Compensated Compactness and Applications to Materials

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1 Overview of the Aims of the Workshop

The workshop "Compensated Compactness and Applications to Materials" brought together experts from the theory of compensated compactness with researchers in material science. The workshop focused on the recent progress in the compensated compactness theory involving measures or L^1-maps. These developments have enabled many new results, for instance in the theories of microstructure, shape optimization, dislocation theory, fracture, and plasticity. The workshop aimed to balance theoretical and more applied talks to give the participants the opportunity to exchange the latest ideas, learn new methods, and start new collaborations.

Over the last years, much progress was made in the quest to understand the structure of L^1-maps or singular measures solving linear partial differential constraints. Deep new theorems have been discovered as a result of novel interactions within the pure mathematics community between PDE theory, geometric measure theory and harmonic analysis. The potential of this approach is far-reaching and has already led to the resolution of several long-standing conjectures as well as opened up new avenues to understand the fine structure of singularities.

The main reason for the added difficulty in the L^1 or measure context is that in addition to dealing with fast oscillation, concentrations also have to be considered. In particular, for vector-valued maps or measures the interaction between oscillations and concentrations causes a whole host of new phenomena. This leads to relaxation results of underlying energies whose explicit expression is very difficult to obtain.

This avenue of research has turned out to be intimately connected to the comprehension of the convergence properties of nonlinear quantities under weak convergence of maps, leading in the 1970s to the notion of polyconvexity by John Ball in [19] and to compensated compactness theory by François Murat and Luc Tartar [49, 46, 45, 47]. However, only after the recent progress has it become possible to take on problems which require a similar theory for "rough" objects like L^1-maps and measures.

The theoretical questions at the core of the emerging field of "linear-growth compensated compactness" are very challenging. Since they promise new insights into the formation of singularities in merely L^1-bounded sequences of maps, some of these questions are being pursued for their own sake. On the other hand, it has become clear by now that the potential impact on more "applied" questions is even more intriguing than the abstract work.

The main objective of the workshop was to bring together specialists of both theoretical and applied communities working on questions related to compensated compactness theory for "rough" objects like L^1-maps and measures. There were three main goals: (a) To take stock of and to disseminate the available results showing an interaction between L^1-bounds, linear partial differential constraints, and compensated compactness, (b) To define questions originating from plasticity, dislocation theory, and shape optimization

which require a theory for "rough" objects, and (c) To survey applications of the theory to microstructure, shape optimization, dislocation theory, fracture, and plasticity. The workshop facilitated new collaborations and exchange of ideas.

2 Presentations

Altogether, 24 leading experts in the field as well as early-career researchers presented research talks. The range of topics was very wide, although accessible for all the audience.

- *Geometric measure theory:* Connor Mooney focused on the Bernstein problem, a reformulation of the problem of minimal surfaces; Giovanni Alberti and Adolfo Arroyo-Rabasa were interested in the development of measure theoretic tools to understand the interaction between oscillation and concentration phenomena, in connection with the vanishing mass conjecture introduced by Guy Bouchitté about 20 years ago in [21].

- *(Multi)scale asymptotic analysis:* Irene Fonseca discussed the effect of oscillating wells in the context of phase transition, Annika Bach presented works related to the interplay between homogenization and phase field models of fracture, and Caterina Zeppieri discussed about the homogenization of variational integrals involving randomly distributed holes. Elisa Davoli presented new results related to the (non) stability through linearization of the noninterpenetration of matter constraint in fractured materials.

- *Nonlinear elasticity*: Duvan Henao discussed the formation of cavitation in hyperelastic media, André Guerra presented new results related to the connections between rank-one convexity and quasi-convexity, while Carolin Kreisbeck extended lower semicontinuity results of quasiconvex integrals to the case of nonlocal energies. Ian Tobasco presented curvature dependent elastic energies to address wrinkling and folding of confined elastic sheets.

- *Plasticity*: Amit Acharya presented a dual variational formulation of dislocation in connection to equations of fluid dynamics, Tom Hudson proposed a model which explains dislocation motions in a nonlinear setting, and Adriana Garroni presented results which explain how a 3D variational model involving incompatible fields can lead to a line tension dislocation model in the mesoscale limit. Elise Bonhomme rather focussed on a macroscale formulation and presented results showing that the presence of irreversibility in a quasistatic evolution setting can lead to non stability phenomenon of Γ-converging damage models.

- *Shape optimization*: Heiner Olberman presented new results on the shape optimization of Willmore type energies under an area constraint expressed through a penalizing Lagrange multiplier. Giuseppe Buttazzo studied shape optimization problems involving the torsional rigidity and the eigenvalue of the Laplace operator and their connection through the Blaschke-Santaló diagram. Mariapia Palombaro constructed optimal microstructures of a 3D polycrystalline material, where the degree of freedom is given by the orientation of the crystal.

- *Optimal transportation*: Guy Bouchitté presented new results about short-range interaction functionals for models of many interacting particles such as in crowd motion.

- *Hyperbolic systems*: Paolo Bonicatto presented a unified measure theoretic framework to address the well-posedness of transport type equations with a Lipschitz vector field where the unknown is a lower dimensional object. Andrew Lorent presented how the use of entropies can lead to rigidity results in the context of the Aviles-Giga functional and differential inclusions. Benoît Merlet spoke about nonoriented Aviles-Giga and rigidity results for small energy configurations. Michaël Goldman discussed about the structure of defect measures associated to energies penalizing oscillations in oblique directions. Finally, Gilles Francfort presented results related to uniqueness issues for non strictly convex linear growth variational problems by the use of conservation laws associated to the Euler-Lagrange equations.

2.1 First Day

The first day of the workshop began with Irene Fonseca delivering a talk on "Phase Separation in Heterogeneous Media." She discussed how certain modern technologies and biological systems utilize natural heterogeneities of the medium or engineered inclusions to produce composite materials with specific physical properties. She also mentioned that to model such situations, a variational approach based on the gradient theory should be used. In addition, Fonseca explained how different regimes should be considered when the potential and the wells depend on the spatial position, even in a discontinuous way. This was joint work with Riccardo Cristoferi and Likhit Ganedi, based on previous results also obtained with Adrian Hagerty and Cristina Popovici (see [31, 28, 25, 30, 29]).

Amit Acharya's talk titled "Field Dislocation Mechanics, Ideal MHD, and a Dual Variational Formulation" reviewed the fully nonlinear system of Field Dislocation Mechanics to establish an exact analogy with the equations of ideal magnetohydrodynamics (ideal MHD) under suitable physically simplifying circumstances. He talked about how weak solutions with various conservation properties have been established for ideal MHD recently by Faraco, Lindberg, and Szekelyhidi (2021) using the techniques of compensated compactness and convex integration. By the established analogy, these results would seem to transfer directly to the idealization of Field Dislocation Mechanics considered. A dual variational principle was also demonstrated for this system of PDE (see [2, 3, 4, 5, 6, 1, 11, 10, 9]).

Elise Bonhomme delivered a talk entitled "Can quasi-static evolutions of perfect plasticity be derived from brittle damage evolutions?". She addressed the question of the interplay between relaxation and irreversibility through evolution processes in damage mechanics. Bonhomme inquired whether the quasi-static evolution of an elastic material undergoing a process of plastic deformation can be derived as the limit model of a sequence of quasi-static brittle damage evolutions. She talked about how the interplay between relaxation and irreversibility of the damage is not stable through time evolutions and how the obtained effective quasi-static damage evolution may not be of perfect plasticity type, in contrast with the static case considered in [14].

Duvan Henao delivered a talk titled "Harmonic dipoles in elasticity." He discussed how whenever the stored energy density of a hyperelastic material has slow growth at infinity, it may undergo cavitation under large hydrostatic tension, constituting a failure of quasiconvexity and a challenge for the existence theory in elastostatics. Henao also talked about how this obstacle has been overcome under certain coercivity hypotheses, which, however, fail to be satisfied by the paradigmatic example in elasticity, that of 3D neo-Hookean materials. He presented a joint work with Marco Barchiesi, Carlos Mora-Corral, and Rémy Rodiac, where this borderline case was solved for hollow axisymmetric domains.

Andrew Lorent delivered a talk on "Differential inclusions, entropies, and the Aviles Giga functional." He outlined some elementary questions and theorems about differential inclusions. Lorent also talked about the concept of entropies from scalar conservation laws and the adaptation of this concept to the Aviles Giga functional. He showed how these topics are connected and how the connection has application to both differential inclusions and to the Aviles Giga functional (see [41, 40]).

To close this first day workshop, Caterina Zeppieri discussed the homogenization of vectorial integral functionals with q growth in a bounded domain of $\mathbb{R}^n$, $n > q > 1$, which is perforated by a random number of small spherical holes with random radii and centres. The goal was to show that for a class of stationary short-range correlated processes for the centres and radii of the holes, in the homogenized limit, we obtain a nonlinear averaged analogue of the "strange term" obtained by Cioranescu and Murat in 1982 (see [27, 26]), in the periodic case. The speaker demonstrated that the clustering holes do not have any impact on the homogenization procedure and the limit functional, despite the fact that there are holes which overlap with probability one. This was achieved by requiring only that the random radii have finite $n - q$-moment, which is the minimal assumption to ensure that the expectation of the nonlinear capacity of the balls is finite. The talk was based on joint work with K. Zemas and L. Scardia.

2.2 Second Day

On the second day of the workshop, Elisa Davoli presented her work [8] on the passage from nonlinear to linearized Griffith-fracture theories under non-interpenetration constraints. She characterized sequences of deformations satisfying a Ciarlet-Necas condition in SBV^2 and showed that they admit asymptotic represen-

tations in $GSBD^2$ satisfying a suitable contact condition. With an explicit counterexample, she proved that this result fails if convergence of the energies does not hold. She further proved that each limiting displacement satisfying the contact condition can be approximated by an energy-convergent sequence of deformations fulfilling a Ciarlet-Neç as condition. This was joint work with Stefano Almi (Naples) and Manuel Friedrich (Erlangen).

Michael Goldman presented his results on a family of energies penalizing oscillations in oblique directions. These functionals control second order derivatives rather than first order ones and actually have mixed (or oblique) derivatives given by bounded measures. The main focus of his talk was the study of the rectifiability properties of these "defect" measures. He also drew connections with branched transportation, PDE constrained measures, and Aviles-Giga type differential inclusions (see [35, 33, 34]).

Adolfo Arroyo-Rabasa talked about how PDE constraints interact with the formation of mass concentrations (which are necessary for the formation of measure singularities). He discussed how the formation of "strong mass concentrations" along a sequence of PDE-constrained functions is "fully unconstrained" as long as the expectation of its values belongs to the wave cone associated with the PDE. He also explained the "gluing technique" behind the proof as well as some interesting applications.

André Guerra presented his work on quasiconvexity and nonlinear elasticity. Quasiconvexity is the fundamental existence condition for variational problems. He showed that rank-one convexity, a simple necessary condition, implies quasiconvexity in two dimensions in a special class of isotropic energies. He also proved existence theorems for quasiconvex energies in the context of nonlinear elasticity. His proof combined complex analysis with the theory of gradient Young measures. On the way to the main result, he established quasiconvexity inequalities for the Burkholder function, which yielded many sharp higher integrability results. This was joint work with Kari Astala, Daniel Faraco, Aleksis Koski, and Jan Kristensen (see [37, 12]).

Heiner Olbermann discussed the Willmore functional on graphs with an additional penalization of the area where the curvature is non-zero. Sending the penalization parameter to infinity and rescaling suitably, he derived the limit functional in the sense of Γ-convergence.

Finally, Giovanni Alberti presented the "Vanishing Mass Conjecture," which G. Bouchitté formulated about twenty years ago in the context of optimization of light elastic structures (see [15]). He illustrated this conjecture placing the emphasis on its geometric nature and some partial results obtained with Andrea Marchese (University of Trento) and Andrea Merlo (University of the Basque Country).

2.3 Third Day

Giuseppe Buttazzo gave a talk on the representation of Blaschke-Santaló diagrams, an important problem that arises in shape optimization. He discussed the case of torsional rigidity and the first eigenvalue of the Laplace operator, although other cases have been considered in the literature. From a numerical perspective, this involves representing the image of a given map $F : X \to \mathbb{R}^k$, where X is a compact metric space and $k = 2$. He also mentioned the interesting case when X is a subset of an Euclidean space $\mathbb{R}^d$ (with d much larger than k), and the suitable use of Voronoi tessellations plays an important role. His last research was in collaboration with Benjamin Bogosel and Edouard Oudet (see [24, 23, 22, 20]).

Benoit Merlet presented a non-oriented version of the Aviles-Giga functional, which serves as a model for pattern formation, particularly striped patterns in 2D. He showed that sequences with uniformly bounded energy as the scale parameter goes to 0 are relatively compact in L^1_{loc}. He also described the limit configurations in the vanishing energy limit case. These results are similar to their counterparts for the classical Aviles-Giga functional, but new phenomena appear in the non-oriented case, and the proofs require new ideas. The work was done in collaboration with Michaël Goldman, Marc Pegon, and Sylvia Serfaty (see [36]).

Carolin Kreisbeck discussed a class of variational problems with integral functionals involving nonlocal gradients, motivated by new nonlocal models in hyperelasticity. She addressed several aspects of the existence theory of these problems and their asymptotic behavior. Her analysis relied on suitable translation operators that allowed her to switch between classical, fractional, and nonlocal gradients. She showed that quasiconvexity characterizes weak lower semicontinuity in the fractional and nonlocal setting, and derived relaxation and homogenization results from a general Γ-convergence statement. The limiting behavior as the fractional order tends to 1 yields localization to a classical model. Her work was done jointly with Javier Cueto and Hidde Schönberger (see [32, 39]).

Ian Tobasco discussed the wrinkling and folding of confined elastic sheets. He presented recent progress

towards an effective variational theory for wrinkles and folds. In the first part of his talk, he discussed the wrinkling of curved shallow shells that float on top of a flat water bath. He derived and solved the Γ-limit of a rescaled shallow shell model, which explained the patterns that arise in a given floating shell. He found that the wrinkles of positively and negatively curved shells are linked. In the second part of his talk, he discussed recent work on folds. He used a fully nonlinear model for a plate confined to a planar cavity and laterally squeezed and proved a scaling law and compactness result involving a BV-type energy that arose to control the length of the folds. The limiting mid-plane was shown to deform by a length-preserving map that can change orientation across a singular set containing the folds. His work was done in collaboration with Eleni Katifori, Joey Paulsen, and Samuel Wallace (see [50, 51]).

2.4 Fourth Day

On the fourth day of the workshop, Paolo Bonicatto gave a talk entitled "Existence and uniqueness for the Lipschitz transport of normal currents". He discussed efforts to extend classical theory to the case where the unknown is k-currents in $\mathbb{R}^d$, or generalized k-dimensional surfaces, which involve the Lie derivative L_b of currents in direction b and read $\partial_t T_t + L_b T_t = 0$. Bonicatto proved the existence and uniqueness of the equation in the class of normal currents under the natural assumption of Lipschitz regularity of the vector field b. He also talked about the notion of decomposability bundle introduced recently by Alberti and Marchese and how it is crucial to the argument.

Annika Bach presented a talk titled "Γ-convergence and homogenisation of singularly-perturbed elliptic functionals". She studied the asymptotic behavior of a general class of singularly-perturbed elliptic functionals of Ambrosio-Tortorelli type as the perturbation parameter vanishes. Bach showed that the functionals Γ-converge (up to subsequences) to a free-discontinuity functional of brittle type, under mild regularity assumptions and suitable super-linear growth conditions on the integrands. She also provided asymptotic formulas for the limiting volume and surface integrands, which showed that the volume and surface contributions decouple in the limit. Bach's work is based on joint works with T. Esposito, R. Marziani, and C. I. Zeppieri (see [18, 16]).

Tom Hudson gave a talk titled "Elastoplastic evolution of single crystals driven by dislocation flow". He discussed how dislocation motion is a key feature of crystal plasticity at the smallest scales, and how many mathematical challenges must be overcome to establish a well-posed theory that accurately couples dislocation motion and continuum theories in a three-dimensional nonlinear setting. Hudson presented joint work with Filip Rindler [38] on a model that proposes a novel geometric language built on the concepts of space-time currents, or "slip trajectories" and the "crystal scaffold" to describe the movement of discrete dislocations. The model recovers several laws that were known in special cases before, such as the equation for the Peach-Koehler force on a dislocation in a linearized context.

Connor Mooney presented a talk titled "The anisotropic Bernstein problem". He discussed how the Bernstein problem asks whether entire minimal graphs in $\mathbb{R}^{n+1}$ are necessarily hyperplanes, and how the answer is positive if and only if $n < 8$. The anisotropic Bernstein problem asks the same question about minimizers of parametric elliptic functionals, which are natural generalizations of the area functional that both arise in material science and offer important technical challenges. Mooney discussed the recent solution of this problem (the answer is positive if and only if $n < 4$), based on joint work with Y. Yang (see [42, 44, 43]).

Mariapia Palombaro gave a talk titled "Optimal microstructures using infinitely many rotations". She discussed how composite materials display a wide range of conduction properties depending on the geometric configuration of the phases, and how a classical problem is to find the range of the effective conductivity, a constant but in general anisotropic matrix describing the overall electrical behavior of the composite. Palombaro presented optimal bounds for isotropic mixtures and provided new anisotropic optimal microgeometries in the case of a three-dimensional polycrystalline material, where the principal conductivities of the basic crystal are given, but the orientation of the crystal is allowed to change from point to point (see [7]).

Guy Bouchitté conducted research on a class of short-range interaction functionals for a model of many particles interacting in $\mathbb{R}^d$, such as in Density Functional Theory or crowd motion. The energy cost of the particles was usually considered to be repulsive and described by a two-point function, $c_\varepsilon(x, y) = \ell(\frac{|x-y|}{\varepsilon})$, where $\ell : \mathbb{R}_+ \to [0, \infty]$ was decreasing to zero at infinity, and the small parameter $\varepsilon > 0$ scaled the interaction distance. Bouchitté reviewed the link between this model and multimarginal optimal transport before focusing on new results obtained in collaboration with R. Mahadevan from the University of Concepcion, Chile. The

study identified the mean-field energy of the model in the short-range regime $\varepsilon \ll 1$, assuming only that $\int_{r_0}^{\infty} \ell(r) r^{d-1}\, dr < +\infty$. This extends and simplifies existing results in the homogeneous case $\ell(r) = r^{-s}$ where $s > d$.

2.5 Fifth Day

On the last day of the conference, two talks were presented. In the first talk titled "Hyperbolicity as a possible path to the uniqueness of minimizers for energies with linear growth," Gilles Francfort presented a work in collaboration with Jean-François Babadjian [13] where a functional of the gradient that exhibited linear growth at infinity is investigated. The relaxed functional has BV minimizers. In 2D, their uniqueness was tied to the properties of a spatial continuity equation, for which regular Lagrangian flows did not apply. However, techniques related to the work of Jabin-Otto-Perthame on 2D Ginzburg-Landau models allowed a better understanding of the associated characteristic flow from which uniqueness followed.

In the last talk of the workshop, Adriana Garroni spoke about "Three-dimensional line-tension limits for line singularities and applications". She presented a derivation of a line tension model for dislocations in 3D, starting from a variational model that accounted for the elastic energy induced by incompatible elastic fields. Under a kinematic constraint that forced the dislocations to be diluted on a mesoscopic scale, via Γ-convergence, they deduced energies concentrated on 1-rectifiable lines which could be interpreted as line tension energies for dislocations in a single crystal. The result was based on a recent paper in collaboration with S. Conti and R. Marziani. They treated a quite general framework that included several different regularized variational models present in the literature, ranging from linear elastic energies with core regularization to non-linear elastic energies with sub-quadratic regularization.

3 Outcome of the Meeting

The final structure of the workshop was a focussed gathering of 27 participants (12 belonging to minorities), among which 8 were online. This small size made it easier to meet all the physically present participants and start mathematical discussions. It was particularly helpful for young researchers to easily meet senior researchers. Out of the 20 physically present participants (10 belonging to minorities), 1 talk was given by a graduate student (Elise Bonhomme), 5 by postdoctoral fellows or assistant professors (Adolfo Arroyo-Rabasa, Annika Bach, Paolo Bonicatto, André Guerra, Ian Tobasco) and 5 by young faculty (Carolin Kreisbeck, Andrew Lorent, Heiner Olbermann, Mariapia Palombaro, Caterina Zeppieri). These were complemented by talks from worldwide experts in different areas of calculus of variations and partial differential equations including Amit Acharya (dislocation mechanics), Giovanni Alberti (geometric measure theory), Guy Bouchitté (shape optimization, optimal transport), Irene Fonseca (Γ-convergence, epitaxial growth), Gilles Francfort (fracture mechanics, plasticity), Adriana Garroni (dislocations).

The presence of so many young participants and the wonderful environment of the Banff International Research Center contributed to a very informal, friendly, and unique atmosphere. We kept the talks to 40 minutes which gave plenty of time for informal discussions and networking. New friendships were formed and collaborations were initiated during the meeting. From the feedback we received, the workshop was very successful and the participants enjoyed the meeting and specifically the pleasant atmosphere of the workshop.

4 Comments

We attach below some feedbacks we got from several participants:

Acharya: *"I had very nice interactions with many of the participants: Rindler, Alberti, Bonicatto, Bouchitté, Arroyo-Rabassa, and various others. This was a great workshop which I enjoyed very much."*

Alberti: *"I discussed at length with the following people: Amit Acharya (grain boundaries, justifications of the Read–Shockley formula) Adolfo Arroyo-Rabasa (his work related to concentration Young Measures) Paolo Bonicatto (strong locality property of 1st order linear differential operators). I also have brief conversations with other people, including Adriana Garroni e Ian Tobasco. The only real collaboration is with Paolo*

Bonicatto. I have no comment to add, besides the fact that I really enjoyed the workshop, and specifically the atmosphere of the workshop."

Bach: *"The workshop in Banff was a natural possibility to discuss a bit with Caterina Zeppieri about possible future research questions related to the projects that we have been working on so far. Besides that I am particularly happy that I had the possibility to interact with Carolin Kreisbeck, who some years ago followed the same career path as I am about to do now. It was not only very helpful to learn about her experiences with a tenure-track position in the Netherlands coming from Germany, but we also found the time for a very fruitful discussion about possible future collaborations."*

Fonseca: *"I had very interesting discussions with Gilles Francfort and Guy Bouchitté. The latter is visiting me at CMU for two weeks following the Banff workshop, and we are continuing the project that we initiated while at Banff. This collaboration would have likely not happened if it wasn't for the fact that during one week at BIRS we matured the ideas that we are now pursuing with Leonard Kreutz (postdoc at CMU)."*

Guerra: *"I also had fruitful interactions with several researches in Banff. I would highlight in particular very interesting discussions with Gilles Francfort, Ian Tobasco and Paolo Bonicatto; I initiated new collaborations with the last two researchers. Overall the conference atmosphere was pleasant and prone to interesting mathematical discussions. The relatively small size of the workshop made it easier to meet everyone quickly and to discuss with those with research interests closer to mine."*

Kreisbeck: *"The interactions with the other participants were inspiring and most valuable. Particularly, the week at BIRS has opened up the opportunity to explore a future collaboration with Ian Tobasco on the analysis of high-contrast checkerboard composites and kirigami-type structures. An exchange between our groups is already in planning."*

Lorent: *"I enjoyed it very much and learned a lot. In particular, I valued meeting (or getting to know better) younger mathematicians in the field and reconnecting with old friends and colleagues."*

Palombaro: *"During the workshop I had fruitful discussions about G-closure and quasiconvexity/polyconvexity problems with André Guerra, Gilles Francfort, Carolin Kreisbeck and Caterina Zeppieri, who have worked on closely related problems."*

Zeppieri: *"I had some exchanges with Mariapia Palombaro on certain G-closure problems and optimal bounds."*

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