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Quantum search on noisy intermediate-scale quantum devices

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Outline

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- Introduction (Grover's algorithm)
- Depth-optimized and divide-and-conquer quantum search algorithms
- Implementations of quantum search algorithms on quantum computers
- Conclusions and outlooks

NISQ era

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We have entered the noisy intermediate-scale quantum (NISQ) era. Some NISQ computers are open to researchers, for example:

- Quantum Azure of Microsoft.
- IBM quantum (IBMQ): superconducting quantum computers up to 127 qubits.
- Rigetti computing: superconducting quantum computers up to 32 qubits.
- Honeywell Quantum Solution (Quantinuum): trapped-ion quantum computers up to 12 qubits.
- IonQ: trapped-ion quantum computers up to 21 qubits.

Limitations of NISQ computers

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- NISQ computers already have enough qubits (width), over a hundred.
- It is the number of consecutive operations (depth) limits the power of quantum computers.The gates are noisy.
- Our research aims to reduce the depth (errors) of quantum search algorithm.

Search problem

- Each item of the database corresponds to a basis vector in the Hilbert space.
- One-way function (oracle) f:

$$f : \{0,1\}^n \rightarrow \{0,1\}$$

Quantum query checks all the witnesses simultaneously.

- Evaluate the one-way function $f(x)$ is polynomial fast. Evaluation of the inverse $f^{-1}(1) = t$ is exponentially slow (like the telephone book).
- Search problem is to find $t \in \{0,1\}^n$ which gives $f(t) = 1$, given by the one-way function. We assume one target item (also called marked item or solution).

Grover's algorithm

- Initial state (wave function) is the average of all the items in the database: $|s_n\rangle = H^{\otimes n}|0\rangle^{\otimes n}$. Here H is the Hadamard gate.
- Query to the oracle (phase kickback): $U_t = \mathbb{1}_{2^n} - 2|t\rangle\langle t|$
  Assume one target item $|t\rangle$.
- (Global) diffusion operator uses the n-qubit Toffoli gate:

$$D_n = 2|s_n\rangle\langle s_n| - \mathbb{1}_{2^n}$$

  Reflection in the average. This will be modified. We shall separate the database into several blocks of equal size and replace $|s_n\rangle$ by averages in blocks.
- (Global) Grover operator: $G_n = D_n U_t$

Motivation

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Grover's algorithm is optimal in the number of oracle. There are different measures of complexity. Can we improve it performance on actual devices?

Yes! Because oracle is NOT the only operations in Grover's algorithm. Diffusion uses a lot of gates, wihc create noise.

Local diffusion operator

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- Simplest version. Divide the database into blocks of the equal size (2^{n-m} blocks):

$$D_m = \mathbb{1}_{2^{n-m}} \otimes (2|s_m\rangle\langle s_m| - \mathbb{1}_{2^m})$$

simultaneously in each block. Can be realised by Toffoli gates.

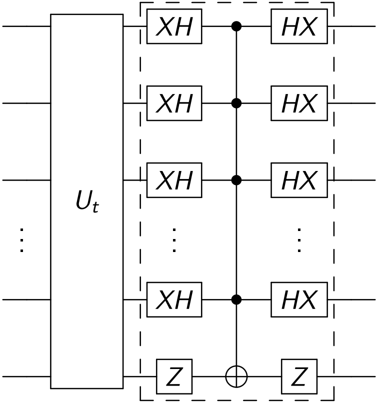
- Local Grover operator

$$G_m = D_m U_t$$

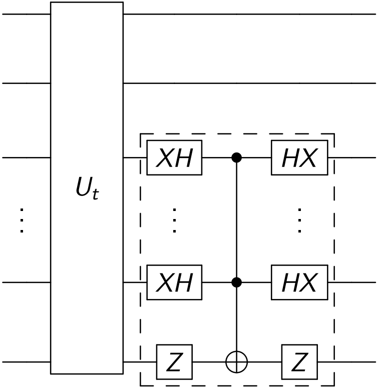
- Operators G_n and G_m can be represented as elements of $O(3)$ group. They do not commute if $m \neq n$.

- Over the course of the algorithm we can use different m.

Local diffusion operator

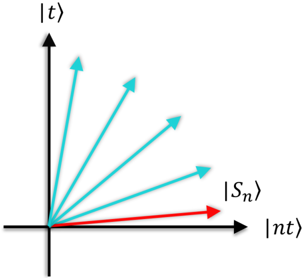


D_n

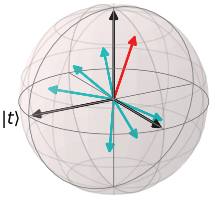


D_m

Non-Abelian search



Grover's algorithm: $O(2)$ rotation.



Search with global and local diffusion operators. It is the $O(3)$ rotation. Use of local diffusion operators of different sizes with increase the dimension of the sphere.

Strategy I

Hybrid classical-quantum search:

- We can guess first several bits of the target item address. We initialize the state [sequence of bits] as $|t_1'\rangle \otimes |s_m\rangle$. The success probability of $t_1' = t_1$ is $P(t_1' = t_1)$.
- Apply G_m^j on the initial state.
- The total success probability is $P_n'(j) = P(t_1' = t_1)P_m(j)$ with

$$P_m(j) = \sin^2((2j+1)\theta_b)$$

with the new angle: $\sin\theta_b = 1/\sqrt{b}$ and $b = 2^m$ is the number of items in each block. When we shall get several local diffusions then we shall get several different angles.

Strategy II

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- Design the operator

$$S_{n,m}(\tilde{j}) = G_n^{j_1} G_m^{j_2} \cdots G_n^{j_{q-1}} G_m^{j_q}$$

The G_n and G_m are non-commuting elements of $O(3)$ group.

- The success probability is

$$P_{n,m}(\tilde{j}) = |\langle t|S_{n,m}(\tilde{j})|s_n\rangle|^2$$

- The depth of the operator $S_{n,m}(\tilde{j})$ is $d(S_{n,m}(\tilde{j}))$. The expected depth of the circuit is

$$\langle d_{n,m}(\tilde{j})\rangle = \frac{d(S_{n,m}(\tilde{j}))}{P_{n,m}(\tilde{j})}$$

- Depth optimization

$$\langle d_n \rangle = \min_{m,\tilde{j}} \langle d_{n,m}(\tilde{j})\rangle$$

Strategy III

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Divide-and-conquer strategy with depth optimization:

- Operator $S^{(1)}_{n,m}(\tilde{j})$ is to find t_1. The success probability is $P^{(1)}_{n,m}(\tilde{j})$.
- Operator $S^{(2)}_{n,m'}$ is to find t_2. The success probability is $P^{(2)}_{n,m}(\tilde{j}')$.
- The expected depth of the circuit is

$$\langle d_{n,m,m'}(\tilde{j},\tilde{j}')\rangle = \frac{d(S^{(1)}_{n,m}(\tilde{j})) + d(S^{(2)}_{n,m'}(\tilde{j}'))}{P^{(1)}_{n,m}(\tilde{j})P^{(2)}_{n,m}(\tilde{j}')}$$

- Depth optimization

$$\langle d_{n,2}\rangle = \min_{m,m',\tilde{j},\tilde{j}'} \langle d_{n,m,m'}(\tilde{j},\tilde{j}')\rangle$$

Critical depth ratio

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- Circuit/operator depth is the number of consecutive elementary operations required to run a circuit/operator on quantum hardware.

- Define the depth ratio

$$\alpha = \frac{\mathrm{d}(U_t)}{\mathrm{d}(D_n)}.$$

- Consider the critical ratio α_c, below which Grover's algorithm is not optimal in depth. We find $\alpha_c = \mathcal{O}(n^{-1}2^{n/2})$.

- If the circuit is divided into two stages, the critical ratio is a constant at large n: $\lim_{n\to\infty} \alpha_c = 1 + \sqrt{3}$.

Remarks

- Three strategies can be jointly applied.
- Different problems (oracles), different quantum devices, different error rates, et al., all can give different optimal circuits. We can choose different local diffusion over the course of the algorithm [different m].
- Divide-and-conquer can also be applied to parallel running.

Depth-optimized quantum search algorithm

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We test different realizations of three- and four-qubit quantum search algorithm on different IBMQ devices.

- We choose the toy oracle which is single-qubit-gate equivalent to the *n*-qubit Toffoli gate.
- We choose the target states randomly.
- Probabilities are obtained on 30 trails with 8192 shots.

Notations

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For example, the 3-qubit search algorithm can have the circuits:

- D3M3: one 3-qubit diffusion operator followed by 3-qubit measurements.
- D2M3: one 2-qubit diffusion operator followed by 3-qubit measurements.
- G1D2M2: random guess one qubit then one 2-qubit diffusion operator followed by 2-qubit measurements.
- D2M1|D2M2: first stage is one 2-qubit diffusion operator followed by 1-qubit measurement; second stage is one 2-qubit diffusion operator followed by 2-qubit measurement.

Metrics

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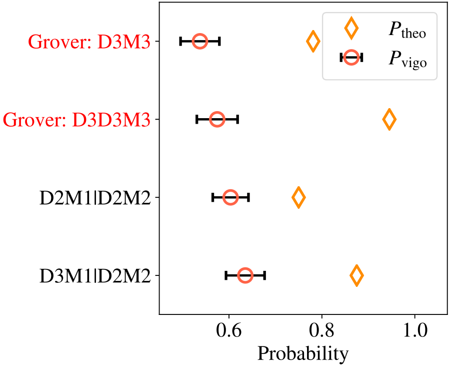
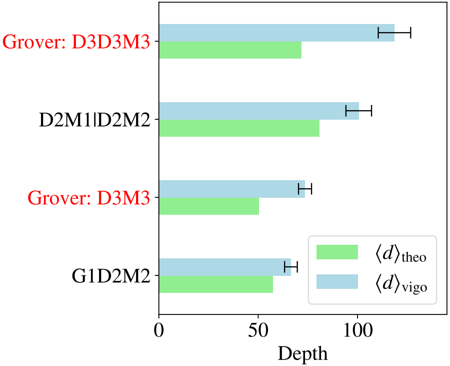
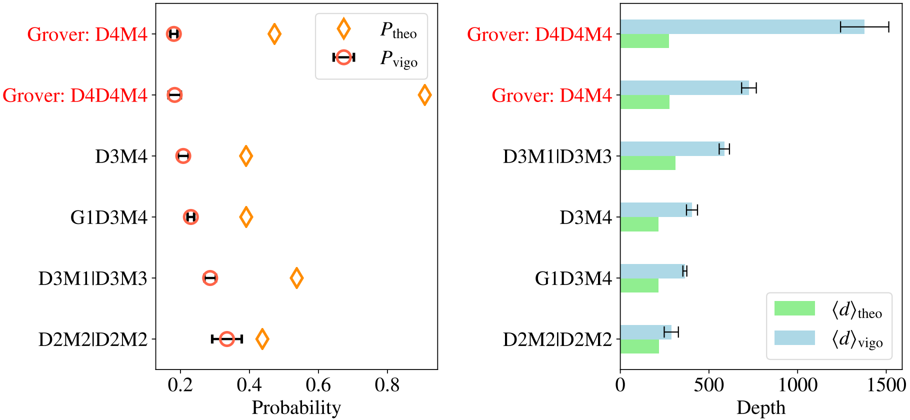
- Success probability finding the target state.
- Expected depth is the averaged depth needed in order to find the target state. It also characterizes the expected time finding the target state.
- There are more metrics, such as selectivity and circuit fidelity.

Three-qubit search



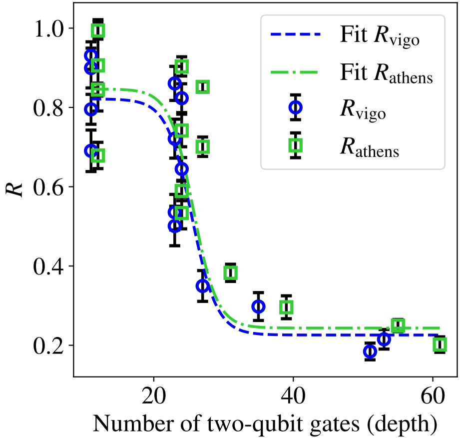


Four-qubit search



Degraded ratio

NISQ devices favors shallow depth circuits. $R = P_{\text{real}}/P_{\text{theo}}$

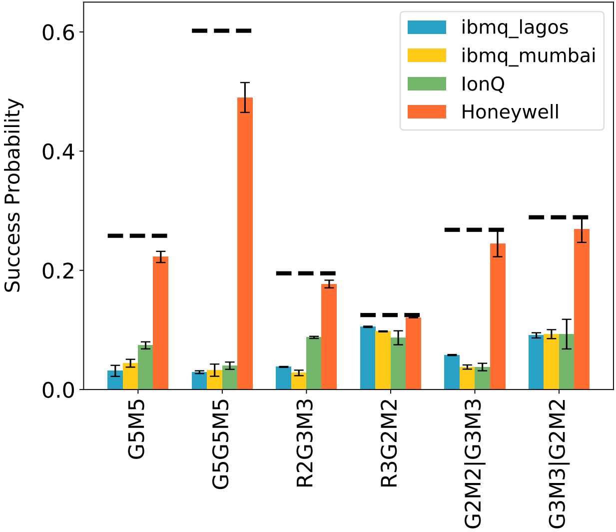


Setup for benchmarks

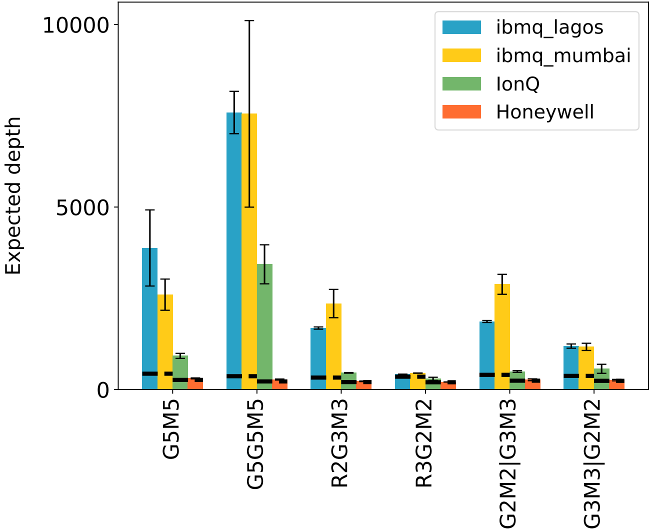
We also benchmark the five-qubit quantum search algorithms on different platforms of quantum computing: IBMQ, IonQ and Honeywell quantum devices.

- We choose the toy oracle which reflects the phase of state $|01011\rangle$.
- Each circuit runs 3×400 shots to estimate the output distributions.
- There is always a trade-off between circuit width and depth. We choose the implementations with the least depth, which requires one ancillary qubit (total six qubits for the five-qubit search algorithm).

Success probability



Expected depth



Expected depth

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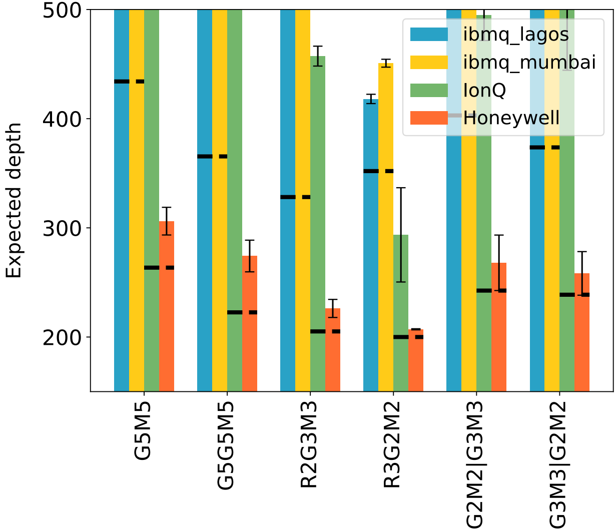

Noisy simulation

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- We also simulate the noisy implementations. The depolarizing channel

$$\mathcal{L}(\rho) = (1-\varepsilon)\,\rho + \frac{\varepsilon}{2^n}\mathbb{1},$$

is applied after each step. The error rate ranges $\varepsilon \in [0, 1]$.

- The steady state of depolarizing channel is the classical states with the uniform distribution. The actual errors in quantum computers are much more complicated.

Conclusions and outlooks

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- Grover's algorithm is not optimal in real implementations. Designing the optimal-depth quantum search circuits are highly nontrivial.
- Current quantum processors can barely run the five-qubit search algorithms.
- Although NISQ devices have enough number of qubits, the quality of qubits limits the circuit depth. It is hard to find practical usefulness of circuits with shallow depths.
- Designing noisy-resilience quantum circuits would be extremely important in practice.

Disclaimer

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The material of the this talk is based on papers:

- K. Zhang and V. Korepin, *Depth optimization of quantum search algorithms beyond Grover's algorithm*, Phys. Rev. A, **101**, 032346 (2020).
- K. Zhang, P. Rao, K. Yu, H. Lim, and V. Korepin, *Implementation of improved quantum search algorithms on IBM quantum processors*, Quantum Inf. Process. 20, 233 (2021).
- K. Zhang, K. Yu, and V. Korepin, *Quantum search on noisy intermediate-scale quantum devices*, arXiv:2202.00122 (2022).

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Thanks you for your attention!