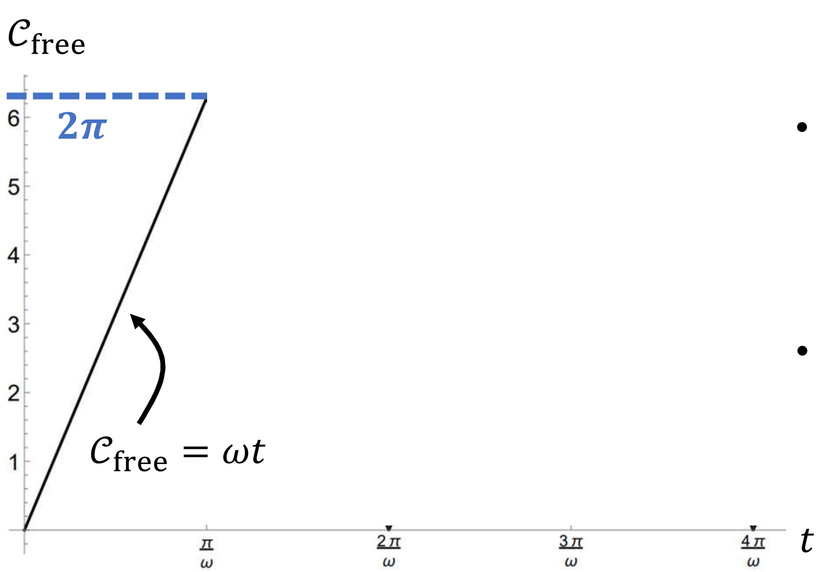
Complexity: Free Harmonic Oscillator

<u>Example</u>: Free Harmonic Oscillator

$$|\psi_T\rangle = \hat{\mathcal{U}}_{\text{target}}\,|\psi_R\rangle$$

$$|\psi_T\rangle = e^{-i\hat{H}_0 t}\,|\psi_R\rangle$$

$$\hat{H}_0 = \frac{\omega}{2}\left(\hat{a}^\dagger \hat{a} + \hat{a}\,\hat{a}^\dagger\right)$$ Haque, Jana, **BU**



<u>Operator Circuit Complexity</u>

- Model as continuous application of operators

$$\hat{\mathcal{U}}_{\text{target}} = \overleftarrow{P}\exp\left[\int_0^1 V^I(s)\,\hat{\mathcal{O}}_I\,ds\right]$$

$\{\hat{\mathcal{O}}_I\}$: basis of gates
$V^I(s)$: tangent vectors

$$\left\{\hat{\mathcal{O}}_1 = \frac{\hat{a}^2 + \hat{a}^{\dagger 2}}{4} \quad \hat{\mathcal{O}}_2 = i\,\frac{\hat{a}^2 - \hat{a}^{\dagger 2}}{4} \quad \hat{\mathcal{O}}_3 = \frac{\hat{a}^\dagger \hat{a} + \hat{a}\,\hat{a}^\dagger}{4}\right\}$$

- Characterize gates by structure constants $[\hat{\mathcal{O}}_I, \hat{\mathcal{O}}_J] = i\,f_{IJ}^K \hat{\mathcal{O}}_K$

$$[\hat{\mathcal{O}}_1, \hat{\mathcal{O}}_2] = -i\hat{\mathcal{O}}_3, \quad [\hat{\mathcal{O}}_3, \hat{\mathcal{O}}_1] = i\hat{\mathcal{O}}_2, \quad [\hat{\mathcal{O}}_2, \hat{\mathcal{O}}_3] = i\hat{\mathcal{O}}_1$$ **su(1,1)**

- Minimization:
  ⇒ Euler-Arnold eq on group manifold ($G_{IJ} = \delta_{IJ}$)

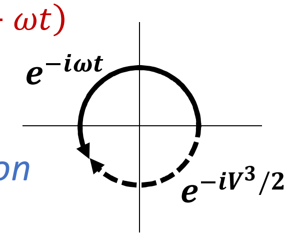
$$G_{IJ}\frac{dV^J}{ds} = f_{IJ}^K\, G_{KL}\, V^J\, V^L \longrightarrow$$

$$V^1 = 0, \quad V^2 = 0, \quad V^3 = \min\begin{cases} 2(\omega t - 2\pi n) \\ 2(2\pi n - \omega t)\end{cases}$$

- Complexity

$$\mathcal{C}_{\text{free}} = \sqrt{(V^1)^2 + (V^2)^2 + (V^3)^2} = |V^3|$$

V^3 is a compact direction



Complexity: Free Scalar Field

Free scalar field ϕ in $(d+1)$-dimension, mass m, box L

Haque, Jana, **BU**

$\Lambda = N_{\max}\pi/L$ UV cutoff

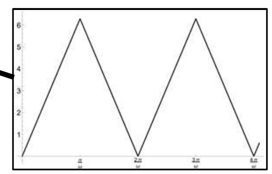
$$\hat{\phi} = \sum_{\vec{n}}^{N_{\max}} \frac{1}{\sqrt{2\,E_{\vec{n}}}}\left(\hat{a}_{\vec{n}}\, e^{i\vec{p}_{\vec{n}}\cdot\vec{x}} + \hat{a}_{\vec{n}}^{\dagger} e^{-i\vec{p}_{\vec{n}}\cdot\vec{x}}\right)$$

Mode expansion: $\begin{cases} \vec{p}_{\vec{n}} = \vec{n}\pi/L \\ E_{\vec{n}} = \sqrt{p_{\vec{n}}^2 + m^2} \end{cases}$

Target Unitary

$$\mathcal{U}_{\text{target}} = \prod_{\vec{n}}^{N_{\max}} e^{-i\frac{1}{2}E_{\vec{n}}\left(\hat{a}_{\vec{n}}^{\dagger}\hat{a}_{\vec{n}} + \hat{a}_{\vec{n}}\hat{a}_{\vec{n}}^{\dagger}\right)}$$

copies of free oscillator for each mode



Complexity of free scalar field

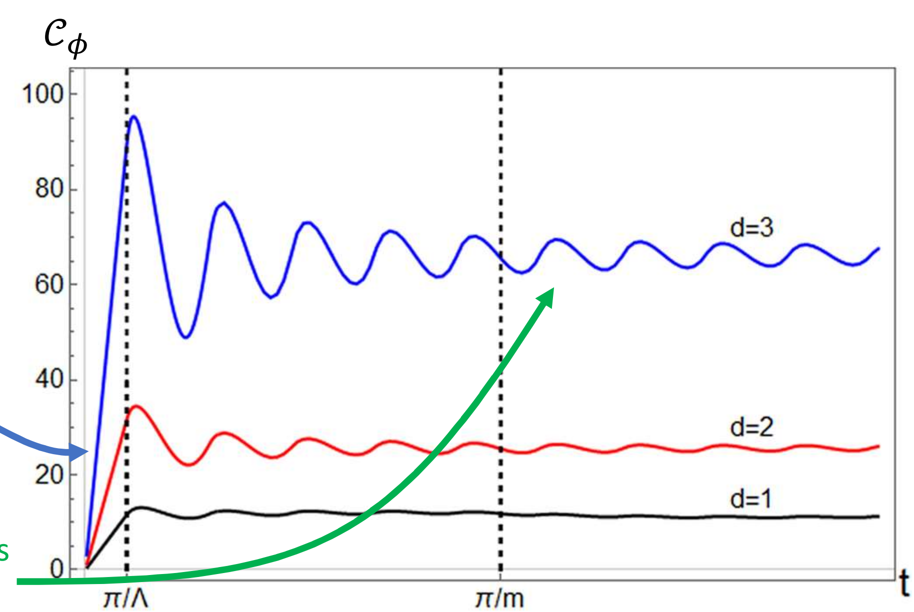
$$\mathcal{C}_{\phi} = \sqrt{\sum_{\vec{n}}^{N_{\max}} \left(V_{\vec{n}}^{3}\right)^2} \sim L^{d/2}\sqrt{\int^{\Lambda}\left(V^{3}(p)\right)^2\, d^d p}$$

continuum limit

$$\sim \begin{cases} L^{d/2}\Lambda^{d/2}\,(\Lambda t) & \text{early times} \quad t \ll \pi/\Lambda \\ L^{d/2}\Lambda^{d/2} & \text{late times} \quad t \gg \pi/\Lambda \end{cases}$$

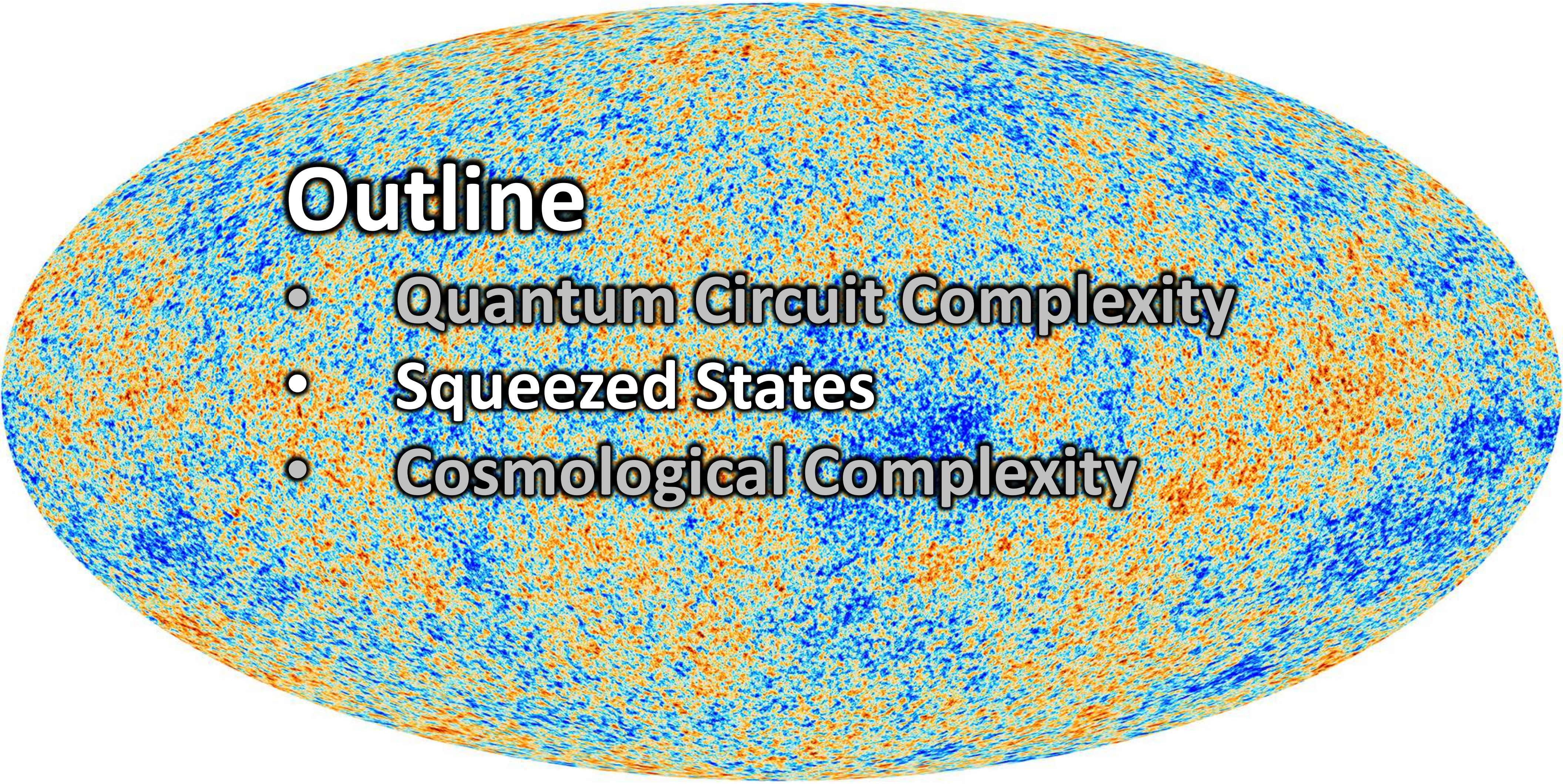
Linear Growth:
complexity of only one mode growing

Saturation:
complexity of all modes oscillating, average out



Outline

- Quantum Circuit Complexity
- Squeezed States
- Cosmological Complexity



ESA and Planck Collaboration

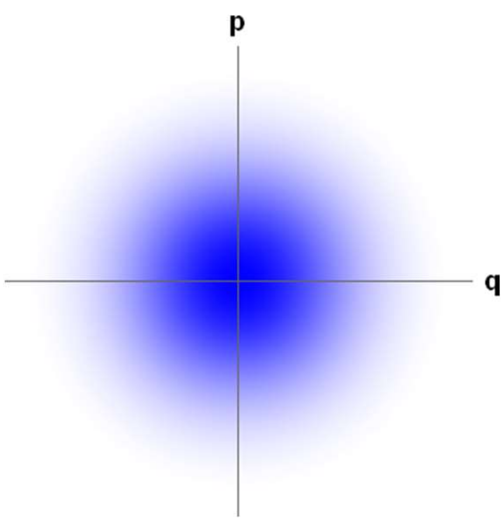
Squeezed States

Vacuum States

$$\psi(q) \sim e^{-\frac{q^2}{2}}, \qquad \varphi(p) \sim e^{-\frac{p^2}{2}}$$

$$\langle \Delta\hat{q}^2 \rangle = \frac{1}{2} \qquad \langle \Delta\hat{p}^2 \rangle = \frac{1}{2}$$

$$\langle \Delta\hat{q}^2 \rangle \langle \Delta\hat{p}^2 \rangle = \frac{1}{4}$$



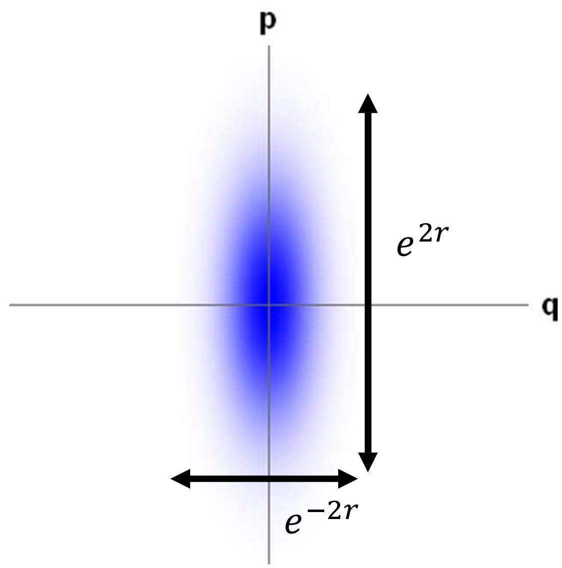
$r = 0$

Squeezed Vacuum State

$$\psi(q) \sim e^{-\frac{q^2}{2}e^{2r}}, \qquad \varphi(p) \sim e^{-\frac{p^2}{2}e^{-2}}$$

$$\langle \Delta\hat{q}^2 \rangle = \frac{1}{2}e^{-2r} \qquad \langle \Delta\hat{p}^2 \rangle = \frac{1}{2}e^{2r}$$

$$\langle \Delta\hat{q}^2 \rangle \langle \Delta\hat{p}^2 \rangle = \frac{1}{4}$$

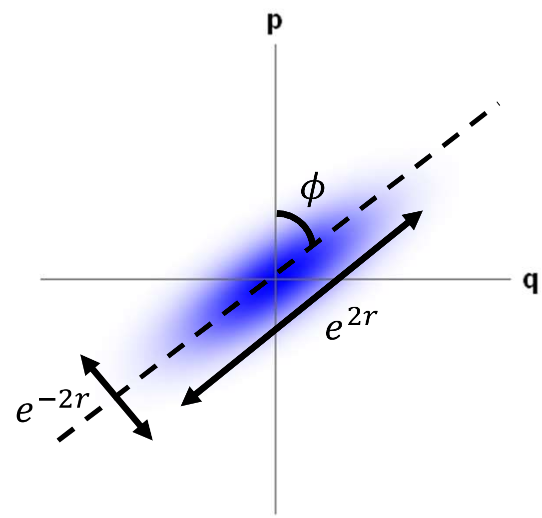


$r = 0.5$
$\phi = 0$

Squeezed, ***Rotated*** Vacuum State

$$\hat{q}_+ = \hat{p}\sin\phi + \hat{x}\cos\phi$$
$$\hat{q}_- = \hat{p}\cos\phi - \hat{x}\sin\phi$$

$$\langle \Delta\hat{q}_+{}^2 \rangle \langle \Delta\hat{q}_-{}^2 \rangle = \frac{1}{4}$$



$r = 0.5$
$\phi = \pi/3$

Squeezed States

Described by **squeezing parameter r, squeezing angle ϕ**, and **rotation angle θ**

$$|r,\phi,\theta\rangle = \hat{\mathcal{S}}(r,\phi)\,\hat{\mathcal{R}}(\theta)\,|0\rangle$$

$$\hat{\mathcal{U}} = \hat{\mathcal{S}}(r,\phi)\hat{\mathcal{R}}(\theta)$$

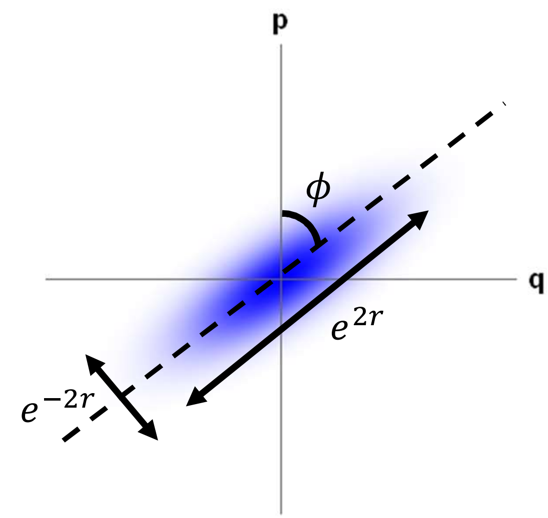
where $\hat{\mathcal{S}}(r,\phi) \equiv \exp\left[\frac{r(t)}{2}\left(e^{-2i\phi}\,\hat{a}^2 - e^{2i\phi}\,\hat{a}^{\dagger 2}\right)\right]$ **squeezing operator**

$\hat{\mathcal{R}}(\theta) \equiv \exp\left[-i\theta\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right)\right]$ **rotation operator**

Squeezed, ***Rotated*** Vacuum State

$$\hat{q}_+ = \hat{p}\sin\phi + \hat{x}\cos\phi$$
$$\hat{q}_- = \hat{p}\cos\phi - \hat{x}\sin\phi$$

$$\langle\Delta\hat{q}_+{}^2\rangle\langle\Delta\hat{q}_-{}^2\rangle = \frac{1}{4}$$



$r = 0.5$
$\phi = \pi/3$

"World record" laboratory squeezing
$r \approx 1.7$
Vahlbruch, et al, 2016

Squeezed States found in:
- Quantum Optics
- Gravitational Wave Detection
- Cosmological Perturbations

Squeezed States in Cosmological Perturbations

Cosmological Perturbations

$$ds^2 = a(\eta)^2\,(-d\eta^2 + (1 - 2\,\mathcal{R})\,d\vec{x}^2)$$

scale factor → $a(\eta)$; conformal time → η; curvature pert → $\mathcal{R}$

Mukhanov variable

$$v = z\,\mathcal{R}, \qquad z = a\sqrt{2\epsilon}$$

Canonical Quantization

$$\hat{v} = \int \frac{d^3k}{(2\pi)^3}\,\hat{v}_k\,e^{i\vec{k}\cdot\vec{x}} \qquad \hat{v}_{\vec{k}} = \frac{1}{\sqrt{2k}}\left(\hat{a}_{\vec{k}} + \hat{a}^{\dagger}_{-\vec{k}}\right)$$

$$\hat{H} = \int d^3k\,\hat{H}_{\vec{k}} = \int d^3k\,\frac{1}{2}\left[k\,\left(\hat{a}_{\vec{k}}\,\hat{a}^{\dagger}_{\vec{k}} + \hat{a}^{\dagger}_{-\vec{k}}\hat{a}_{-\vec{k}}\right) - i\,\frac{z'}{z}\,\left(\hat{a}_{\vec{k}}\,\hat{a}_{-\vec{k}} - \hat{a}^{\dagger}_{\vec{k}}\hat{a}^{\dagger}_{-\vec{k}}\right)\right]$$

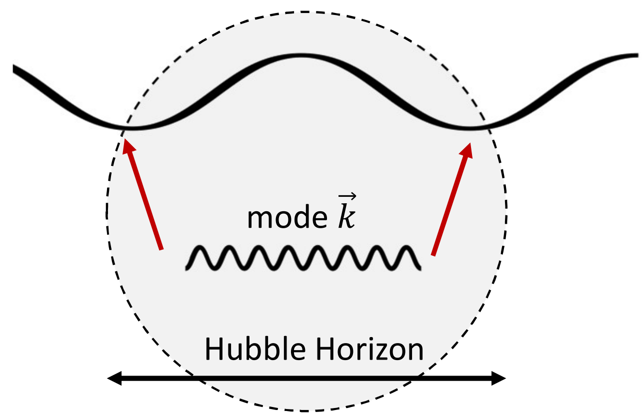
Free-particle | Inverted Oscillator

Time-dependent frequency → $\frac{z'}{z}$

Two-mode squeezed state $(\vec{k}, -\vec{k})$

Grishchuk, Sidorov
Albrecht, Ferreira, Joyce, Prokopec

Accelerating background stretches modes outside horizon



Squeezed States in Cosmological Perturbations

Squeezed Cosmological Perturbations

$$|r_k, \phi_k, \theta_k\rangle = \hat{\mathcal{S}}(r_k, \phi_k)\, \hat{\mathcal{R}}(\theta_k)\, |0_{\vec{k}}, 0_{-\vec{k}}\rangle$$

Squeezing Parameters:

Squeezing strength $r_k = r_k(\eta)$

Squeezing angle $\phi_k = \phi_k(\eta)$

Rotation angle $\theta_k = \theta_k(\eta)$

} Time-dependence from expanding background

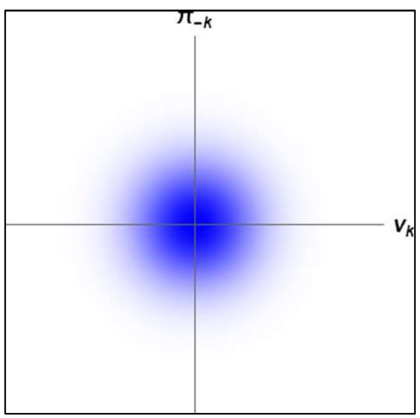
Example:

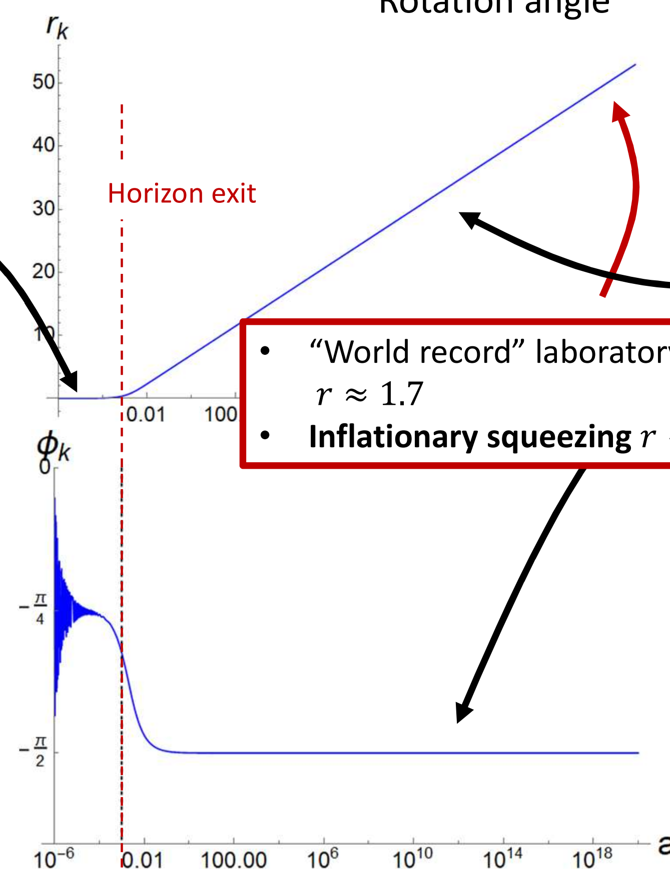
de Sitter (inflation)

Inside Horizon:

$$k \gg \frac{a'}{a}$$

- $r_k \ll 1$
- ϕ_k, θ_k evolve



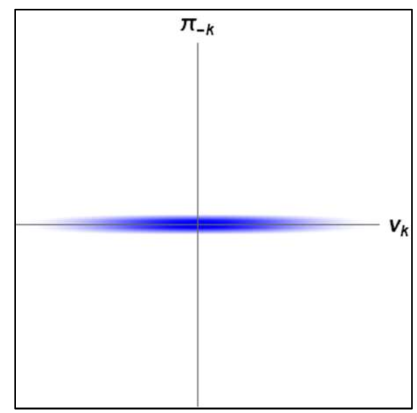


- "World record" laboratory squeezing $r \approx 1.7$
- **Inflationary squeezing** $r \sim N_e \sim 60!$

Outside Horizon:

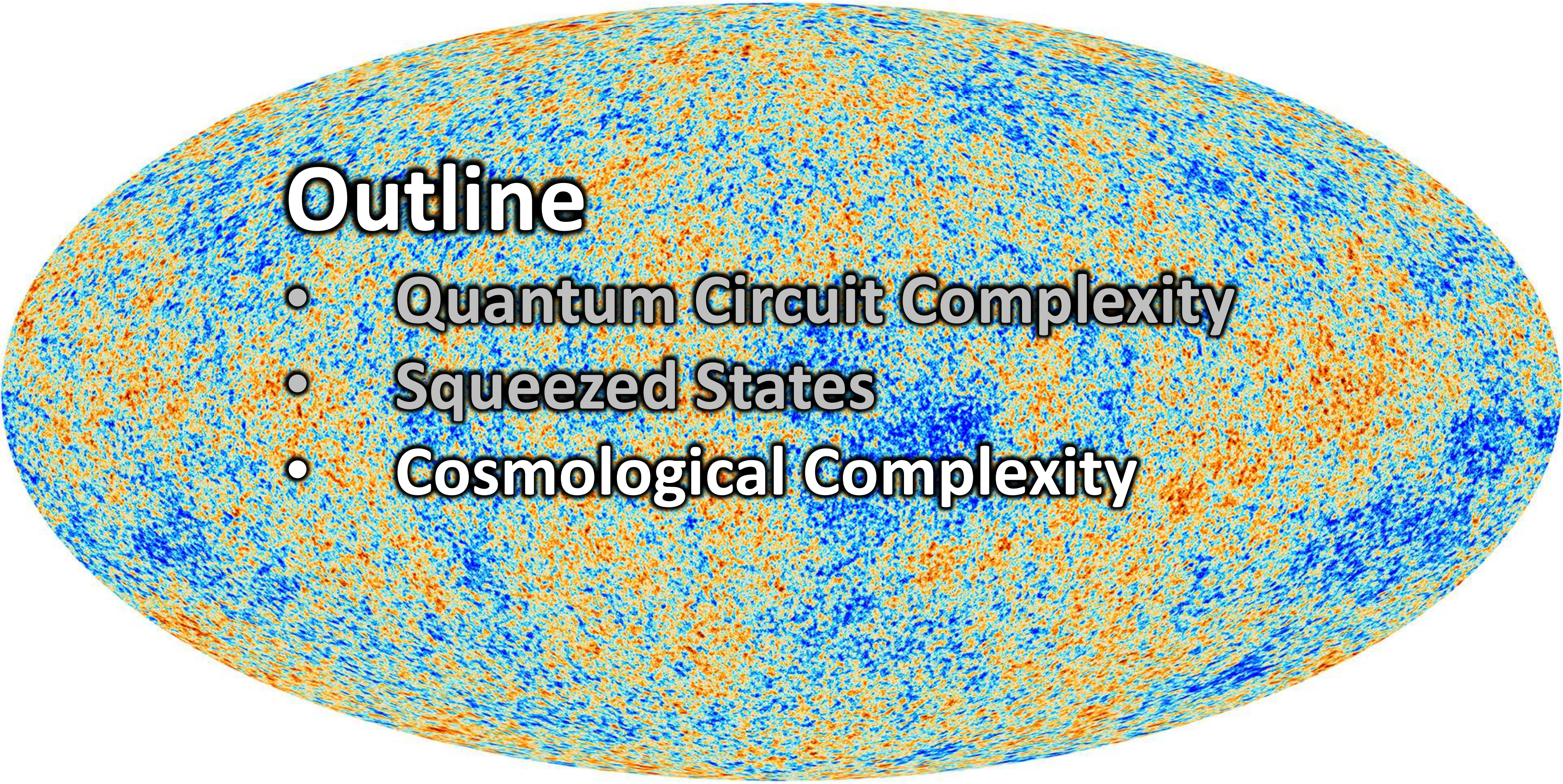
$$k \ll \frac{a'}{a}$$

- $r_k \sim \ln a$
- $\phi_k \sim \pi/2$ (constant)
- θ_k constant



Outline

- Quantum Circuit Complexity
- Squeezed States
- Cosmological Complexity



ESA and Planck Collaboration

Cosmological Complexity

the complexity of quantum cosmological perturbations

$$|\psi_T\rangle = \hat{\mathcal{U}}_{\text{cosmo}}\,|\psi_R\rangle$$

Squeezed Cosmological Perturbations

$$|r_k, \phi_k, \theta_k\rangle = \hat{\mathcal{S}}(r_k, \phi_k)\,\hat{\mathcal{R}}(\theta_k)\,\left|0_{\vec{k}}, 0_{-\vec{k}}\right\rangle$$

Operator Circuit Complexity

Haque, Jana, **BU**

$$\hat{\mathcal{U}}_{\text{target}} = \overleftarrow{P}\,\exp\left[\int_0^1 V^I(s)\,\hat{\mathcal{O}}_I\,ds\right]$$

$$\hat{\mathcal{S}} = \exp\left[\frac{r_k}{2}\left(e^{-2i\phi_k}\,\hat{a}_{\vec{k}}\hat{a}_{-\vec{k}} - e^{2i\phi_k}\hat{a}^\dagger_{\vec{k}}\hat{a}^\dagger_{-\vec{k}}\right)\right]$$ **squeezing operator**

$$\hat{\mathcal{R}} = \exp\left[-i\theta_k\left(\hat{a}_{\vec{k}}\,\hat{a}^\dagger_{\vec{k}} + \hat{a}^\dagger_{-\vec{k}}\hat{a}_{-\vec{k}}\right)\right]$$ **rotation operator**

- Characterize gates by structure constants

$$\left[\hat{\mathcal{O}}_I, \hat{\mathcal{O}}_J\right] = i\,f_{IJ}^K\hat{\mathcal{O}}_K$$

$$\left.\begin{aligned}
\hat{\mathcal{O}}_1 &= \frac{\hat{a}_{\vec{k}}\,\hat{a}_{-\vec{k}} + \hat{a}^\dagger_{\vec{k}}\hat{a}^\dagger_{-\vec{k}}}{2}\\
\hat{\mathcal{O}}_2 &= i\,\frac{\hat{a}_{\vec{k}}\,\hat{a}_{-\vec{k}} - \hat{a}^\dagger_{\vec{k}}\hat{a}^\dagger_{-\vec{k}}}{2}\\
\hat{\mathcal{O}}_3 &= \frac{\hat{a}_{\vec{k}}\,\hat{a}^\dagger_{\vec{k}} + \hat{a}^\dagger_{-\vec{k}}\hat{a}_{-\vec{k}}}{2}
\end{aligned}\right\}\mathrm{SU}(1,1)$$

- Minimization:
  ⇒ Euler-Arnold eq on group manifold

$$G_{IJ}\frac{dV^J}{ds} = f_{IJ}^K\,G_{KL}\,V^J\,V^L$$

$V^I(s)$: tangent vectors

⇒ solve for $V^J(s)$, construct $\mathcal{U}_{\text{target}}$

- Operator Circuit Complexity

$$\mathcal{C}^{(o)} = \sqrt{G_{IJ}V^IV^J}$$

Cosmological Complexity

the complexity of quantum cosmological perturbations

$$|\psi_T\rangle = \hat{\mathcal{U}}_{\text{cosmo}}\,|\psi_R\rangle$$

Squeezed Cosmological Perturbations

$$|r_k, \phi_k, \theta_k\rangle = \hat{\mathcal{S}}(r_k, \phi_k)\,\hat{\mathcal{R}}(\theta_k)\,\left|0_{\vec{k}}, 0_{-\vec{k}}\right\rangle$$

Operator Circuit Complexity Haque, Jana, **BU**

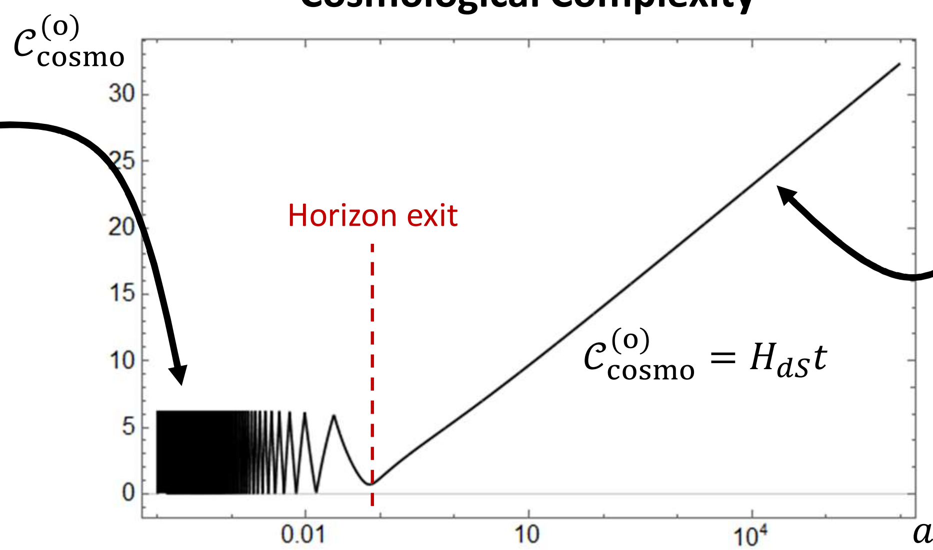
$$\hat{\mathcal{U}}_{\text{target}} = \overleftarrow{P}\exp\left[\int_0^1 V^I(s)\,\hat{\mathcal{O}}_I\,ds\right]$$

$$\hat{\mathcal{S}} = \exp\left[\frac{r_k}{2}\left(e^{-2i\phi_k}\,\hat{a}_{\vec{k}}\hat{a}_{-\vec{k}} - e^{2i\phi_k}\hat{a}^\dagger_{\vec{k}}\hat{a}^\dagger_{-\vec{k}}\right)\right]$$ **squeezing operator**

$$\hat{\mathcal{R}} = \exp\left[-i\theta_k\left(\hat{a}_{\vec{k}}\,\hat{a}^\dagger_{\vec{k}} + \hat{a}^\dagger_{-\vec{k}}\hat{a}_{-\vec{k}}\right)\right]$$ **rotation operator**

Inside Horizon:
- $r_k \ll 1$
- Complexity oscillates $\mathcal{C}^{(\mathrm{o})}_{\text{cosmo}} \leq 2\pi$ due to rotation phase θ

Cosmological Complexity



Outside Horizon:
- Grows with squeezing, e-folds $\mathcal{C}^{(\mathrm{o})}_{\text{cosmo}} \approx r_k \approx \ln\left(\frac{a}{a_{\text{exit}}}\right)$
- Growth is linear in cosmic time t, $a(t) = e^{H_{dS}t}$

Cosmological Complexity

the complexity of quantum cosmological perturbations

$$|\psi_T\rangle = \hat{\mathcal{U}}_{\text{cosmo}} \, |\psi_R\rangle$$

Squeezed Cosmological Perturbations $$|r_k, \phi_k, \theta_k\rangle = \hat{\mathcal{S}}(r_k, \phi_k)\, \hat{\mathcal{R}}(\theta_k)\, \left|0_{\vec{k}}, 0_{-\vec{k}}\right\rangle$$

(Gaussian) State Circuit Complexity

Bhattacharyya, Das, Haque, **BU**

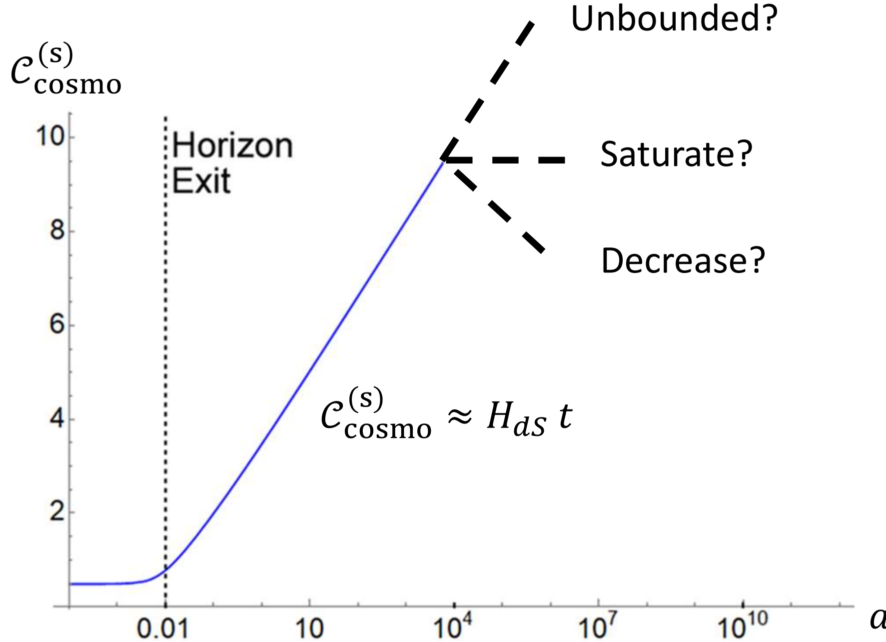
- Characterize $\hat{\mathcal{U}}_{\text{cosmo}}$ by its action on Gaussian states

$$\Psi_{\text{sq}} = \langle q_{\vec{k}}, q_{-\vec{k}} | r_k, \phi_k, \theta_k\rangle \sim e^{A\left(q_{\vec{k}}^2 + q_{-\vec{k}}^2\right) - B\, q_{\vec{k}}\, q_{-\vec{k}}}$$

What is the long-term behavior of cosmological complexity?

Inside Horizon:

- $r_k \ll 1$
- Gaussian State Complexity insensitive to phase $\mathcal{C}^{(s)}_{\text{cosmo}} \ll 1$



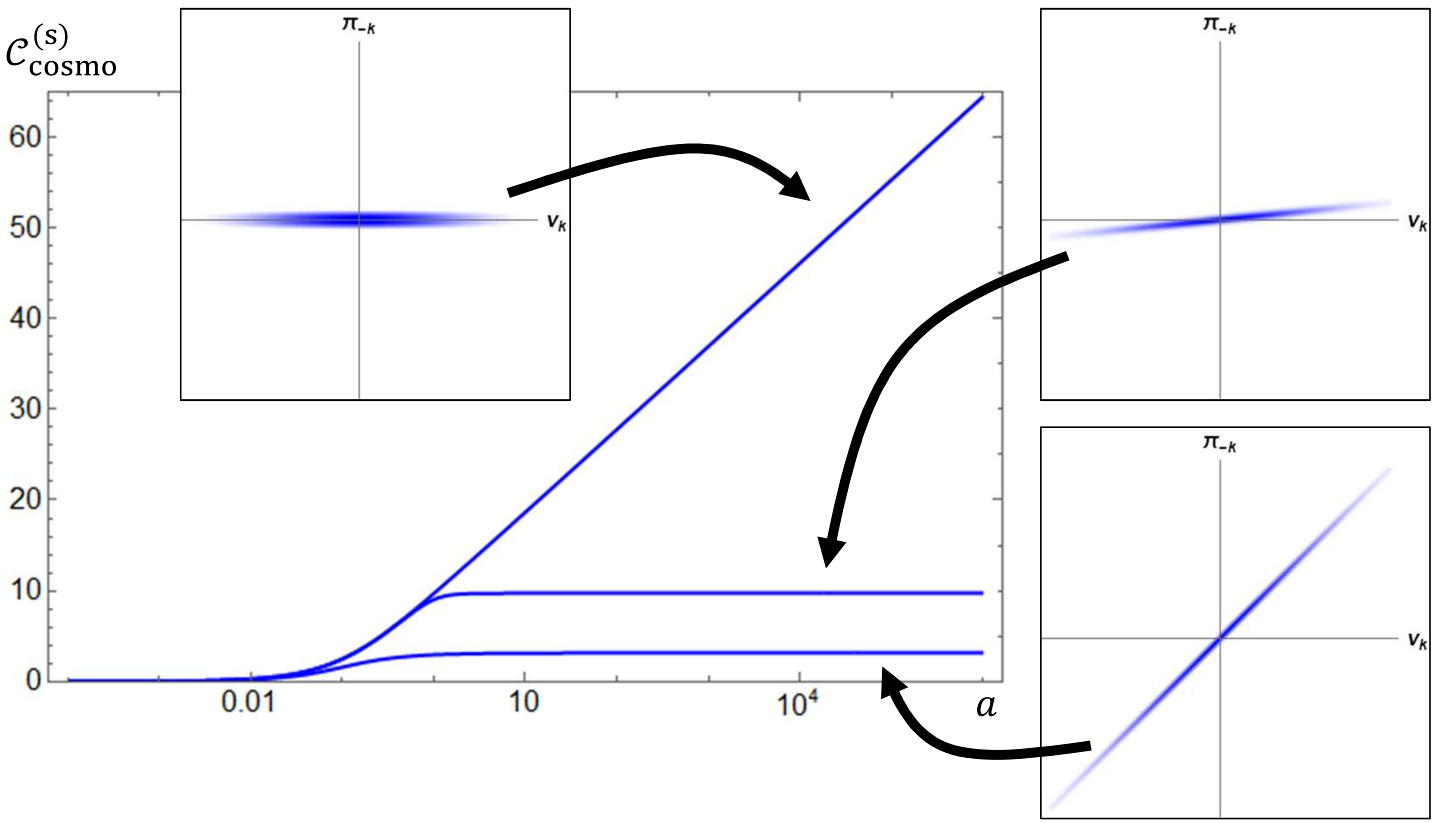
Outside Horizon:

- Grows with squeezing, e-folds
  $$\mathcal{C}^{(s)}_{\text{cosmo}} \approx r_k \approx \ln\left(\frac{a}{a_{\text{exit}}}\right)$$
- Growth is linear in cosmic time t, $a(t) = e^{H_{dS} t}$

Cosmological Complexity

the complexity of quantum cosmological perturbations

Squeezed Cosmological Perturbations $|r_k, \phi_k, \theta_k\rangle = \hat{\mathcal{S}}(r_k, \phi_k)\,\hat{\mathcal{R}}(\theta_k)\,|0_{\vec{k}}, 0_{-\vec{k}}\rangle$

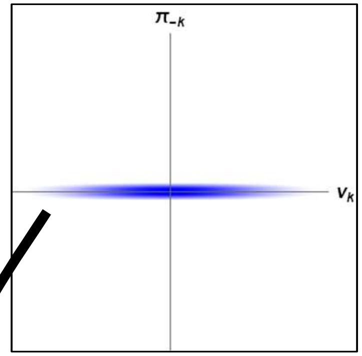


Unbounded growth of complexity depends sensitively on squeezing angle ϕ

- ❑ Complexity of dS is **maximal** w.r.t. ϕ
  Why?

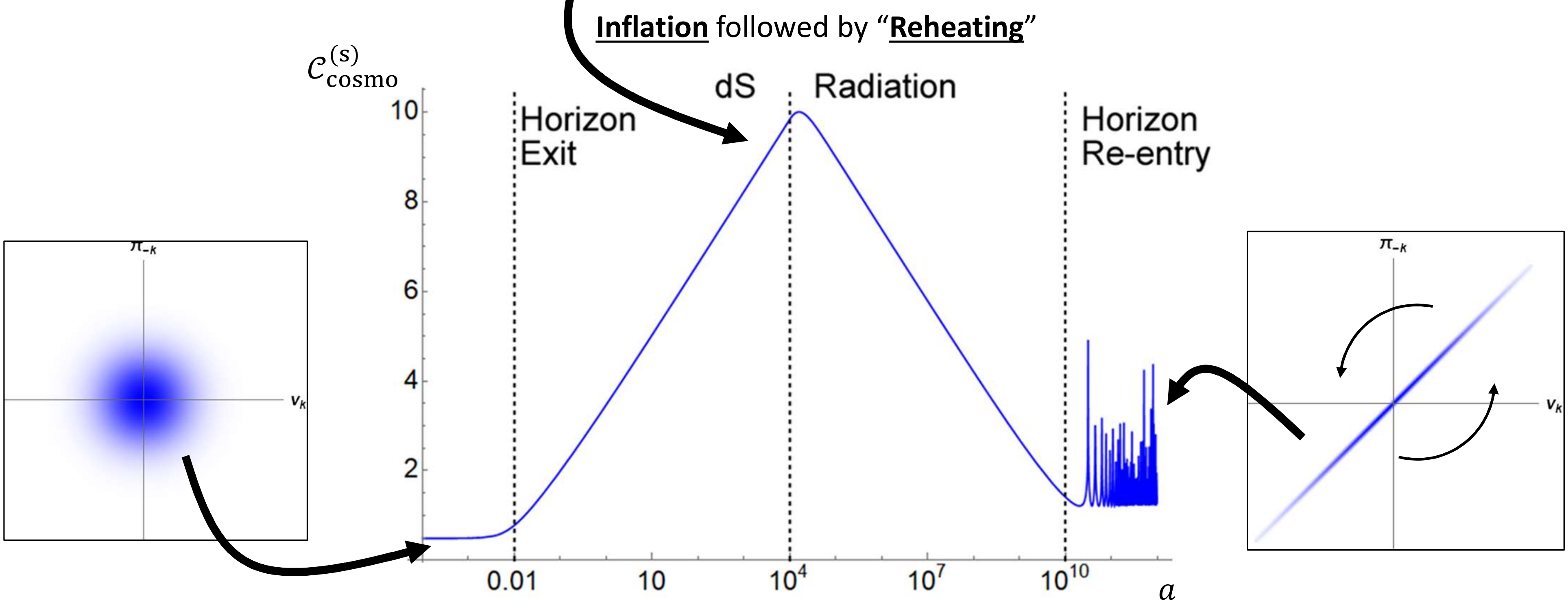
Cosmological Complexity

the complexity of quantum cosmological perturbations



"De-Complexification":

- Complexity **decreases** for radiation-dominated, then "freezes-in" upon horizon re-entry!
- Modes are still highly squeezed
- dS → radiation transition cuts off complexity growth



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Cosmological Complexity

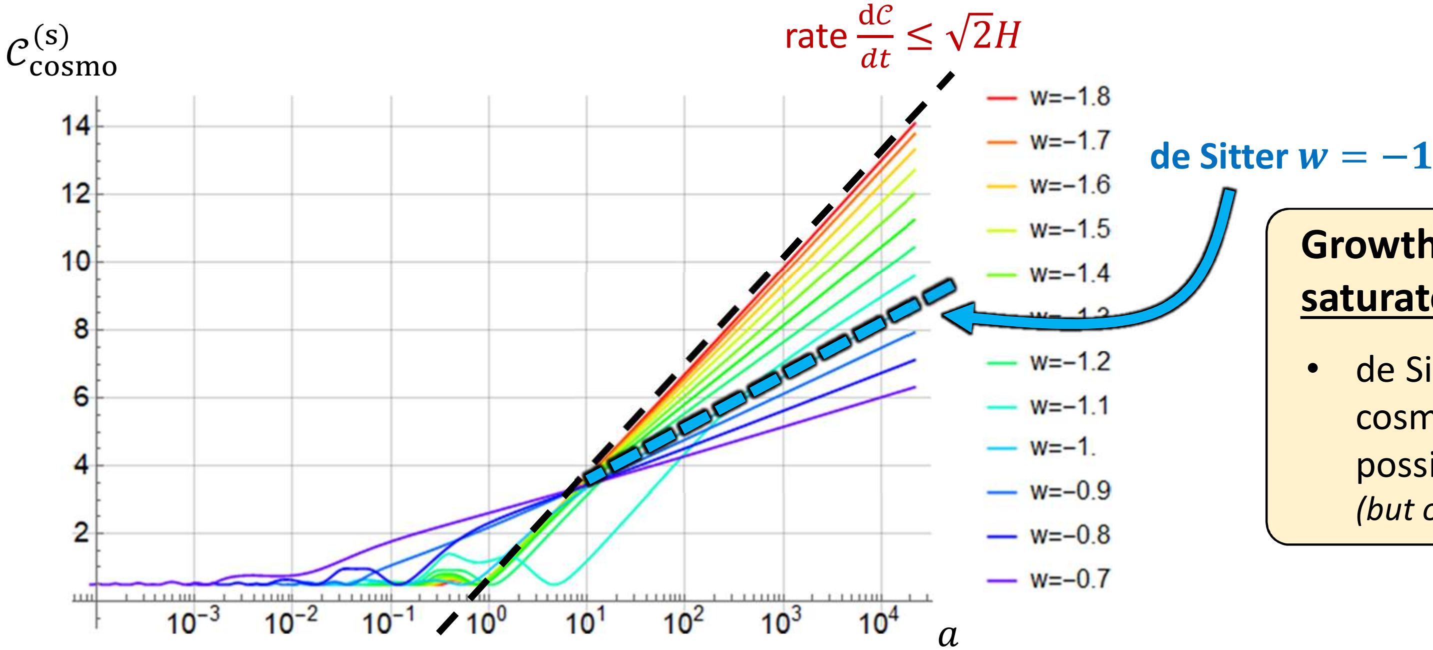
the complexity of quantum cosmological perturbations

Accelerating, Expanding Backgrounds

$$ds^2 = a(\eta)^2\,(-d\eta^2 + d\vec{x}^2) \qquad a(\eta) = \left(\frac{\eta_0}{\eta}\right)^{-2/(1+3w)}$$

Equation of state $p = w\rho$

Bound on growth rate $\frac{d\mathcal{C}}{dt} \leq \sqrt{2}H$



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Growth rate of complexity **saturates** at $w = -5/3$

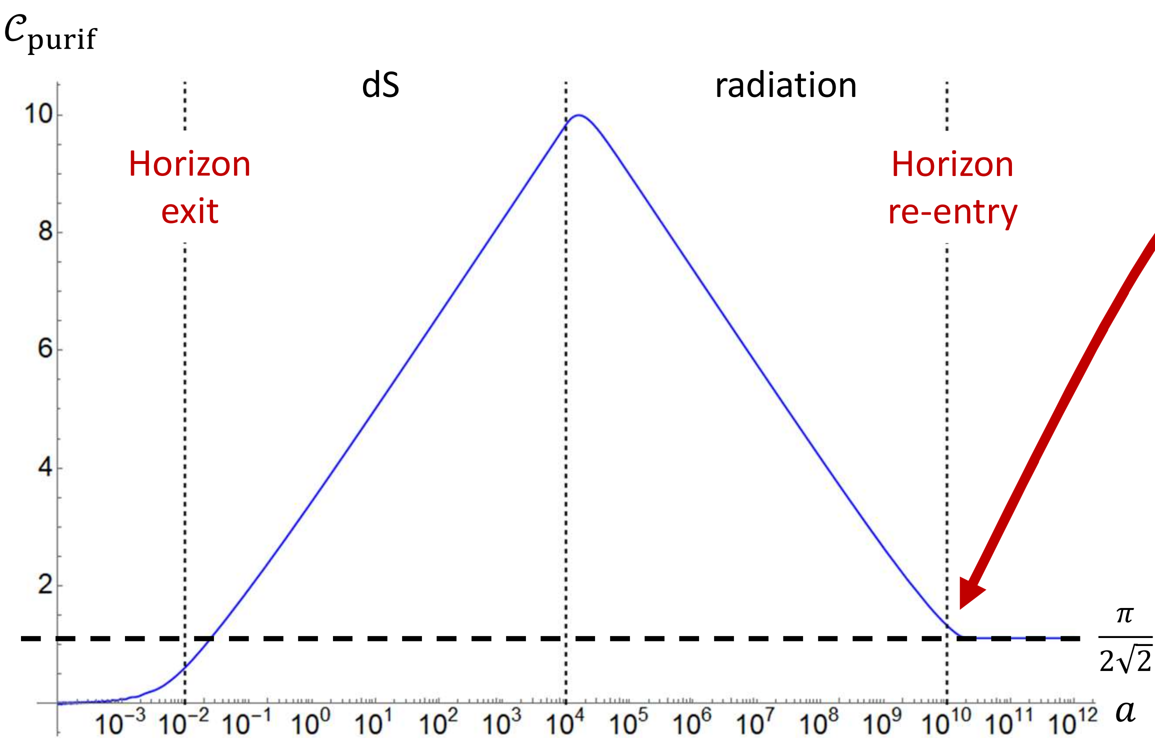
- de Sitter is not fastest growth in cosmological complexity among all possible accelerating backgrounds… *(but others violate NEC)*

Cosmological Complexity

Decoherence

Pure State → **Thermal Density Matrix**

$$\hat{\rho}_{\rm pure} = |r_k,\phi_k,\theta_k\rangle\langle r_k,\phi_k,\theta_k| \longrightarrow \hat{\rho}_{\rm red} = \frac{1}{\cosh^2 r_k}\sum_{n=0}^{\infty} \tanh^{2n} r_k\ |n_{\vec{k}}, n_{-\vec{k}}\rangle\langle n_{\vec{k}}, n_{-\vec{k}}|$$



<u>Complexity of Purification</u>

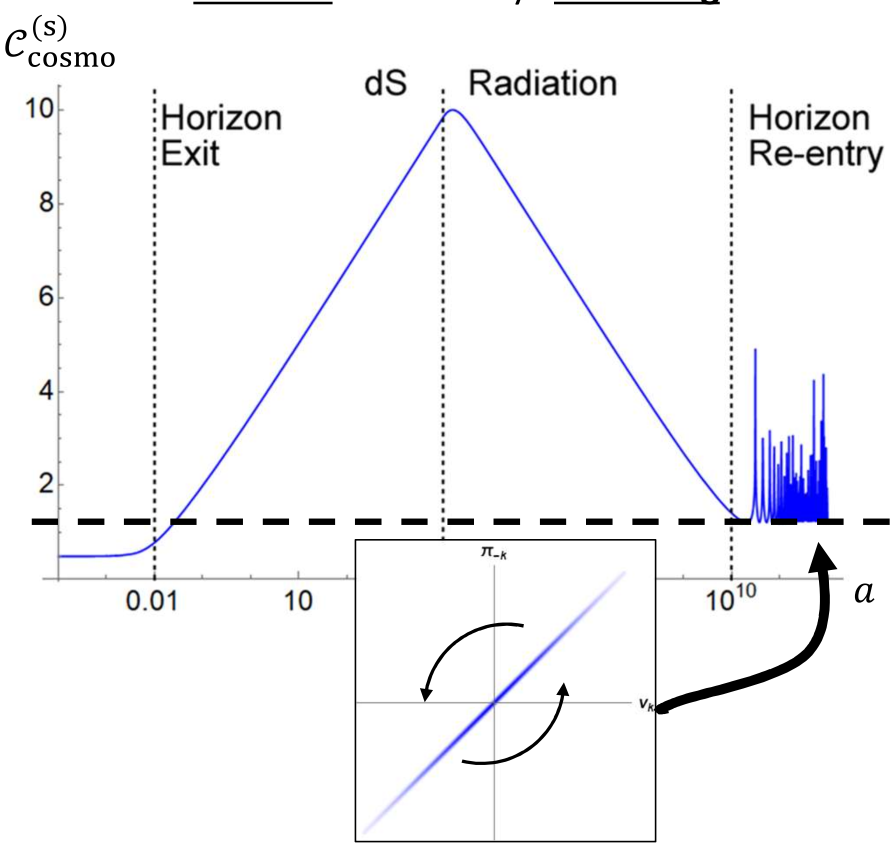
- Assume decoherence occurs at re-entry
- Purification with ancillary dof
  $\mathcal{H} \to \mathcal{H} \otimes \mathcal{H}_{\rm anc}$
- Minimize complexity over purification
  $\mathcal{C}_{\rm purif} = \min_{\{\rm anc\}} \mathcal{C}_{\rm tot}$
- Complexity of purification $\mathcal{O}(1)$
  $\mathcal{C}_{\rm purif} \approx \frac{\pi}{2\sqrt{2}}$

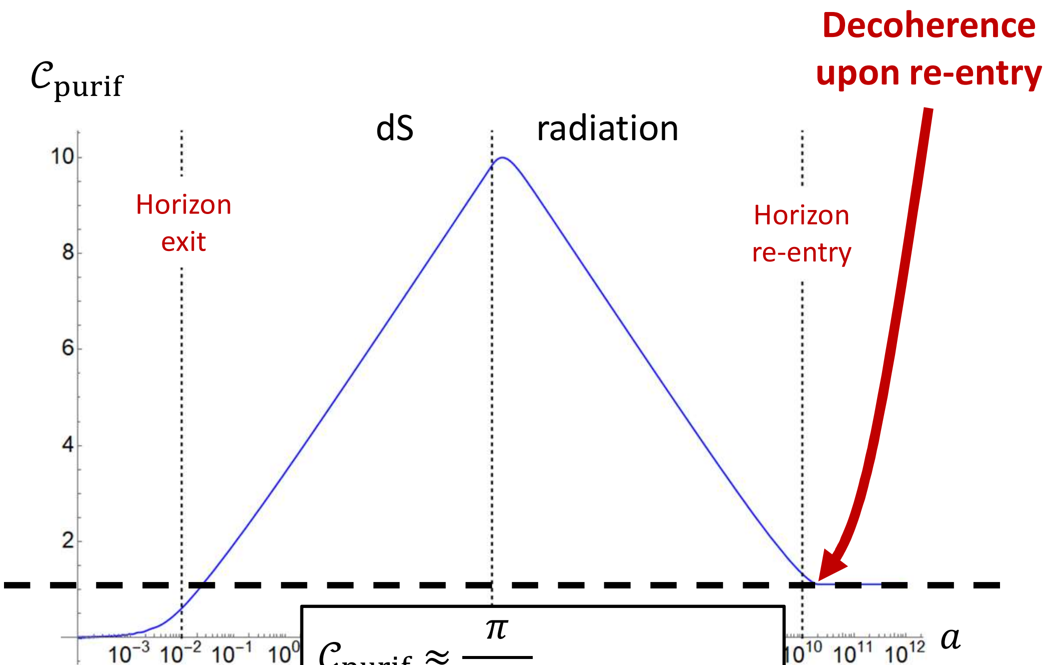
Haque, Jana, **BU**

Cosmological Complexity

Decomplexification vs Decoherence

Inflation followed by "**Reheating**"





$$\mathcal{C}_{\rm purif} \approx \frac{\pi}{2\sqrt{2}}$$

Complexity of Purification

Rapidly changing squeezing angle $\approx$ Complexity of decohered mixed state

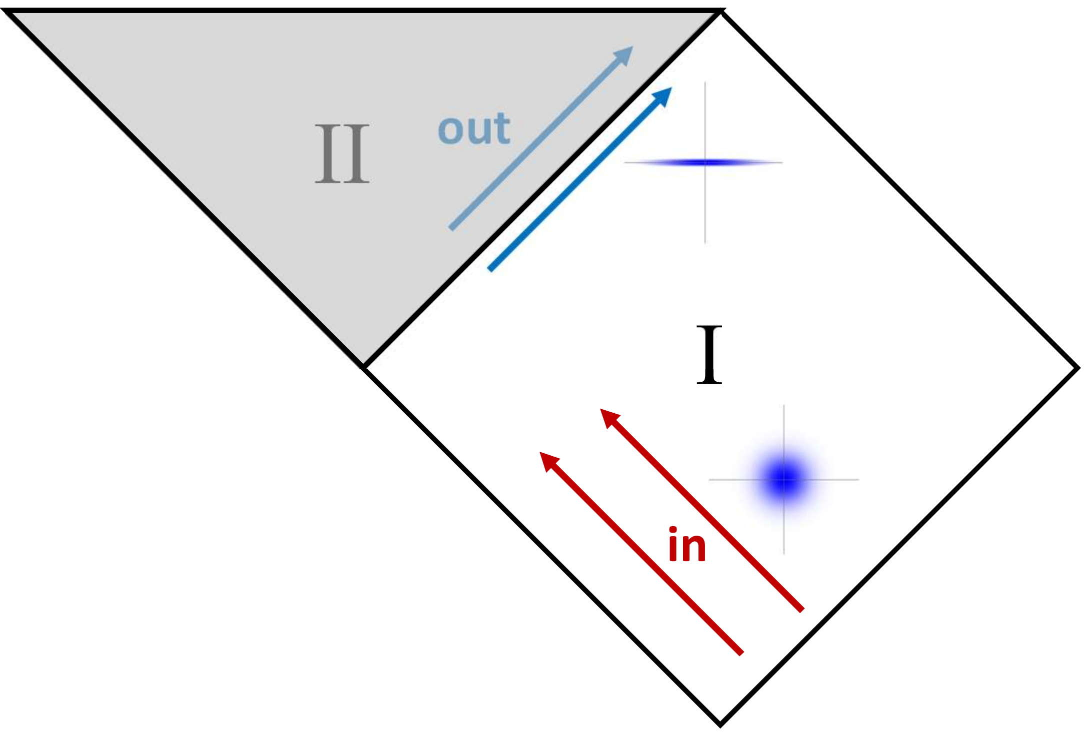
Aside: Complexity of Hawking Radiation

Hawking radiation: two-mode squeezed states

$$|0\rangle_{\rm in} = \mathcal{N}_k \sum_{n_k} e^{-(4\pi GMk)\, n_k}\ |n_k\rangle_{\rm I} \otimes |n_k\rangle_{\rm II}$$

$$\tanh r_k = e^{-4\pi GM} \qquad \phi_k = \pi/2$$

$$\begin{cases} r_k \ll 1 & k \gg (GM)^{-1} \ \ \text{high freq} \\ r_k \gg 1 & k \ll (GM)^{-1} \ \ \text{low freq} \end{cases}$$



- Complexity of Hawking Radiation

$$\mathcal{C}_{\rm Hawk}(k) = \frac{1}{\sqrt{2}} \ln\left(\frac{1+e^{-4\pi GMk}}{1-e^{-4\pi GM}}\right)$$

$$\approx \begin{cases} 0 & k \gg (GM)^{-1} \ \ \text{high freq} \\ \ln\left(\frac{1}{GMk}\right) & k \ll (GM)^{-1} \ \ \text{low freq} \end{cases}$$

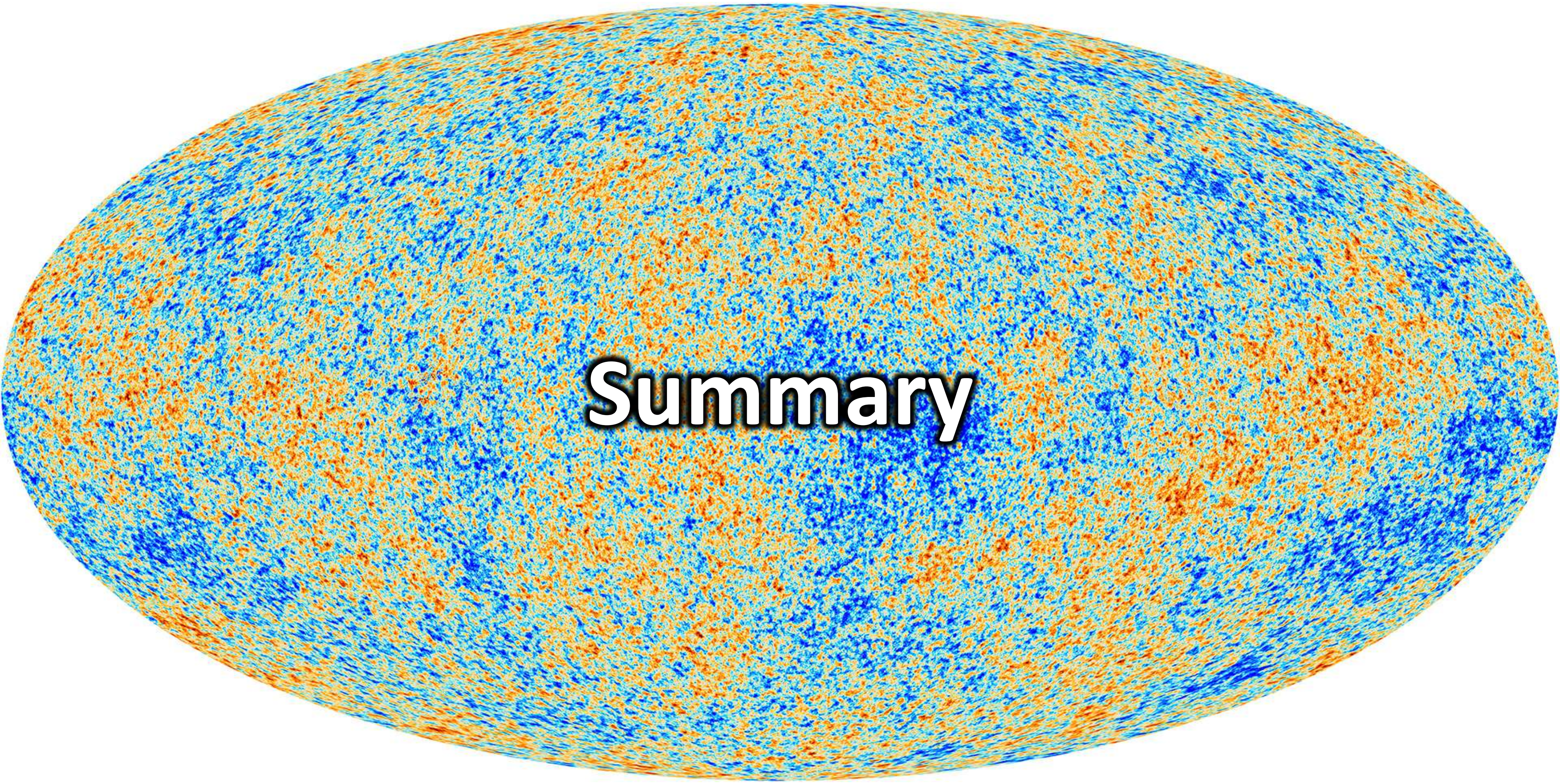
❑ Complexity of Hawking radiation is maximal w.r.t. ϕ Why?

- Tracing out modes inside horizon
  - Thermal density matrix
  - Complexity of purification

$$\mathcal{C}_{\rm purif}(k) \approx \frac{\pi}{2\sqrt{2}}$$

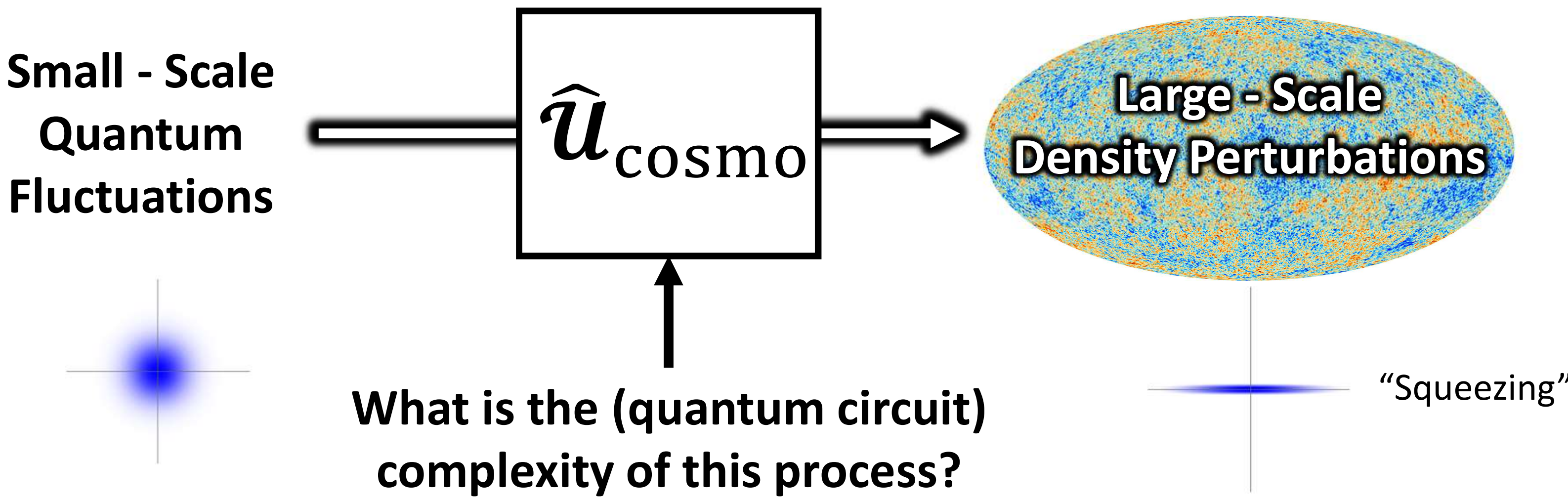
❑ Complexity of reduced state smaller than pure state

Summary



ESA and Planck Collaboration

Summary



What is the (quantum circuit) complexity of this process?

- ❑ Cosmological Complexity in dS grows linearly with time $\mathcal{C}_{\text{cosmo}} = H_{dS}\, t$
- ❑ Complexity depends sensitively on squeezing angle ϕ
  - Complexity of dS is **maximal** w.r.t. ϕ. Why?
- ❑ Growth rate of complexity $\frac{d\mathcal{C}}{dt}$ is bounded from above for accelerating backgrounds
- ❑ ***Decomplexification*** during radiation-domination phase
  - Connection between decomplexification and decoherence?