New trends in fluids and collective dynamics

Theodore Drivas (Stony Brooke University)
Nancy Rodriguez (The University of Colorado, Boulder)
Roman Shvydkoy (University of Illinois at Chicago)
Eitan Tadmor (University of Maryland, College Park)

July 24-28, 2023

1 Overview of the subject

In the subject of partial differential equations (PDEs) which includes a broad range of models describing various physical phenomena, two distinct and seemingly unrelated branches may find themselves intricately connected. Such connections result in bursts of development as ideas are transferred between the subjects. An example of such merger was implementation of dispersive techniques into incompressible fluids in the 2000s. More recently, the geometric ideas of the Nash-Gromov immersion theory were brought to solve the long standing Onsager conjecture for energy conservation in the incompressible Euler system.

Currently, the subject of collective behavior, which spans studies of many biological, technological and social sysytems, is undergoing a similar development spurred by newly found links with fluid dynamics. The thread that runs through these fields stems from statistical mechanics of many particle systems – when the number of agents increases to infinity, the evolution equations describing collective behavior start to resemble equations of hydrodynamics.

One of the famous and notoriously challenging problems in collective studies is called *emergence*. Emergence is a paradigm stating that many naturally occurring global phenomena, such as swarming in biological systems or clustering in social networks, appear as a result of many locally communicating agents. Depending on the context and particular phenomenon in question, this paradigm can be formulated in precise mathematical terms. For example, occurrence of milling patterns in schools of fish governed by a 3Zone communication protocol is an emergent phenomenon.

A wide and recently popular class of alignment models such as Cucker-Smale and its variants has received a lot of attention in recent years. With the introduction of models with singular communication protocol new prospectives on collective behavior emerged with them new challenges in the analysis of fractional parabolic equations. Attempts to explain alignment through relaxation in kinetic equations brought adaptations of the hypocoercivity techniques from collisional models. Ideas coming from mixing and enhanced dissipation are now being implemented to chemotactic propulsion of micro-organisms. Methods of the regularity theory of fluid models such as modulus of continuity technique, generation of small scales, DiGiorgi-Nash-Moser iteration, are finding useful interpretations in the context of collective behavior. Models incorporating multiple scales of description and merging the classical hydrodynamical systems with systems of collective behavior have been very prominent in studies of immersed flocks where agents influence dynamics of the ambient fluid through a properly modeled stress. Applications to traffic control, droplet formation, cell migration, etc, are numerous.

2 Objectives planned and achieved

Such a rapid development of the new field necessitates active collaboration of specialists in various areas of PDEs. This makes our workshop timely and impactful. The main goal of this workshop was to provide a platform for an exchange of ideas between the subjects that can transformative to our understanding of collective phenomena and fluids.

This goal was achieved by bringing a diversified group of researchers from primarily two subjects of our focus – fluid dynamics and applied collective behavior. We gathered speakers involved in recent breakthroughs in regularity theory for fractional parabolic equations, aggregation, and chemotaxis models, as well as speakers more involved in implementation of analytical tools to specific models. The list of participants spans a broad range of carrier ages from young researchers to more senior and established ones.

Through careful planning and providing plenty of time for off-talk conversations we set the stage for active collaboration and exchange of knowledge between the participants, in person and virtual alike.

3 Format of the meeting

The meeting has been conducted in the hybrid format with most of the talks being delivered in survey / overview style accessible to participants in other fields and young researchers. We scheduled 4 one-hour talks on Monday, 6 talks on Tuesday, 2 talks on Wednesday, 6 talks on Thursday and 1 talk on Friday morning. Wednesday featured the traditional half-day schedule. Most talks have been recorded.

In total we hosted 9 in person talks and 7 virtual talks via Zoom. All talks, virtual and in person, have been broadcast to all the participants. Among the 16 speakers 6 were women of various seniority.

4 Abstracts and day-by-day activities

The first day featured four talks related to data assimilation, particle approximation methods and application to specific problems in collective droplet dynamics. These developments are important to understanding how and in what ways one can use the fundamental equations in real life measurements and how these measurements help calibrate the systems in question to improve their predictability. Mean-field limits and particle methods are crucial in justification and numerical implementation of some of the macroscopic models of fluid and collective phenomena. A surprising connection between non-local particle approximations and renormalization of fields in turbulence theory was discovered.

The schedule on this day was rather relaxed. We provided plenty of free discussion time.

Franziska Weber: *On Bayesian data assimilation for PDEs with ill-posed forward problems.*

> We consider Bayesian data assimilation for time-evolution PDEs, for which the underlying forward problem may be very unstable or ill-posed. We formulate assumptions on the forward solution operator of such PDEs under which stability of the posterior measure with respect to perturbations of the noisy measurements can be proved. We also provide quantitative estimates on the convergence of approximate Bayesian filtering distributions computed from numerical approximations. For the Navier-Stokes equations, our results imply uniform stability of the filtering problem even at arbitrarily small viscosity, when the underlying forward problem may become ill-posed, as well as the compactness of numerical approximations in a suitable metric on time-parametrized probability measures. This is a joint work with Samuel Lanthaler and Siddhartha Mishra.

Hangjie Ji: *Coarsening of thin films with weak condensation.*

> A lubrication model can be used to describe the dynamics of a weakly volatile viscous fluid layer on a hydrophobic substrate. Thin layers of the fluid are unstable to perturbations and break up into slowly evolving interacting droplets. In this talk, we will present a reduced-order dynamical system derived from the lubrication model based on the nearest-neighbour droplet interactions in the weak condensation limit. Dynamics for periodic arrays of identical drops and pairwise

droplet interactions are investigated which provide insights to the coarsening dynamics of a large droplet system. Weak condensation is shown to be a singular perturbation, fundamentally changing the long-time coarsening dynamics for the droplets and the overall mass of the fluid in two additional regimes of long-time dynamics. This is joint work with Thomas Witelski.

Pierre-Emmanuel Jabin:*The mean-field limit of non-exchangeable integrate and fire systems*

We investigate the mean-field limit of large networks of interacting biological neurons. The neurons are represented by the so-called integrate and fire models that follow the membrane potential of each neurons and captures individual spikes. However we do not assume any structure on the graph of interactions but consider instead any connection weights between neurons that obey a generic mean-field scaling. We are able to extend the concept of extended graphons, introduced in Jabin-Poyato-Soler, by introducing a novel notion of discrete observables in the system. This is a joint work with D. Zhou.

Katy Craig: *Nonlocal particle approximations of constrained transport and diffusion, with applications to sampling.*

Given a desired target distribution and an initial guess of its samples, what is the best way to evolve the locations of the samples so that they accurately represent the desired distribution? A classical solution to this problem is to evolve the samples according to Langevin dynamics, a stochastic particle method for the Fokker-Planck equation. In today's talk, I will contrast this classical approach with two novel deterministic approaches based on nonlocal particle methods: (1) a nonlocal approximation of dynamic optimal transport, with state and control constraints, and (2) a nonlocal approximation of degenerate diffusion equations. I will present recent work analyzing the convergence properties of each method, alongside numerical examples illustrating their behavior in practice. This is based on joint works with Karthik Elamvazhuthi, Matt Haberland, Matt Jacobs, Harlin Lee,and Olga Turanova.

Second day was mode intense in terms of schedule of talks. We had 6 talks delivered altogether and active discussions after some of the talks. Most of the talks focused on regularity and long time behavior of mascroscopic hydrodynamic models of incompressible fluids which also play a central role for multi-scale modeling of self-organization of suspended active agents. The last talk featured a very recent groundbreaking development in regularity of solutions to the incompressible 3D Navier-Stokes system. This talk invoked active discussion afterwards.

Yao Yao: *Small scale formation for the 2D Boussinesq equation.*

In this talk, we consider the 2D incompressible Boussinesq equation without thermal diffusion, and aim to construct rigorous examples of small scale formations as time goes to infinity. In the viscous case, we construct examples of global-in-time smooth solutions where the H^1 norm of density grows to infinity algebraically in time. For the inviscid equation in the strip, we construct examples whose vorticity grows at least like t^3 and gradient of density grows at least like t^2 during the existence of a smooth solution. These growth results work for a broad class of initial data, where we only require certain symmetry and sign conditions. As an application, we also construct solutions to the 3D axisymmetric Euler equation whose velocity has infinite-in-time growth. This is a joint work with Alexander Kiselev and Jaemin Park.

Daniel Lear: *Time periodic solutions near shear flows for 2D Euler.*

In this talk we will consider the existence of time periodic solutions arbitrarily close to shear flows for the 2D incompressible Euler equations. In particular, we will present some results concerning the existence of such solutions near Couette, Taylor-Couette and Poiseuille flows. In the first part of the talk, we will introduce the problem and review some well-known results on this subject. In the second we will outline some of the ideas underlying the construction of our time periodic solutions. Joint work with Angel Castro.

Changhui Tan: *On the well-posedness of the Euler-alignment system.*

In this talk, I will provide an overview of recent advancements in the field of local and global well-posedness theories concerning the compressible Euler system with singular velocity alignment. This system, commonly referred to as the Euler-alignment system, serves as a mathematical model for studying the collective behavior of animal flocks. My discussion will primarily focus on two specific scenarios: (1) the establishment of global well-posedness for small rough initial data that reside within critical Besov spaces, and (2) the achievement of global well-posedness for large smooth initial data in suitable multi-dimensional settings.

Joonhyun La: *Singular coherent structures in 2D Euler equation and hydrodynamic limits toward them.*

In this talk, we will see that certain singular coherent structures in 2D Euler equation propagates. Also, we will see that these solutions can be approximated by solutions of Boltzmann equation. The talk is based on a joint work with Theodore Drivas and Tarek Elgindi, and a joint work with Chanwoo Kim.

Mikhail Perepelitsa: *Kinetic modeling of Myxobacteria motion with nematic alignment*

Motivated by motion of myxobacteria, we review several kinetic approaches for modeling motion of self-propelled, interacting rods. We will focus on collisional models of Boltzmann type and discuss the derivation of the governing equations, the range of their validity, and present some analytical and numerical results. We will show that collisional models have natural connection to classical mean-field models of nematic alignment.

Hussain Ibdah: *Bypassing Holder super-criticality barriers in viscous, incompressible fluids*

We will go over the main ideas used in showing that as long as a super-critical Holder semi norm of the classical solution to the incompressible Navier-Stokes system is under control, the solution remains smooth. The key idea is exploiting an enhanced regularity effect coming from the transport term at the level of a simple one-dimensional drift-diffusion equation, allowing us to break the criticality barrier. We then employ ideas introduced by Kiselev, Nazarov, Volberg, and Shterenberg to propagate this to abstract drift-diffusion equations, providing to our knowledge the very first reasonable extension of the celebrated parabolic regularity result of Nash to an equation that is not in divergence form. Such an approach coupled with subtle pressure estimates due to Silvestre also allows us to rigorously treat the incompressible Navier-Stokes as a perturbation of drift-diffusion, obtaining a super-critical regularity criterion.

Wednesday's schedule was half-day talks / half-day free time. We heard two excellent talks on recent results in regularity theory of systems governing chemotactic diffusion and macroscopic system of alignment dynamics.

Alexander Kiselev: *Suppression of chemotactic blow up by buoyancy*

Chemotactic blow up in the context of the Patlak-Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that the presence of fluid advection can arrest singularity formation given that the fluid flow possesses mixing or diffusion enhancing properties and its amplitude is sufficiently strong - an effect that is conjectured to hold for more general classes of nonlinear PDE. In this talk, I will discuss Patlak-Keller-Segel equation coupled with fluid flow obeying Darcy's law via buoyancy force. It turns out that in this case, the singularity formation is suppressed by arbitrarily weak coupling. The talk is based on joint work with Zhongtian Hu and Yao Yao.

Trevor Leslie: *Limiting configurations for solutions to the 1D Euler Alignment System*

The Euler Alignment system is a hydrodynamic PDE version of the celebrated Cucker-Smale ODE's of collective behavior. Together with Changhui Tan (University of South Carolina), we developed a theory of weak solutions in 1D, which provide a uniquely determined way to evolve the dynamics after a blowup. Inspired by Brenier and Grenier's work on the pressureless Euler equations, we show that the dynamics of interest are captured by a nonlocal scalar balance law, the unique entropy solution of which we generate through a discretization involving the "sticky particle Cucker-Smale" system. In this talk, we will discuss the formation of clusters of mass in the Euler Alignment system, and we will describe how to predict these clusters using the flux from the associated scalar balance law.

On Thursday we had a very diverse program in terms of subjects discussed in the talks. The day featured a talk on implementation of machine learning to improving privacy in rapidly spreading applications of Artificial Intelligence. Alignment modeling, another take on chemotaxis-fluid systems, hydrodynamic limit of kinetic models, mass distribution of solution to variational problems, are the topics featured on this day. Interesting connections were found between these subjects. For example, the relative entropy method discussed by Aneta Wroblewska-Kaminska can potentially be implemented to handle more general system with friction/self-propulsion forces. The talk of Siming He presented a result on enhanced dissipation rates in a chemotaxis system that may be implemented to obtain optimal relaxation rate in a kinetic model of collective behavior based on Cucker-Smale communication protocol. Talk of Choi presented a very clever attempt to resolve the long standing problem of justifying kinetic Vlasov-Alignment model with singular communication.

Nicolas Garcia Trillos: *Adversarial machine learning, clustered federated learning, and how the analysis of particle dynamics can help implementing them.*

Machine learning and its applications in AI have entered into a new stage in their development: while the use of AI algorithms is widespread and will continue expanding, it is imperative to ask how can we guarantee that as these algorithms penetrate into more domains of our lives they will also be sensitive to privacy concerns, make fair decisions, and be both reliable and robust to data corruption. Are we ready to certify when a given algorithm complies with specific requirements and behaves in the way it is intended to?

In this talk, I will discuss adversarial machine learning in supervised learning and clustered federated learning, two examples of ML settings where model accuracy is not the sole criterion for training learning systems. I will present novel approaches for the training of models in these two settings that rely on the use of particle dynamics and their analysis. Our solution to the first problem is inspired by the literature of gradient flows in the space of probability measures under the Wasserstein-Fisher-Rao geometry, and our solution to the second problem is inspired by the literature of consensus-based optimization. With this talk I hope to convey the multiple opportunities for mathematicians to participate in the conversation about pressing societal questions in the development of AI.

Ruiwen Shu: *Interaction energy minimizers on bounded domains*

I will discuss the behavior of interaction energy minimizers on bounded domains. When the interaction potential is more singular than Newtonian, the mass does not tend to concentrate on the boundary; when it is Newtonian or less singular, the mass necessarily concentrates on the boundary for purely repulsive potentials. We also draw a connection between bounded-domain minimizers and whole-space minimizers.

Aneta Wroblewska-Kaminska: *Relative entropy method for hydrodynamic models.*

We show that weak solutions of degenerate compressible Navier-Stokes equations converge to the strong solutions of the pressureless Euler system with linear drag term, Newtonian repulsion and quadratic confinement. The proof is based on the relative entropy method using the artificial velocity formulation for the one-dimensional Navier-Stokes system. The result is based on the joint work with Jose A. Carrillo and Ewelina Zatorska.

Moreover we will shortly discuss how to obtain general nonlinear aggregation-diffusion models, including Keller-Segel type models with nonlinear diffusions, as relaxations from nonlocal compressible Euler type hydrodynamic systems via the relative entropy method. This result is based on the joint work with Jose A. Carrillo and Yinping Peng.

Siming He: *Enhanced dissipation and blow-up suppression in a chemotaxis-fluid system*

In this talk, we will present a coupled Patlak-Keller-Segel-Navier-Stokes (PKS-NS) system that models chemotaxis phenomena in the fluid. The system exhibits critical threshold phenomena. For example, if the total population of the cell density is less than 8π, then the solutions exist globally in time. Moreover, finite time blowup solutions exist if this population constraint is violated. We further show that globally regular solutions with arbitrary large cell populations exist. The primary blowup suppression mechanism is the shear flow mixing induced enhanced dissipation phenomena.

Jan Peszek: *Singular alignment dynamics*

I will present the latest results and ideas related to the micro- to meso- and macroscopic limit for singular alignment dynamics. This includes the heterogeneous gradient flows related to weakly singular alignment (joint with David Poyato, University of Granada) with matrix valued communication, and a monokineticity estimate for strongly singular alignment (joint with Michal Fabisiak, University of Warsaw). In particular, I will show that any weakly continuous solution to strongly singular Cucker-Smale kinetic equation is monokinetic. This information can be used to obtain (via direct micro- to macroscopic mean-field limit) existence of measure-valued solutions to the fractional Euler-alignment system in the whole space for general initial data admitting vacuum.

Young-Pil Choi: *ON THE EXISTENCE OF SOLUTIONS FOR THE VLASOV-ALIGNMENT MODEL WITH SINGULAR COMMUNICATION WEIGHTS*

In this talk, we discuss the existence theory for the Vlasov-alignment model with singular communication weights $\Phi(r) = r^{-\gamma}$ In the case $\gamma \in (0, d)$, we show the local-in-time existence of weak solutions, and the unique- ness is obtained for $\gamma \in (0, d-1]$. We also consider the hypersingular com- munication weight, where $\gamma \in (d, d + 1/4)$, and establish the local-in-time well-posedness for the Vlasov-alignment model .

The the last day, since many speakers had to leave early, we schedule only one morning talk via Zoom. Nonetheless, the talk was very informative as the model discussed is involved in many systems of collective behavior, and is attracted interest of many participants.

Ewelina Zatorska: *Analysis of traffic and collective behaviour models in 1D*

I will present our recent results on the 1-dimensional hydrodynamic models of traffic, lubrication and collective behavior in 1 dimension. I will discuss existence results, interesting two-velocity reformulations, singular limits (hard congestion) and long time behavior of solutions.

5 Conclusion

As organizers we feel like this workshop was a successful experiment, one of the first attempted, to bring together two disconnected groups of researchers from the fields of collective behavior and fluid dynamics. It also provided a platform for many young mathematicians to present their talks and connect with more senior participants. The workshop resulted in active collaboration and exchange of knowledge between the groups. New prospectives were found on different problems and new collaborations were initiated during the event.

This workshop will set an example for other similar meetings we will be planing in the near future.