Harmonic Analysis and Convexity

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1 Overview of the Field

Convexity is a very old topic which can be traced at very least to Archimedes. These days the area is especially active due to its numerous applications to the linear programming, tomography, medicine, information theory, to name a few. For the last twenty five years Harmonic Analysis has been the major tool for solving the most challenging open problems in Convex Geometry. Since convexity is a very natural notion and an excellent choice for graduate students and postdocs, the workshop brought together a number of top and junior researchers with the aim of discussing most recent developments in the areas.

The topics of the workshop included harmonic analysis in $\mathbb{R}^n$ and on the sphere, spherical operators and special classes of bodies, geometric inequalities, discrete and differential geometry, topology, probability and random matrices.

2 Presentation Highlights

We start our highlights with the results of *Hermann König*. His talk was about *non-central sections of the ℓ_1^n-ball and the regular simplex*. Let $n \in \mathbb{N}$, $n \geq 2$, let S^{n-1} be a unit sphere in $\mathbb{R}^n$ and let K be a convex body in $\mathbb{R}^n$. Denote by $A(a,t) = A_K(a,t)$ its t-section function,

$$A(a,t) = \mathrm{vol}_{n-1}(K \cap (a^\perp + ta)), \qquad t \in \mathbb{R}, \quad a \in S^{n-1}$$

where $a^\perp$ is the hyperplane passing through the origin and orthogonal to a unit vector $a \in S^{n-1}$. Given K and $t \in \mathbb{R}$, how to find $a_{max}, a_{min} \in S^{n-1}$ so that

$$A(a_{min}, t) \leq A(a,t) \leq A(a_{max}, t) \qquad \forall a \in S^{n-1} \ ?$$

For $k \in \mathbb{N} \cap [1,n]$ let $a^{(k)} = \frac{1}{\sqrt{k}}(1, \ldots, 1, 0, \ldots, 0)$, where the coordinate 1 is taken k times. Then, for the unit cube $K = \frac{1}{2}B_\infty^n = [-\frac{1}{2}, \frac{1}{2}]^n$ and $t = 0$ one has the classical results that

$$1 = A(a^{(1)}, 0) \leq A(a,0) \leq A(a^{(2)}, 0) = \sqrt{2}.$$

also, if $p > 0$ and $K = B_p^n = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : \sum_{j=1}^{n} |x_j|^p \le 1\}$, then it is known that

$$A(a^{(n)}, 0) \le A(a, 0) \le A(a^{(1)}, 0), \qquad 0 < p < 2.$$

What about non-central sections of the bodies, i.e., what if $t \neq 0$? If $K = \frac{1}{2}B_\infty^n$, $n \ge 3$, and $t \in (\frac{\sqrt{n-1}}{2}, \frac{\sqrt{n}}{2}]$, then $A(a,t) \le A(a^{(n)}, t)$ $\forall a \in S^{n-1}$, and the same result holds for $n \ge 5$ and $t \in (\frac{\sqrt{n-2}}{2}, \frac{\sqrt{n-1}}{2}]$. However, for $K = B_1^n$, $t \in (\frac{1}{\sqrt{2}}, 1]$, one has

$$A(a,t) \le A(a^{(1)}, t) = \frac{2^{n-1}}{(n-1)!}(1-t)^{n-1} \qquad \forall a \in S^{n-1}.$$

Also, for $n \ge 4$ and $t \in (\frac{1}{\sqrt{3}}, 1]$, for $n = 3$ and $t \in (\sqrt{2} - 1 - \sqrt{5 - \frac{7}{\sqrt{2}}}, 1]$, and for $n = 2$, $t \in (\frac{3}{4}, 1]$, it is shown that the same result holds. Moreover, the explicit formula for the t-sections of $K = B_1^n$, $1 > a_1 > a_2 > \cdots > a_n > 0$, is given,

$$A(a,t) = \frac{2^{n-1}}{(n-1)!} \sum_{j=1}^{n} \frac{a_j^{n-2}(a_j - t)_+^{n-1}}{\prod\limits_{k=1, k\neq j}^{n} (a_j^2 - a_k^2)}.$$

If

$$\Delta^n = \{x \in \mathbb{R}_+^{n+1} : \sum_{j=1}^{n+1} x_j = 1\}$$

is the n-dimensional simplex of side-length$\sqrt{2}$, then its centroid is $c = \frac{1}{n+1}(1, \dots, 1)$. Assuming that the hyperplane $x \cdot a = 0$ passes through c, one has

$$A(a, 0) \le A(\bar{a}, 0) = \frac{\sqrt{n+1}}{(n-1)!} \frac{1}{\sqrt{2}} \qquad \forall a \in S^{n-1} \; : \quad \sum_{j=1}^{n+1} a_j = 0,$$

where $\bar{a} = \frac{1}{\sqrt{2}}(1, -1, 0, \dots, 0)$. Several other results that allow to determine the maximal and minimal sections of the simplex were given for some $t \neq 0$.

Carsten Schütt presented results related to measuring *the Banach-Mazur distance* $d(l_r^{n^2}, l_p^n \otimes_\varepsilon l_q^n)$ *between the spaces* $l_r^{n^2}$ *and* $l_p^n \otimes_\varepsilon l_q^n$ in the cases $r = 1, 2$ and $1 \le p, q \le \infty$. Here

$$d(X, Y) = \inf\{\|T\| \|T^{-1}\| : T : X \to Y\},$$

where T is an isomorphism between the Banach spaces X and Y, the norm of the vector (the matrix) $A_{n\times n}$ in $l_p^n \otimes_\varepsilon l_q^n$ is defined as

$$\|A\|_{l_p^n \otimes_\varepsilon l_q^n} = \|A\|_{L(l_{p^*}^n, l_q^n)} = \sup_{\|x\|_{p^*}=1, \, \|y\|_q = 1} \sum_{i,j=1}^{n} A_{ij} x_i y_j,$$

and p^* is defined via $\frac{1}{p^*} + \frac{1}{p} = 1$. It is shown that

$$d(l_2^{n^2}, l_p^n \otimes_\varepsilon l_q^n) = \begin{cases} n^{\frac{1}{p} + \frac{1}{q} - 1}, & \frac{3}{2} \le \frac{1}{p} + \frac{1}{q} \\ \sqrt{n}, & 1 \le p, q \le 2 \quad \text{and} \quad \frac{3}{2} \ge \frac{1}{p} + \frac{1}{q} \\ n^{\frac{1}{p^*}}, & 1 \le q \le 2 \le p \le q^* \\ n^{\frac{1}{q}}, & 1 \le q \le q^* \le p \\ n^{\frac{1}{q^*}}, & 2 \le q \le p, \end{cases}$$

and

$$d(l_1^{n^2}, l_p^n \otimes_\varepsilon l_q^n) = \begin{cases} n^{\frac{5}{2} - \frac{1}{p} - \frac{1}{q}}, & \frac{3}{2} \le \frac{1}{p} + \frac{1}{q} \\ n, & 2 \le p, q. \end{cases}$$

Tomasz Tkocz gave a talk titled *Hardwired... to Szarek and Ball* about his joint results with Alexandros Eskenazis and Piotr Nayar. Let $\{\varepsilon_k\}_{k=1}^{\infty}$ be a sequence of i.i.d random variables which are $Unif(\{-1,1\})$, and let $\{\xi_k\}_{k=1}^{\infty}$ be a sequence of i.i.d random variables which are $Unif(S^{n-1})$. In 1976 Szarek proved that

$$\mathbb{E}\Big|\sum_{j=1}^{n} a_j\varepsilon_j\Big| \geq \mathbb{E}\Big|\frac{\varepsilon_1+\varepsilon_2}{\sqrt{2}}\Big|.$$

On the other hand, in 1986 K. Ball showed that

$$\mathbb{E}\Big|\sum_{j=1}^{n} a_j\xi_j\Big|^{-1} \leq \mathbb{E}\Big|\frac{\xi_1+\xi_2}{\sqrt{2}}\Big|^{-1};$$

both results hold for all $a=(a_1,\ldots,a_n)\in S^{n-1}$ and all $n\geq 1$. Such inequalities were also known only for some other distributions such as $Unif(S^d)$, $Unif(B_2^d)$, GMs and the marginals of l_p-balls. The main result is that the above estimates hold for essentially all distributions sufficiently close to ε/ξ, provided $\|a\|_\infty \leq \frac{1}{\sqrt{2}}$.

Dylan Langharst delivered a lecture *On the measures satisfying a monotonicity of the surface area with respect to Minkowski sum* on his joint results with M. Fradelizi, M. Madiman and A. Zvavitch. One of the theorems of Fradelizi, Madiman and Zvavitch says that

$$\mathrm{vol}_n(A)+\mathrm{vol}_n(A+B+C)\geq \mathrm{vol}_n(A+B)+\mathrm{vol}_n(A+C).$$

Does a similar result hold for other measures? On a similar note, recently, G. Saracco and G. Stefani proved that if μ on $\mathbb{R}^n$ is absolutely continuous with respect to the Lebesgue measure, and for any two convex bodies one has $\mu^+(\partial K)\leq \partial^+(\partial L)$, then μ must be a constant multiple of the Lebesgue measure. Here

$$\mu^+(\partial K)=\lim_{\varepsilon\to 0}\frac{\mu(K+\varepsilon B_2^n)-\mu(K)}{\varepsilon}.$$

The main result is that if μ is absolutely continuous with respect to the Lebesgue measure such that for any convex bodies K and L one has $\mu^+(\partial(K+L))\leq \partial^+(\partial K)$, then μ is a constant multiple of the Lebesgue measure.

Bartłomiej Zawalski solved a problem of Louis Montejano by showing that *the star-convex bodies with rotationally invariant sections* are the bodies of revolution. One of the most famous questions in Banach space theory belongs to Banach himself and asks the following. Let B^n be a Banach space of finite dimension n and let $k\in\mathbb{N}$ be such that $1<k<n$. If all the k-dimensional subspaces of B^n are isometrically isomorphic to each other, is B^n a Hilbert space? In the 30's of the last century H. Auerbach, S. Mazur and S. Ulam solved the case $n=3$. At the end of 1960's M. Gromov settled it for odd n. More recently, J. Bracho and L. Montejano obtained several results on a complex version of Problem 1, S. Ivanov, D. Mamaev and A. Nordskova solved the case $n=4$, and G. Bor, L. Hernández-Lamoneda, V. Jiménez de Santiago and L. Montejano solved the question $n=4k+2\geq 6$, $n\neq 134$. One of the key elements of the proof of the last authors was to show that the hyperplane sections of the unit ball of B^n must be the body of revolution, which prompted the authors to ask the following. Let $K\subset\mathbb{R}^n$, $n\geq 4$, be a convex body containing the origin in its interior. If every hyperplane section of K passing through this point is a body of affine revolution, is K necessarily a body of affine revolution? The main result of Zawalski gives an affirmative answer to this question, provided the boundary of K is C^3 and K is origin-symmetric.

Maud Szusterman presented her results on *Vector balancing and lattice coverings: inequalities via the Gaussian measure*. Let U,V be origin-symmetric convex bodies in $\mathbb{R}^n$ and let

$$\beta(U,V)=\inf\{\beta>0:\ \forall u_1,u_2,\ldots,u_n\in U, \exists\varepsilon\in\{\pm1\}^n:\ \sum_{i=1}^{n}\varepsilon_i u_i\in\beta V\}=$$

$$=\max_{u_1,\ldots,u_n\in U}\ \min_{\varepsilon\in\{\pm1\}^n}\|\sum_{i=1}^{n}\varepsilon_i u_i\|_V.$$

It is known that $\beta(B_2^n, B_2^n) = \sqrt{n}$, $\beta(B_\infty^n, B_2^n) = c_n n$ (with $c_n = 1$ if Hadamard matrices exist in $\mathbb{R}^n$), $\beta(B_1^n, B_\infty^n) \le 2$, $\beta(B_\infty^n, B_\infty^n) \le c\sqrt{n}$. Komlos conjecture asks if $\beta(B_2^n, B_\infty^n) = O_n(1)$. The best result related to this conjecture belongs to Banaszczyk, who proved that $\beta(B_2^n, B_\infty^n) \le 5\sqrt{2}\sqrt{\log n}$. It is also known $\beta(B_2^n, V) \le 5$ if $\gamma_n(V) \ge \frac{1}{2}$, where γ_n is a Gaussian measure on $\mathbb{R}^n$. Let

$$\mu(L, V) = \inf\{t > 0 : \, L + tV = \mathbb{R}^n\}$$

be the covering radius of the lattice L with respect to V (if $L = \mathbb{Z}^n$, $V = B_2^n$, then $\mu(\mathbb{Z}^n, B_2^n)) = \frac{\sqrt{n}}{2}$) and let

$$\lambda_k(L, V) = \inf\{t > 0 : \, \dim(\text{span})(tV \cap L) \ge k\}.$$

If

$$\alpha(U, V) = \sup_L \frac{\mu(L, V)}{\lambda_n(L, V)} = \sup_{L, \lambda_n(L,V)=1} \mu(L, V),$$

then $\alpha(B_2^n, V) \le \frac{1}{\psi^{-1}(\frac{1}{2})}$ for any convex body V satisfying $\gamma_n(V) \ge \frac{1}{2}$. Here

$$\psi(x) = 2\Phi(x) - 1, \qquad \Phi(x) = \gamma_1((-\infty, x]).$$

It is also shown that for any convex body $V \subset \mathbb{R}^n$ one has $\alpha(B_2^n, V) \le \frac{1}{\psi^{-1}(\gamma_n(V))}$.

Oscar Adrian Ortega Moreno spoke about *The complex plank problem, revisited.* Balls complex plank theorem states that if $v_1, \ldots, v_n$ are unit vectors in $\mathbb{C}^d$, and $t_1, \ldots, t_n$, non-negative numbers satisfying $\sum_{k=1}^n t_k^2 = 1$, then there exists a unit vector v in $\mathbb{C}^d$ for which $|\langle v_k, v\rangle| \ge t_k$ for every k. Oscar presented an elegant version of Balls original proof. He started with the case when all t_k's are $\frac{1}{\sqrt{n}}$. In this case, he shows that if $v_1, \ldots, v_n$ are unit vectors in $\mathbb{C}^d$ and u maximizes $\prod_{k=1}^n |\langle v_k, u\rangle|$ among unit vectors, then $|\langle v_k, u\rangle| \ge \frac{1}{\sqrt{n}}$ for every k. For general positive t_k's satisfying $\sum_{k=1}^n t_k^2 = 1$ and unit vectors $v_1, \ldots, v_n$ in $\mathbb{C}^d$ one proves that the vector u maximizes $\prod_{k=1}^n |\langle v_k, u\rangle|^{t_k^2}$ among unit vectors, then $u = \sum_{k=1}^n \frac{t_k^2}{\langle v_k, u\rangle} v_k$. Then the proof is reduced to the previous case, provided one analyzes the function $p(z) = \prod_{k=1}^n \left|\frac{\langle v_k, zv_1 + u\rangle}{\langle v_k, u\rangle}\right|^{t_k^2}$ applying the maximum principle (the function $\log|p(z)|$ is subharmonic).

Mark Rudelson delivered a talk *Approximately Hadamard matrices and random frames* about his joint results with his student *Xiaoyu Dong*. Let A be an $N \times n$ matrix with i.i.d. symmetric non-degenerate entries and let A_I be its submatrix such that its columns belong to a subset $I \subset \{1, 2, \ldots, N\}$. Then there exist constants c, C, α, β depending on the distribution of entries of A with the following property: if $N \ge e^{Cn}$, then there exists $L \ge e^{cn}$ such that the probability of the existence of disjoint subsets $I_1, \ldots I_L$ of $\{1, \ldots, N\}$ with $|I_j| = n$ and $\frac{s_{max}(A_{I_j})}{s_{min}(A_{I_j})} < \alpha \; \forall j \in \{1, \ldots, L\}$, is greater or equal to $1 - e^{-e^{\beta n}}$. Here $s_j(A) = \sqrt{\lambda_j(AA^T)}$, λ_j are the eigenvalues of AA^T, and s_{max}, s_{min} are the maximal and minimal singular values among s_j's. It is also shown that there exist constants $0 < c < C$ such that for any $n \in \mathbb{N}$ one can find an $n \times n$ matrix V with ± 1 entries satisfying $c\sqrt{n} \le s_{min}(V) \le s_{max}(V) \le C\sqrt{n}$. Finally, it is proved that finding a sub-matrix with a bounded condition number requires an exponential number of columns for matrices with sub-gaussian entries.

Alexander Litvak brought up a discussion about volume ratio of convex bodies and spoke on *volume ratio between projections of convex bodies*. The volume ratio of two convex bodies $K, L \subset \mathbb{R}^n$ containing the origin in their interior is defined as $vr(K, L) = \inf(\frac{|K|}{|TK|})^{\frac{1}{n}}$, where the infimum is taken over all affine maps $T : \mathbb{R}^n \to \mathbb{R}^n$ and $|K|$ stands for the n-dimensional volume of K. What is the maximal possible volume ratio? Giannopoulus and Hartzoulaki proved that $vr(K, L) \le c\sqrt{n}\log n$, while Khrabrov showed that there are bodies K and L such that $vr(K, L) \ge c\sqrt{\frac{n}{\log\log n}}$. One sees that there is a log gap between these results. Another well-known measurement of the distance between bodies is the Banach-Mazur distance $d(K, L)$ mentioned in the first talk,

$$d(K, L) = \inf\{\lambda > 0 : \, K - x \subset T(L - y) \subset \lambda(K - x), \quad x, y \in \mathbb{R}^n\}$$

where $T : \mathbb{R}^n \to \mathbb{R}^n$ is linear. If K and L are origin-symmetric, i.e., $K = -K$ and $L = -L$, then Gluskin proved that $d(K, L) \geq cn$, while Rudelson showed that $d(K, L) \leq cn^{\frac{4}{3}} \log^9 n$. It is open if Rudelson's result can be improved to $n \log^\alpha n$ for some $\alpha > 0$. Another Rudelson's result states that for origin-symmetric convex bodies K and L one has $\delta_k(K, L) \leq C \max\{\frac{k^2}{n}, \sqrt{k} \log n\}$, where $\delta_k(K, L) = \inf\{d(PK, QL) : rank(P) = rank(Q) = k\}$, $1 \leq k \leq n$, and the infimum runs over all projections P, Q of rank k. Here the threshold is $k = n^{\frac{2}{3}}$ (up to logs). The main result is that for any K and $k \geq n$ there exists $L = -L$ such that

$$\delta'_k(K, L) = \inf\{vr(PK, QL) : rank(P) = rank(Q) = k\} \geq \frac{ck}{\sqrt{n \log n}}.$$

Moreover, if $k \geq n^{\frac{2}{3}}$, then $\delta'_k(K, L) \leq \frac{ck}{\sqrt{n}}$ and this result (sharp for $K = B_1^n$) holds for any L.

Piotr Nayar presented several joint results with J. Melbourne and C. Roberto on *Minimum entropy of a log-concave random variable for fixed variance*. Let $Var(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2$, $h(X) = h(f) = -\int f \log f$, $h_\alpha(X) = h_\alpha(f) = \frac{1}{1-\alpha} log\Big(\int f^\alpha\Big)$, where $\alpha \in (0, +\infty) \setminus \{1\}$. It is proved that for $\alpha \in (\frac{1}{3}, +\infty) \setminus \{1\}$, $h_\alpha(f)$ is maximal under fixed variance for $f(x) = c_0(1 + (1-\alpha)(c_1 x)^2)_+^{\frac{1}{\alpha-1}}$. For $\alpha = 1$, $h(X) \leq \frac{1}{2} \log Var(X) + \frac{1}{2} \log(2\pi e)$. M. Bialobrzeski, M. Madiman, P. Nayar and M. Fradelizi showed that for a symmetric log-concave random variable and α^* solving $\frac{1}{1-\alpha} \log \alpha = \frac{1}{2} \log 6$ one has

$$h_\alpha(X) \geq \begin{cases} \frac{1}{2} \log Var(X) + \frac{1}{2} \log 12, & \alpha \leq \alpha^* \\ \frac{1}{2} \log Var(X) + \frac{1}{2} \log 2 + \frac{\log \alpha}{\alpha - 1}, & \alpha \geq \alpha^*. \end{cases}$$

J. Melbourne, P. Nayar and C. RobertoIt proved that for a log-concave real random variable one has $h(X) \geq \frac{1}{2} \log Var(X) + 1$ with equality for $f(x) = e^{-x} 1_{[0,+\infty)}$, and for $\alpha \geq 1$, $h_\alpha(X) \geq \frac{1}{2} \log Var(X) + \frac{\log \alpha}{\alpha - 1}$. The case $\alpha < 1$ is open.

Eli Putterman talked about *Small-ball probabilities for mean widths of random polytopes* and presented his joint work with J. Haddad, D. Langharst, M. Roysdon, and D. Ye. Given a convex body $K \subset \mathbb{R}^n$, define the l-th higher order projection body $\Pi^l K \subset \mathbb{R}^{nl}$ via its support function $h_{\Pi^l K}(\theta) = \max\limits_{\{x \in \Pi^l K\}} x \cdot \theta$, $\theta = (\theta_1, \ldots, \theta_l) \in \mathbb{R}^{nl}$, as

$$h_{\Pi^l K}(\theta_1, \ldots, \theta_l) = \int \max_{i=1,\ldots,n} u \cdot \theta_i \, dS_K(u),$$

where S_K is the surface area measure of K. The above expression is equal to $n\text{vol}_n(C_\theta, K[n-1])$, the multiple of the mixed volume of the simplex $C_\theta = \text{conv}(0, \theta_1, \ldots, \theta_l)$ and $n-1$ copies of K. One shows that for $\Pi^{0,l} K = (\Pi^l K)^*$, where K^* stands for the polar of K, one has

$$\text{vol}_n(K)^{(n-1)l} \text{vol}_{nl}(\Pi^{0,l} K) \leq \text{vol}_n(B_2^n)^{(n-1)l} \text{vol}_{nl}(\Pi^{0,l} B_2^n).$$

Eli poses the following problem. Let $(\theta_1, \ldots, \theta_l)$ be i.i.d. random vectors uniformly distributed on S^{nl-1} and let $W_\theta = W(C_\theta)$ be the mean width of C_θ. Compute $\mathbb{E}_\theta(W_\theta^{-nl})$. So far, it has been evaluated up to a constant, $\mathbb{E}_\theta(W_\theta^{-nl})^{\frac{1}{nl}} \approx \max(\sqrt{\frac{nl}{\log l}}, \sqrt{l})$, the transition between the quantities occurs when $l \sim c^n$.

Orli Herscovici presented her joint results with Galyna Livshyts, Liran Rotem and Alexander Volberg on *the stability and the equality cases in the Gaussian B-inequality*. They considered the B-inequality of D. Cordero-Erausquin, M. Fradelizi, and B. Maurey, staying that $\gamma(\sqrt{ab}K) \geq \sqrt{\gamma(aK)\gamma(bK)}$ for all $a, b > 0$ and any convex set $K = -K$, i.e., the function $t \to \gamma(e^t K)$ is log concave with respect to t. Here Γ is the Gaussian measure on $\mathbb{R}^n$. The stability result asserts that if $0 \leq a < b < \infty$, $K = -K$, convex and satisfying $\gamma(\sqrt{ab}K) \geq \sqrt{\gamma(aK)\gamma(bK)}(1+\varepsilon)$ for some $\varepsilon > 0$, then for the inradius $r(K)$ (the largest r such that $rB_2^n \subset K$) one has $r(K) \geq \frac{1}{b}\sqrt{\log \frac{c \log(\frac{b}{a})^2}{n^2 \varepsilon}}$ and $r(K) \leq \frac{c\sqrt{n}}{a} \varepsilon^{\frac{1}{n+1}} (\log \frac{b}{a})^{-\frac{2}{n+1}}$. It is shown that the lower bound is sharp, while the sharpness of the upper bound is open.

Michael Roysdon discussed his joint results with Alexander Koldobsky and Artem Zvavitch related to *Comparison problems for Radon Transforms*. Inspired by the Busemann-Petty problem in Convex Geometry, they examined similar tomography questions concerning estimates of the L_p-norms of even continuous functions given information about their Radon-type transforms. In particular, they studied comparison problems for the spherical and classical Radon transforms by introducing families of functions which extended the class of intersection bodies of star bodies due to Lutwak. Michael also discussed comparison problems for the $(n-k)$-dimensional Radon and spherical Radon transforms. One of the results is that for an even infinitely smooth positive function g on S^{n-1} and $p>1$ there exists an infinitely smooth f on S^{n-1} such that $\|f\|_{L^p(S^{n-1})} \geq \|g\|_{L^p(S^{n-1})}$, provided $g^{p-1}(\theta)\frac{1}{r}$ is not positive definite in $\mathbb{R}^n$ (here one puts $x = r\theta \in \mathbb{R}^n$). He explained why the L^p-analogue of the implication

$$\int\limits_{\theta^\perp + t\theta} f \leq \int\limits_{\theta^\perp + t\theta} g \quad \forall t \in \mathbb{R},\, \theta \in S^{n-1} \quad \Longrightarrow \quad \|f\|_{L^1(S^{n-1})} \geq \|g\|_{L^1(S^{n-1})}$$

(with the L^1-norm replaced by the L^p-norm) does not hold. He also gave a technical sufficient condition on the function g being a "section function of f" (a generalization of the notion of a convex body being the intersection body of a convex body) in terms of the Fourier transform of distributions.

Wen Rui Sun and his advisor *Beatrice-Helen Vritsiou* talked about *Illumination Conjecture for Convex Bodies with many Symmetries*. Suppose one wanted to illuminate a solid object with convex shape, that is, illuminate its surface, by placing a number of light sources around it. What is the smallest number of light sources one would need? This seemingly innocent question has actually turned into a longstanding conjecture in Convex and Discrete Geometry, called the Illumination Conjecture. The conjecture states that for an n-dimensional object, one should need less than $2n$ light sources, except if the object "looks like" a cube (which then needs $2n$). The result is known on the plane and for symmetric bodies in $\mathbb{R}^3$. Tikhomirov proved the result for 1-symmetric bodies (that is, convex bodies with the symmetries of the cube) in all sufficiently large dimensions. The main results show that the illumination conjecture is now verified (along with its equality cases) for 1-symmetric convex bodies in all dimensions and some cases of 1-unconditional convex bodies as well (that is, convex bodies with the symmetries of a rectangular box). In particular, let $B \subset \mathbb{R}^n$ be a 1-unconditional convex body with the boundary ∂B, and let

$$m_B = \max\{k \in \{1,\dots,n\} :\, e_1 + e_2 + \cdots + e_k \text{ or some permutation of it } \in \partial B\}.$$

If $m_B = n-1$ or $m_B = n-2$ and B contains all the permutations of $e_1 + e_2 + \cdots + e_{n-2}$, then B is not an image of the cube and number of the light sources does not exceed $2^n - 2$.

Grigoris Paouris presented his joint results with Kavita Ramanan on a *probabilistic approach to the geometry of p-Schatten balls*. Let $K \subset \mathbb{R}^n$ be a convex body and let X be an anisotropic random vector on $\tilde{K} = \frac{K}{\mathrm{vol}_n(K)^{\frac{1}{n}}}$, i.e., the correlation matrix of X is a multiple of identity. The vector X is called sub-Gaussian if $\langle X,\theta\rangle$ satisfies

$$\Big(\mathbb{E}(|\langle X,\theta\rangle|^q)\Big)^{\frac{1}{q}} \leq k\sqrt{q}\Big(\mathbb{E}(|\langle X,\theta\rangle|^2)\Big)^{\frac{1}{2}} \quad \forall q \geq 2,\, \forall \theta \in S^{n-1},$$

and it is called super-Gaussian if

$$\Big(\mathbb{E}(|\langle X,\theta\rangle|^q)\Big)^{\frac{1}{q}} \geq \frac{\sqrt{q}}{k}\Big(\mathbb{E}(|\langle X,\theta\rangle|^2)\Big)^{\frac{1}{2}} \quad \forall q \leq n,\, \forall \theta \in S^{n-1},$$

For example, $\tilde{B_p^n}$, $p \geq 2$ is sub-Gaussian and $\tilde{B_p^n}$, $p \in (1,2)$ is super-Gaussian. Let $M_{n\times l}$ be a space of matrices, $l \geq n \geq 2$ and let $\langle A,B\rangle_F = \mathrm{tr}(A^T B)$ be the Frobenius scalar product of $A, B \in M_{n,l}$. One says that A belongs to the Schatten class $S_p^{n,l}$ if $\|A\|_{S_p^{n,l}} = \|\Sigma_A\|_p < \infty$, where Σ_A is the finite set of singular numbers of A. If $B(S_p^{n,l})$ is a unit ball in $S_p^{n,l}$ and a random matrix W_p is uniformly distributed in $B(S_p^{n,l})$, one can compute sharp upper and lower bounds for the moments of marginals of this random matrix. If a matrix Γ has singular values $\gamma_1,\dots,\gamma_n$, then

$$\Big(\mathbb{E}|\langle W_p,\Gamma\rangle_F|^q\Big)^{\frac{1}{q}} \simeq \begin{cases} \sqrt{q}\|\Gamma\|_F \\ q^{\frac{1}{p}} l^{\frac{1}{2}-\frac{1}{p}} \Big(\sum\limits_{i=1}^{[\frac{q}{l}]} |\gamma_i^*|^{p'}\Big)^{\frac{1}{p'}} + \Big(\sum\limits_{[\frac{q}{l}]+1}^{n} |\gamma_i^*|^2\Big)^{\frac{1}{2}} & l \leq q \leq nl \\ \sqrt{l} n^{\frac{1}{p}} \|\gamma\|_{p'}, \qquad q \geq nl \end{cases}$$

Here $\|\gamma\|_{p'}$ stands for the $l^n_{p'}$-norm of the vector $(\gamma_1, \dots, \gamma_n)$, $\frac{1}{p'} + \frac{1}{p} = 1$, and $\gamma_1^* \geq \dots, \geq \gamma_n^*$ are the decreasing rearrangements of γ_i's. As a corollary one obtains, for example, that W_p is sub-Gaussian for $p \geq 2$, W_p is super-Gaussian, if $1 < p < 2$.

Galyna Livshyts delivered a lecture *Gaussian principle frequency and convexity* on her joint results with A. Colesanti, E. Francini and P. Salani. Let K be a convex domain. Its principal frequency $\kappa(K)$ is defined as

$$\kappa(K) = \inf_{u \in W^{1,2}(K),\, u|_{\partial K}=0} \frac{\int\limits_K |\nabla u|^2 dx}{\int\limits_K u^2 dx}.$$

Equivalently, $\kappa(K)$ is the smallest positive number such that there exists a non-zero $u \in W^{1,2}(K)$ satisfying $\Delta u = -\kappa(K) u$ on K, $u|_{\partial K} = 0$. G. Faber and E. Krahn proved that $\kappa(K) \leq \kappa(RB_2^n)$, provided $\text{vol}_n(K) = \text{vol}_n(RB_2^n)$. H. Brascamp and E. Lieb showed that $\kappa(tK_0 + (1-t)K_1)^{-\frac{1}{2}} \leq t\kappa(K_0)^{-\frac{1}{2}} + (1-t)\kappa(K_1)^{-\frac{1}{2}}$, provided K_0, K_1 are convex bodies $\mathbb{R}^n$ and $t \in [0,1]$. What are the analogues of these results for $\kappa_\mu(K)$ instead of $\kappa(K)$,

$$\kappa_\mu(K) = \inf_{u \in W^{1,2}(K),\, u|_{\partial K}=0} \frac{\int\limits_K |\nabla u|^2 d\mu}{\int\limits_K u^2 d\mu},$$

where $d\mu(x) = e^{-v(x)} dc$, v is convex on $\mathbb{R}^n$? One can show that $\kappa_\mu(K)$ is the smallest positive number κ_μ such that there exists a non-zero $u \in W^{1,2}(K)$ satisfying $\Delta u - \langle \nabla u, \nabla v \rangle = -\kappa(K) u$ on K, $u|_{\partial K} = 0$. The main result is the following. Let γ be the Gaussian measure on $\mathbb{R}^n$ and let $K \subset \mathbb{R}^n$ be a convex set. If κ_γ is the smallest non-trivial u such that $\Delta u - \langle \nabla u, \nabla v \rangle = -\kappa_\gamma(K) u$ on K, $u|_{\partial K} = 0$, then u is log-concave. It is shown also that $\kappa_\gamma(tK_0 + (1-t)K_1) \leq t\kappa_\gamma(K_0) + (1-t)\kappa(K_1)$.

Julián Haddad presented several results related to *Fiber symmetrization and the Rogers-Brascamp-Lieb-Luttinger inequality*. Given a positive concave function f, $f(x) = \int\limits_0^\infty \chi_{\{f \geq t\}}(x) dt$, its Steiner symmetrization $f^{(v)}$ in the direction v is performed on the level sets and is defined as $f^{(v)}(x) = \int\limits_0^\infty \chi_{S_v\{f \geq t\}}(x) dt$. In 2016 G. Paouris and P. Pivovarov proved that if positive $F_i : \mathbb{R}^{nd} \to \mathbb{R}$ are concave, $1 \leq i \leq k_2$, $f_1, \dots, f_{k_1}$ be non-negative integrable functions on $\mathbb{R}^n$, $a_j^{(i)}$ are real numbers, $j = 1, \dots, d$, $i = 1, \dots, k_1$, then

$$\int\limits_{\mathbb{R}^n} \cdots \int\limits_{\mathbb{R}^n} \prod_{i=1}^{k_2} F_i(x_1, \dots, x_d) \prod_{i=1}^{k_1} f_i\Big(\sum_{j=1}^d a_j^{(i)} x_j\Big) d\mu(x_1) \dots d\mu(x_d) \leq$$

$$\int\limits_{\mathbb{R}^n} \cdots \int\limits_{\mathbb{R}^n} \prod_{i=1}^{k_2} F_i(x_1, \dots, x_d) \prod_{i=1}^{k_1} f_i^{(v)}\Big(\sum_{j=1}^d a_j^{(i)} x_j\Big) d\mu(x_1) \dots d\mu(x_d),$$

provided $\mu \geq 0$ is an absolutely continuous measure on $\mathbb{R}^n$ (with respect to the Lebesgue measure) which is rotationally invariant and with density having convex level sets. Julián explained that inspired by Rogers-Brascamp-Lieb-Luttinger inequality and the above result, he obtained a matrix analogue of several known inequalities. In particular, if $M_{d\times m}(\mathbb{R})$ stands for the space of matrices, $L_i \in M_{d\times m}(\mathbb{R})$, $f_i : M_{n\times m}(\mathbb{R}) \to \mathbb{R}$, $i = 1, \dots, k$, then

$$\int\limits_{M_{n\times d}(\mathbb{R})} \prod_{i=1}^k f_i(xL_i) dx \leq \int\limits_{M_{n\times d}(\mathbb{R})} \prod_{i=1}^k f_i^{(v)}(xL_i) dx,$$

where the "matrix Steiner symmetrization" is properly defined on f_i.

Elisabeth Werner gave a talk *Approximation of convex bodies in Hausdorff distance by random polytopes* about her joint work with J. Prochno, C. Schuett and M. Sonnleitner. While there is extensive literature on approximation, deterministic as well as random, of general convex bodies in the symmetric difference metric, or other metrics coming from intrinsic volumes, very little is known for corresponding random results in the Hausdorff distance. For a polygon Q in the plane, the convex hull of n points chosen at random on the boundary of Q gives a random polygon Q_n. They determine the exact limiting behavior of the expected

Hausdorff distance between Q and a random polygon Q_n as the number n of points chosen on the boundary of Q goes to infinity. More precisely, if the boundary ∂K of a convex body K is smooth, then

$$\lim_{n\to\infty} n^{\frac{2}{n-1}} \int\limits_{\partial K} \dots \int\limits_{\partial K} \delta_\Delta(K, K_n) d\mathbb{P}(X_1)\dots d\mathbb{P}(X_n) = c_d \int\limits_{\partial K} k(x)^{\frac{1}{d+1}} d\mu_K(x).$$

Here μ_K is the affine surface area measure and δ_Δ is the symmetric difference between K and K_n. A similar result is obtained in the case when K is a simple polytope (every vertex meets d facets). In particular, it is shown that the asymptotic behavior of $\mathbb{E}\delta_\Delta(K, K_n)$ (the integral expression in the left-hand side of the above equality) is $n^{-\frac{2}{d+1}}$ in the smooth case and it is $n^{-\frac{d}{d+1}}$ in the polytopal case.

Andrii Arman gave a talk *On covering problems related to Borsuk's conjecture* where he spoke about his recent results with A. Bondarenko and A. Prymak. Borsuks number $b(n)$ is the smallest integer such that any set of diameter 1 in n-dimensional Euclidean space can be covered by $b(n)$ sets of a smaller diameter. K. Borsuk proved that $b(1) = 2$, $b(2) = 3$. In 1993 J. Kahn and G. Kalai showed that $b(n) \geq 1.2^{\sqrt{n}}$ for n large enough. Later, A. Raigorodskii improved it to $b(n) \geq 1.2255^{\sqrt{n}}$ for n large enough. It is unknown what the smallest n is for which $b(n) > n + 1$. Exponential upper bounds on $b(n)$ were first obtained by O. Schramm (1988) and later by J. Bourgain and J. Lindenstrauss (1989), while a lower bound (exponential in $n^{\frac{1}{2}}$) was obtained by J. Kahn and G. Kalai (1993). To obtain an upper bound on $b(n)$, C. Rogers showed that $b(n) \leq (\sqrt{2} + o(1))^n$, M. Lassak proved that $b(n) \leq 2^{n-1} + 1$, while O. Schramm (1988) and J. Bourgain and J. Lindenstrauss (1989) provided exponential upper bound $b(n) \leq (\sqrt{\frac{3}{2}} + o(1))^n$, where O. Schramm considered the case of bodies of constant width and J. Bourgain and J. Lindenstrauss were covering the body by Euclidean balls of smaller diameter. Let $g(K)$ be the smallest number of balls of diameter $< d$ and let $g(n) = \sup\{g(K) : K \subset \mathbb{R}^n, \text{diam}(K) = 1\}$. It is known that $g(n) \geq 1.003^n$ and $g(n) \leq (\sqrt{2}+o(1))^n$. Let $I(K)$ be the minimal number of smaller homothetic translates of a convex body K needed to cover K. It is known that any set of diameter d can be covered by a set of constant width d. Hence, $h(n)$" $=$ "$b(n)$ for constant width and $b(n) \leq h(n)$. Also, $h(n) \geq b(n) \geq 1.2255^{\sqrt{n}}$. Since O. Schramm provided an exponential upper bound on the illumination number of n-dimensional bodies of constant width, G. Kalai (2015) asked for a corresponding lower bound, namely if there exists an n-dimensional convex body of constant width with the illumination number exponential in n, i.e., if there exists $C > 1$ such that $h(n) > C^n$? The result is that $h(n) \geq \frac{c_1}{\sqrt{n}}(\frac{1}{\cos\frac{\pi}{14}})^n$.

Deping Ye presented his (joint with N. Li and B. Zhu) results on *the dual Minkowski problem for unbounded sets*. Let C be a fixed pointed closed convex cone in $\mathbb{R}^n$. A closed convex set $A^* \subset C$ is called C-close if $A = C \setminus A^*$ has positive finite volume. The set A is called C-coconvex. The set A^* is called C-full if the C-coconvex set A is bounded and non-empty. Let also $\Omega_{C^o} = S^{n-1} \cap \text{int}(C^o)$, where C^o is the dual cone of C, i.e., $C^o = \{x \in \mathbb{R}^n : x \cdot y \leq 0 \, \forall y \in C\}$. Schneider posed (and established the existence and uniqueness of the solution to) the Minkowski problem for C-coconvex sets: given a finite Borel measure μ on Ω_{C^o}, does there exist a C-coconvex set A such that $\mu = \bar{S}_{n-1}(A, \cdot)$? Let $\tilde{C}_q(K, \cdot)$ be the q-th dual curvature measure, introduced by Y. Huang, E. Lutwak, D. Yang and G. Zhang in 2016. Let $0 \notin E \subsetneq C$ be a non-empty set and let conv(E, C) be the closed convex hull of E about C, i.e., $\bigcap\{\tilde{E} : \tilde{E}$ is a C-close set such that $E \subset \tilde{E}\}$. One calls $E \subsetneq C$ a C-compatible set if $0 \notin E = \text{conv}(E, C)$. The analogue of Schneider's result is proved for the q-th dual curvature measure of (C, q)-compatible sets (C-compatible with finite q-th dual volume): given a positive Borel measure μ, finite on Ω_{C^o} and satisfying supp$\mu \subset w \subsetneq C^o$, there exists a C-full set A such that $\tilde{C}(A, \cdot) = \mu$ for $0 \neq q \in \mathbb{R}$. Also, for any $q > 0$ there exists a (C, q)-close set $\mathbb{A}$ such that $\tilde{C}(\mathbb{A}, \cdot) = \mu$.

Sergii Myroshnychenko, gave a lecture *Information-theoretic extensions of Kneser-Poulsen conjecture* on his joint results with G. Aishwarya, I. Alam, D. Li and O. Zatarain-Vera. Using methods of rearrangement and majorization, they affirmatively answer the following information-theoretic question that is directly related to the famous Kneser-Poulsen conjecture: suppose Alice wants to communicate with Bob using a collection of points K in space. However, the night is foggy, so Bob receives the random point $x + W$ when Alice sends x, where W is uniformly distributed on the unit ball. Does communication suffer if the points in K are brought pairwise closer together? More precisely, let $\alpha > 0$, $\alpha \neq 1$, and let $h_\alpha(X) = \frac{1}{1-\alpha} \int\limits_{\mathbb{R}^d} f^\alpha dx$, where X is an $\mathbb{R}^d$-valued random variable with distribution having density f with respect to the Lebesgue measure (for $\alpha = 0, 1, \infty$, the corresponding expressions are obtained by passing to the limit). Does one have

$h_\alpha(Tx+W) \le h_\alpha(X+W)$, where $T : \mathbb{R}^d \to \mathbb{R}^d$ is a contraction, W is a random variable with symmetry. It is shown that for any radially symmetric random variable with convex level sets and any contraction, the above inequality holds. The natural behavior of some intrinsic volumes of convex bodies under contractions are also described.

Petros Valettas gave a talk *Probabilistic Pad Problems* about his joint results with S. Dostoglou. It has been observed, by Froissart (1969), that zeros and poles of high order Padé approximants of random perturbations of a deterministic Taylor series tend to form unstable pairs. These pairs appear at loci characteristic of the random part in the coefficients of the Taylor series. While this phenomenon has only been confirmed experimentally, it has been suggested, and indeed broadly used, as a noise detection tool. In his talk Petros explained how one can combine methods from high-dimensional probability and logarithmic potential theory to rigorously establish and quantify this phenomenon for the "pure noise" case, when the coefficients come from some distribution with anti-concentration properties. Recall that, given a power series $F(z) = \sum_{n=0}^{\infty} a_n z^n$, the polynomials $P_m(z) = \sum_{k=0}^{m} p_k z^k$ and $Q_n(z) = \sum_{k=0}^{n} q_k z^k$ give $[m,n]$-Padé approximation of F, provided $F(z) - \frac{P_m(z)}{Q_n(z)} = O(|z|^{m+n+1})$ as $q_0 = 1$. In a probabilistic setup, given a random vector $\xi = (\xi_0, \xi_1, \ldots, \xi_N)$ in $\mathbb{R}^{N+1}$, for $m+n \le N$, one applies $[m,n]$-Padé approximation $\frac{P_m}{Q_n}$ to the random signal $f_\xi(x) = \sum_{k=0}^{N} \xi_k z^k$, where P_m and Q_n are as above. One of the results reads as follows. Let $\xi = (\xi_0, \ldots, \xi_N)$ be a random vector in $\mathbb{R}^N$ which satisfies, $\mathbb{E}|\xi_k| \le K < \infty\ \forall k$, $\sup_{\{v\in\mathbb{R}\}} \mathbb{P}(|\xi_k - v| < \varepsilon) \le \kappa\varepsilon$, $\varepsilon > 0$. Then, given $\varepsilon, \delta \in (0,1)$ and $m \ge C\varepsilon^{-4} n \log(e\kappa K \frac{n}{\delta})$, for every $N \ge m+n$ and for any random vector ξ on $\mathbb{R}^{N+1}$ satisfying the above conditions, the numerator P_m of the $[m,n]$-Padé approximant for $f_\xi(x)k$ satisfies $d_{BL}(\nu_{P_m}, \mu) < \varepsilon$ with probability greater than $1-\delta$. Here d_{BL} stands for the bounded Lipschitz metric.

Paul Simanjuntak discussed his joint results with R. Adamczak and P. Pivovarov concerning *Central Limit Theorem for Volume of Sections of B_p^n*. They established Central Limit Theorem (CLT) for the volumes of intersections of B_p^n, $0 < p < 2$, with uniform random subspaces of fixed co-dimension d as n tends to infinity. The result is obtained using volume representation as sum of Gaussian mixtures: for $p \in (0,2)$. There exist constants $a_{p,d}$, $b_{p,d}$ and $\Sigma_{p,d}^2$ such that

$$\sqrt{n}\Big(\frac{\mathrm{vol}(B_p^n \cap H_n)}{\mathrm{vol}(B_p^{n-d})} - a_{p,d} - \frac{1}{n} b_{p,d}\Big) \xrightarrow{d} N(0, \Sigma_{p,d}^2) \quad \text{as} \quad n \to \infty.$$

As a corollary the higher order approximations for expected volumes are also obtained, refining previous results by Koldobsky and Lifshits and approximation obtained from the Eldan-Klartag version of CLT for convex bodies.

Chase Reuter gave a lecture *The Euclidean ball is locally the only fixed point for the p-centroid body operators*. Characterizing the Euclidean space among all normed spaces is one of the aims of the ten problems formulated in 1956 by Busemann and Petty. These problems lead to the study of certain integral operators on convex bodies, such as the intersection body operator for the first Busemann-Petty problem. In the class of convex bodies, obtaining global statements about the fixed points of such operators is difficult. The local study of these problems appears to be a more approachable initial step, which has yielded local solutions to problems 5 and 8 by M. Alfonseca, F. Nazarov, D. Ryabogin and V. Yaskin. Chase applied similar techniques to study the fixed points up to dilation of the p-centroid body operator in a neighborhood of the Euclidean ball. Given an origin-symmetric convex body K, this body can be identified by its radial function $\rho_K(\theta) = \max\{r > 0 : r\theta \in K\}$ or its support function $h_K(\theta) = \max_{\{x\in K\}} x \cdot \theta$, where $\theta \in S^{n-1}$. For $p \ge 1$, the p-centroid body $\Gamma_p K$ is defined for all $\theta \in S^{n-1}$ as

$$h_{\Gamma_p K}(\theta) = \frac{1}{\mathrm{vol}_n(K)} \int_K |x \cdot \theta|^p dx = \frac{1}{\mathrm{vol}_n(K)} \int_{S^{n-1}} |\sigma \cdot \theta|^p \rho_K^{n+p}(\sigma) d\sigma.$$

When $p = 1$, the boundary of $\Gamma_p K$ can be described physically: If K has density $\frac{1}{2}$ and were allowed to float in a particular orientation, then the boundary of Γ_p is the locus of the centers of masses of the submerged portions for all orientations. In the integral definition, passing to polar coordinates yields $h_{\Gamma_p K} = c_K \mathcal{C}_p \rho_K^{n+p}$, where c_K is some constant depending on the body and $\mathcal{C}_p$ is the p-cosine transform (the integral over S^{n-1} in the right-hand side of the previous integral equality). Since the eigenspaces of the cosine transform are spanned by the spherical harmonics, one uses techniques from harmonic analysis to show that if K is close to the Euclidean ball and $K = c\Gamma_p K$ for some real number c, then K is the Euclidean ball up to a linear transformation.

Katarzyna Wyczesany gave a final talk of the conference titled *A Blaschke-Santalo type inequality for dual polarity*, where she presented her joint results with S. Artstein-Avidan and S. Sadovski. Let T be an order reversing quasi involution acting on all subsets $\mathcal{P}(X)$ of the given set X, i.e., $K \subset TTK$ and $L \subseteq K$ yields $TL \supseteq TL$. Assume also that $C = \{K \subset X : \exists L \subset X : K = TL\}$, then $T|_C$ is a duality (order reversing involution). If $c : X \times X \to (-\infty, \infty]$ is such that $c(x, y) = c(y, x)$, and the c-dual of $K \subset X$ is defined as $K^c = \{y \in X : \forall x \in K, c(x, y) > 0\}$, it is shown that for T as above there exists $c : X \times X \to \{\pm 1\}$ such that for every $K \subset X$ one has $TK = K^c$. The characterization of the order reversing quasi involutions is given and the new Blaschke-Santalo type inequality is proved. Let K be essentially symmetric (for some $e \in S^{n-1}$, one has $x + te \in K$, $x \in e^\perp$, yields $-x + te \in K$) and let $T : \mathcal{P}(\mathbb{R}^n) \to \mathcal{P}(\mathbb{R}^n)$ be an order reversing involution. If $TK = \{x \in \mathbb{R}^n : \forall y \in K, x \cdot y \geq 1\}$, then $\gamma_n(K)\gamma_n(TK) \leq \gamma_n(K_0)^2$ holds. Here $K_0 = \{(x, t) \in \mathbb{R}^{n-1} \times \mathbb{R}^+ : |x|^2 + 1 \leq t^2\}$ and γ_n is the Gaussian measure on $\mathbb{R}^n$.

3 Outcome of the Meeting

The meeting was very successful, we brought together mathematicians from many countries and many research areas, such as convex geometry, discrete geometry, probability, functional and harmonic analysis. Besides the leading scientists, we also had 8 graduate students and 8 postdocs participating in-person in the workshop. Female participation was above 27%. The friendly atmosphere created during the workshop helped many participants not only to identify the promising ways to attack the old problems but also to get acquainted with many open new ones.