Generalised geometry, consistent truncations and the Kaluza-Klein spectrum of string compactifications

Emanuel Malek

Humboldt-Universität zu Berlin

Geometry and Swampland
25th January 2022

with Bobev, Galli, Giambrone, Guarino, Josse, Nicolai, Petrini, Robinson, Samtleben, Sterckx, Trigiante, Vall Camell, van Muiden, Waldram

Consistent truncations

Lower-dimensional theory for compactifications
without scale separation?

Most (all? [Lüst, Palti, Vafa '19]) AdS vacua of string theory

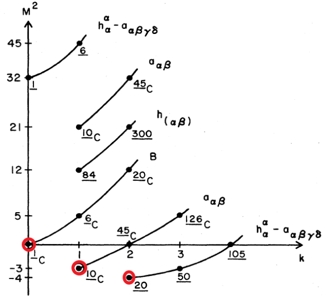
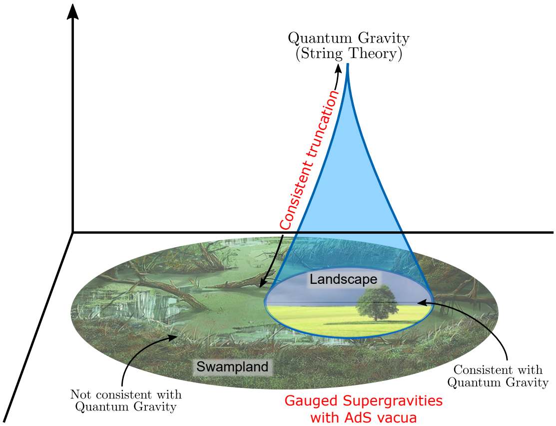


FIG. 2. Mass spectrum of scalars.

Consistent truncation:
All solutions of lower-dim. theory → solutions of 10-d/11-d SUGRA

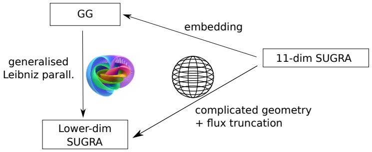
Connection to Swampland

If no AdS vacua have scale separation, only theories with AdS that arise from consistent truncations have higher-dim origin

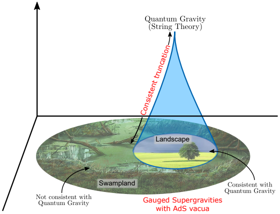


Generalised geometry

& consistent truncations



Swampland of gSUGRA?



Kaluza-Klein spectroscopy

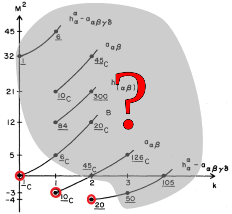


FIG. 2. Mass spectrum of scalars.

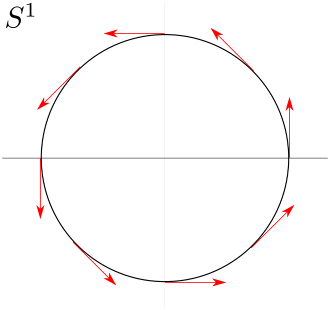
Consistent truncation

Non-linear embedding of lower-dimensional theory
into 10-/11-d supergravity

- All solutions of lower-d SUGRA $\rightarrow$ solutions of 10-/11-d SUGRA
- Non-linearity: highly non-trivial!
- Symmetry arguments crucial

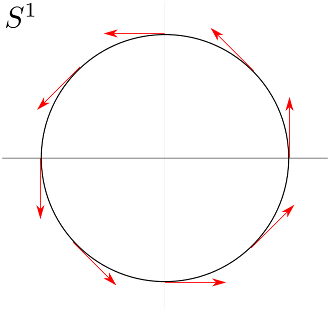
Consistent truncation on group manifold

Symmetry arguments crucial for consistency, e.g. group manifold



Consistent truncation on group manifold

Symmetry arguments crucial for consistency, e.g. group manifold



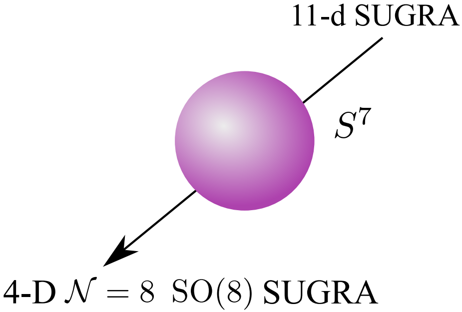
$$U_m{}^\mu \in \mathrm{GL}(D)$$

$$L_{U_m} U_n = f_{mn}{}^p U_p$$

$$g_{\mu\nu}(x, y) = g_{mn}(x)\,(U^{-1})_\mu{}^m(y)(U^{-1})_\nu{}^n(y)$$

Consistent truncations beyond group manifolds

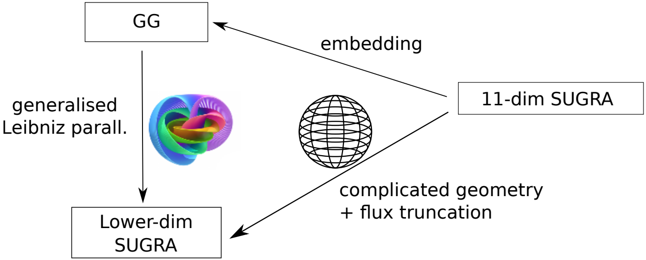
Consistent truncations of 10-d/11-d SUGRA beyond group manifolds?



[de Wit, Nicolai '82]

Generalised geometry and consistent truncations

Consistent truncations captured by
"generalised group manifolds" in GG



$$U_A{}^M \in E_{d(d)}$$

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C$$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

Consistent truncations with less SUSY

Generalised $G \subset E_{d(d)}$ structure
"Singlet intrinsic torsion"

[EM '17], [Cassani, Josse, Petrini, Waldram '19]
Set of well-defined tensors (stabilised by G):

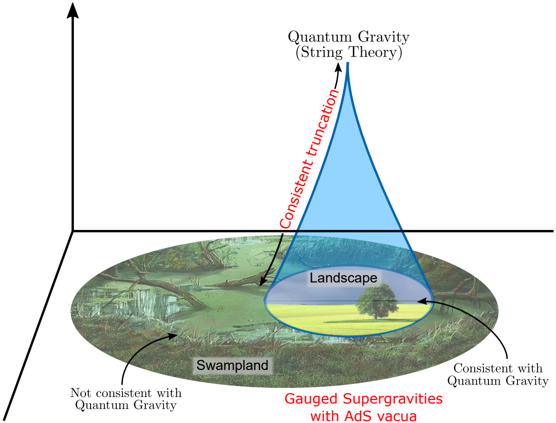
$$\left\{ \mathcal{J}_u{}^M, \ldots \right\}$$

Closed under derivative:

$$\mathcal{L}_{\mathcal{J}_u} \mathcal{J}_v{}^M = f_{uv}{}^w \mathcal{J}_w{}^M$$

Constraints on matter multiplets, gaugings!

Swampland of gSUGRA?



Swampland vs Landscape & consistent truncations

General features of theories from consistent truncations

- Scalar manifold $\rightarrow$ symmetric space
- $M_{\mathrm{scalar}} = \frac{\mathrm{Com}(G, E_{d(d)})}{\mathrm{Com}(G, K_{d(d)})}$
- Compact gauging $\longleftrightarrow$ Killing vectors on compactification

Swampland vs Landscape & consistent truncations

General features of theories from consistent truncations

- Scalar manifold $\rightarrow$ symmetric space
- $M_{\text{scalar}} = \frac{\text{Com}(G, E_{d(d)})}{\text{Com}(G, K_{d(d)})}$
- Compact gauging $\longleftrightarrow$ Killing vectors on compactification

Example

- $\frac{1}{2}$-max theories in $D \geq 4$ dimensions, $G = \text{Spin}(10 - D - N)$

  $\Longrightarrow N \leq 10 - D$ vector multiplets possible

  [EM '17], [Cassani, Josse, Petrini, Waldram '19]

Swampland vs Landscape & consistent truncations

General features of theories from consistent truncations

- Scalar manifold $\rightarrow$ symmetric space
- $M_{\mathrm{scalar}} = \frac{\mathrm{Com}(G, E_{d(d)})}{\mathrm{Com}(G, K_{d(d)})}$
- Compact gauging $\longleftrightarrow$ Killing vectors on compactification

Example

- $\frac{1}{2}$-max theories in $D \geq 4$ dimensions, $G = \mathrm{Spin}(10 - D - N)$
  $\Longrightarrow N \leq 10 - D$ vector multiplets possible
  [EM '17], [Cassani, Josse, Petrini, Waldram '19]
- $D = 5$ $\mathcal{N} = 2$ SUGRA, $G \subset \mathrm{USp}(6)$
  $\Longrightarrow n_{\mathrm{VT}} \leq 14$ vector-tensor multiplets, $n_{\mathrm{H}} \leq 2$ hypermultiplets
  [Josse, EM, Petrini, Waldram '21]

Swampland of AdS gSUGRA

More constraints for gSUGRA with max SUSY AdS, e.g.

- 5-d $\mathcal{N} = 4$ theories: $\leq 10 - D = 5$ vector multiplets

Swampland of AdS gSUGRA

More constraints for gSUGRA with max SUSY AdS, e.g.

- 5-d $\mathcal{N} = 4$ theories: ≤ 3 vector multiplets, handful of gaugings
  No "exotic" RG flows [Bobev, Cassani, Triendl '18]

[EM, Vall Camell '20]

Swampland of AdS gSUGRA

More constraints for gSUGRA with max SUSY AdS, e.g.

- 5-d $\mathcal{N} = 4$ theories: ≤ 3 vector multiplets, handful of gaugings
  No "exotic" RG flows [Bobev, Cassani, Triendl '18]

  [EM, Vall Camell '20]

- 3-d $\mathcal{N} = 16$ theories: compact gauging $\subset$ SO(9)
  c.f. gaugings $E_{8(8)}$, SO(8) $\times$ SO(8), ...

  [Galli, EM – to appear]

Relation to Swampland conjectures?

Kaluza-Klein spectroscopy

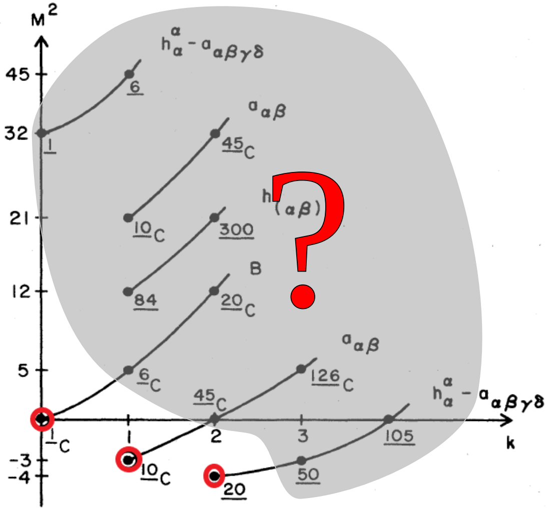


FIG. 2. Mass spectrum of scalars.

Kaluza-Klein spectroscopy

Consistent truncation:

- Lower-dimensional theory
- Compute <u>subset</u> of masses for any vacuum!

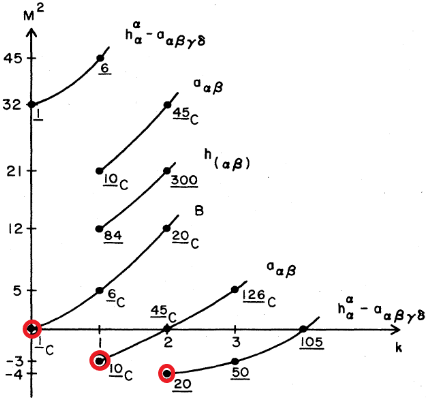
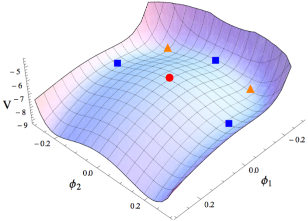


FIG. 2. Mass spectrum of scalars.



Kaluza-Klein spectroscopy

Consistent truncation:

- Lower-dimensional theory
- Compute <u>subset</u> of masses for any vacuum!

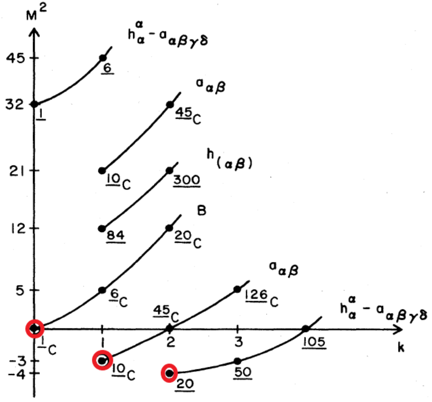
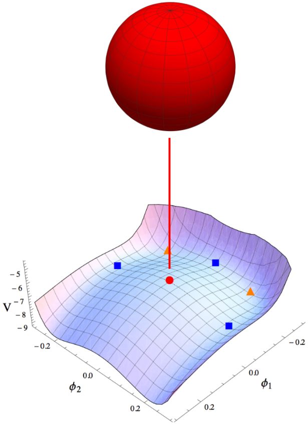


FIG. 2. Mass spectrum of scalars.



Kaluza-Klein spectroscopy

Consistent truncation:

- Lower-dimensional theory
- Compute <u>subset</u> of masses for any vacuum!

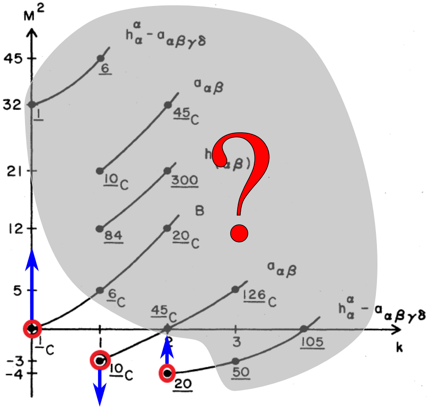
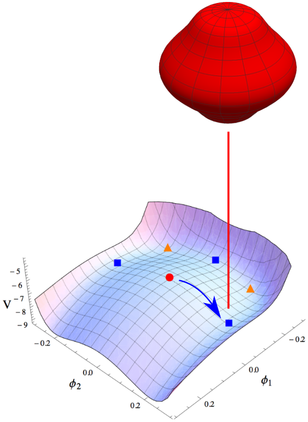


FIG. 2. Mass spectrum of scalars.



Kaluza-Klein spectroscopy

Consiste

- Lo
- Co

[EM, Samtleben '20]

Extend this to full KK spectrum using GG!

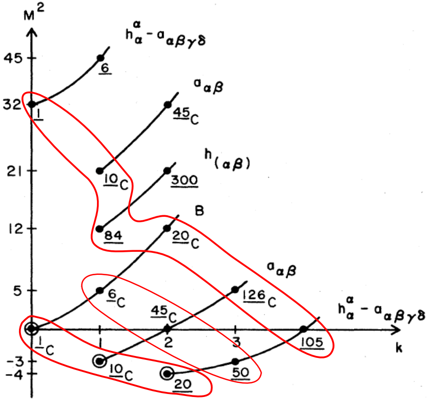
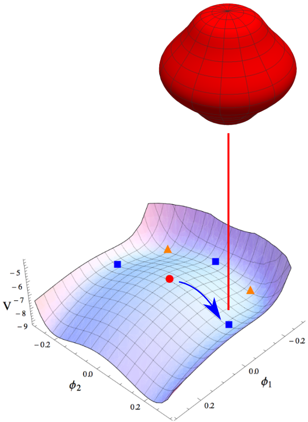


FIG. 2. Mass spectrum of scalars.



Traditional Kaluza-Klein spectroscopy

Traditionally:

- Spin-2 fields [Bachas, Estes '11] ✓
- $M_{int} = \frac{G}{H}$ [Salam, Strathdee '81] ✓

[EM, Samtleben '20]:

- Full spectrum for vacua of maximal gSUGRA
- Compactifications with few or no remaining (super-)symmetries!

KK spectroscopy strategy

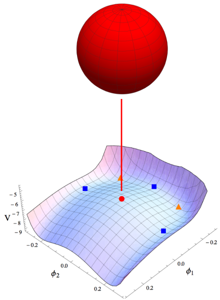
Traditional KK Ansatz: $\phi(x, y) = \phi^{\Sigma}(x) \underbrace{\mathcal{Y}_{\Sigma}(y)}_{\text{harmonics}}$

KK spectroscopy strategy

Traditional KK Ansatz: $\phi(x, y) = \phi^{\Sigma}(x) \underbrace{\mathcal{Y}_{\Sigma}(y)}_{\text{harmonics}}$

GG KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{harmonics}}_{\text{linear}}$

First at max symmetric point:

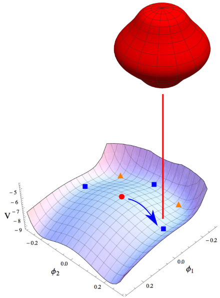


KK spectroscopy strategy

Traditional KK Ansatz: $\phi(x, y) = \phi^{\Sigma}(x) \underbrace{\mathcal{Y}_{\Sigma}(y)}_{\text{harmonics}}$

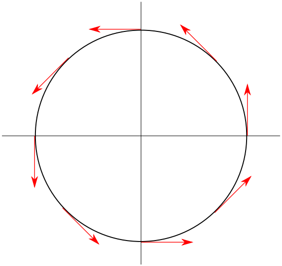
GG KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{harmonics}}_{\text{linear}}$

Then at less symmetric point:



KK spectroscopy at max. symmetric point

${U_A}^M \in E_{d(d)}$ give basis for all fields



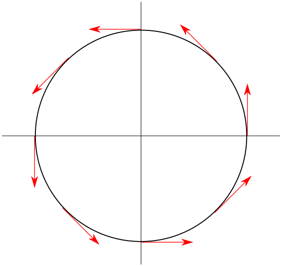
Only need scalar harmonics: $\mathcal{Y}_\Sigma$

c.f. $h_{ij}(x,y) = \sum_\ell h^{(\ell)}(x)\mathcal{Y}^{(\ell)}_{(ij)}(y), \quad b_{ij}(x,y) = \sum_\ell b^{(\ell)}(x)\mathcal{Y}^{(\ell)}_{[ij]}(y)$

"$\mathcal{N}=8$ supermultiplet contains all SUGRA fields"

KK spectroscopy at max. symmetric point

${U_A}^M \in E_{d(d)}$ give basis for all fields

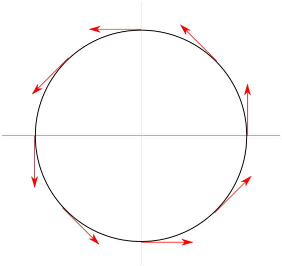


Only need scalar harmonics: $\mathcal{Y}_\Sigma$

$$\mathcal{M}_{MN}(x, Y) \in E_{7(7)}/\mathrm{SU}(8)$$

KK spectroscopy at max. symmetric point

${U_A}^M \in E_{d(d)}$ give basis for all fields



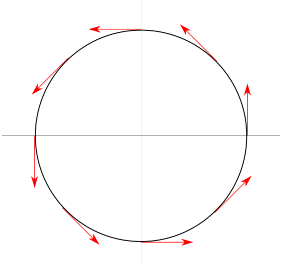
Only need scalar harmonics: $\mathcal{Y}_\Sigma$

$$\mathcal{M}_{MN}(x,Y) = (\delta_{AB} + j_{AB}(x))(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

$$j_{AB} \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

KK spectroscopy at max. symmetric point

${U_A}^M \in E_{d(d)}$ give basis for all fields



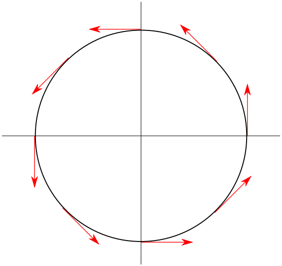
Only need scalar harmonics: $\mathcal{Y}_\Sigma$

$$\mathcal{M}_{MN}(x,Y) = (\delta_{AB} + {j_{AB}}^{\Sigma}(x)\mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

$${j_{AB}}^{\Sigma} \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

KK spectroscopy at max. symmetric point

${U_A}^M \in E_{d(d)}$ give basis for all fields



Only need scalar harmonics: $\mathcal{Y}_\Sigma$

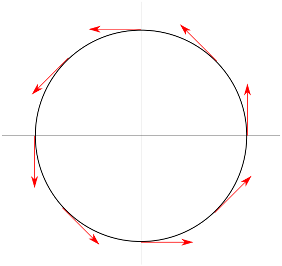
$$\mathcal{M}_{MN}(x,Y) = (\delta_{AB} + {j_{AB}}^\Sigma(x)\mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

$${j_{AB}}^\Sigma \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

KK Ansatz = consistent truncation $\otimes$ scalar harmonics

KK spectroscopy at max. symmetric point

${U_A}^M \in E_{d(d)}$ give basis for all fields



Only need scalar harmonics: $\mathcal{Y}_\Sigma$

$$\mathcal{M}_{MN}(x,Y) = (\delta_{AB} + {j_{AB}}^\Sigma(x)\mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

$${j_{AB}}^\Sigma \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

Immediate mass diagonalisation for any vacuum!

Mass matrix

- Lower-dim info:
$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C \,,$$
- Higher-dim info:
$$\mathcal{L}_{U_A} \mathcal{Y}_\Sigma = L_{K_A} \mathcal{Y}_\Sigma = \mathcal{T}_{A\,\Sigma}{}^\Omega \mathcal{Y}_\Omega \,.$$

Mass matrix:

$$\mathbb{M}^{(\mathrm{scalar})}_{I\Sigma,J\Omega} = \mathbb{M}^{(0)}_{IJ}\,\delta_{\Sigma\Omega} + \delta_{IJ}\,\mathbb{M}^{(\mathrm{spin-2})}_{\Sigma\Omega} + \mathcal{N}_{IJ}{}^C \mathcal{T}_{C,\Omega\Sigma}$$

Mass matrix

- Lower-dim info:
$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C \, ,$$
- Higher-dim info:
$$\mathcal{L}_{U_A} \mathcal{Y}_\Sigma = L_{K_A} \mathcal{Y}_\Sigma = \mathcal{T}_{A\,\Sigma}{}^{\Omega} \mathcal{Y}_\Omega \, .$$

Mass matrix:

$$\mathbb{M}^{(\mathrm{scalar})}_{I\Sigma,J\Omega} = \mathbb{M}^{(0)}_{IJ}\, \delta_{\Sigma\Omega} + \delta_{IJ}\, \mathbb{M}^{(\mathrm{spin-2})}_{\Sigma\Omega} + \mathcal{N}_{IJ}{}^{C} \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}^{(0)}_{IJ} \sim X^2$

Mass matrix

- Lower-dim info:
$$\mathcal{L}_{U_A} U_B = X_{AB}{}^{C} U_C \,,$$
- Higher-dim info:
$$\mathcal{L}_{U_A} \mathcal{Y}_\Sigma = L_{K_A} \mathcal{Y}_\Sigma = \mathcal{T}_{A\,\Sigma}{}^{\Omega} \mathcal{Y}_\Omega \,.$$

Mass matrix:

$$\mathbb{M}^{(\text{scalar})}_{I\Sigma,J\Omega} = \mathbb{M}^{(0)}_{IJ}\,\delta_{\Sigma\Omega} + \delta_{IJ}\,\mathbb{M}^{(\text{spin}-2)}_{\Sigma\Omega} + \mathcal{N}_{IJ}{}^{C}\mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}^{(0)}_{IJ} \sim X^2$
- Spin-2 mass matrix $\mathbb{M}^{(\text{spin}-2)}_{\Sigma\Omega} = \mathcal{T}_{A,\Sigma\Lambda}\mathcal{T}_{A,\Lambda\Omega}$

Mass matrix

- Lower-dim info:

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C \,,$$

- Higher-dim info:

$$\mathcal{L}_{U_A} \mathcal{Y}_\Sigma = L_{K_A} \mathcal{Y}_\Sigma = \mathcal{T}_{A\,\Sigma}{}^\Omega \mathcal{Y}_\Omega \,.$$

Mass matrix:

$$\mathbb{M}^{(\mathrm{scalar})}_{I\Sigma,J\Omega} = \mathbb{M}^{(0)}_{IJ}\,\delta_{\Sigma\Omega} + \delta_{IJ}\,\mathbb{M}^{(\mathrm{spin-2})}_{\Sigma\Omega} + \mathcal{N}_{IJ}{}^C \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}^{(0)}_{IJ} \sim X^2$
- Spin-2 mass matrix $\mathbb{M}^{(\mathrm{spin-2})}_{\Sigma\Omega} = \mathcal{T}_{A,\Sigma\Lambda} \mathcal{T}_{A,\Lambda\Omega}$

Mass matrix

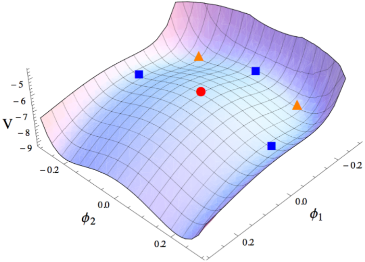
- Lower-dim info:
$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C \,,$$
- Higher-dim info:
$$\mathcal{L}_{U_A} \mathcal{Y}_\Sigma = L_{K_A} \mathcal{Y}_\Sigma = \mathcal{T}_{A\,\Sigma}{}^{\Omega} \mathcal{Y}_\Omega \,.$$

Mass matrix:

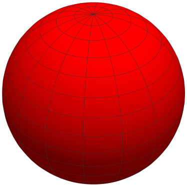
$$\mathbb{M}^{(\mathrm{scalar})}_{I\Sigma,J\Omega} = \mathbb{M}^{(0)}_{IJ}\, \delta_{\Sigma\Omega} + \delta_{IJ}\, \mathbb{M}^{(\mathrm{spin}-2)}_{\Sigma\Omega} + \mathcal{N}_{IJ}{}^C \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}^{(0)}_{IJ} \sim X^2$
- Spin-2 mass matrix $\mathbb{M}^{(\mathrm{spin}-2)}_{\Sigma\Omega} = \mathcal{T}_{A,\Sigma\Lambda} \mathcal{T}_{A,\Lambda\Omega}$
- Key object:
$$\mathcal{N}_{IJ}{}^C \sim X$$

KK spectroscopy at less symmetric point

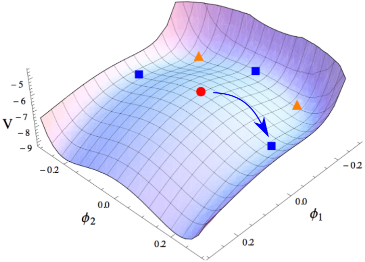


$\longleftrightarrow$

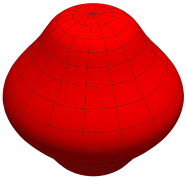


$$\text{KK Ansatz: } \underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$$

KK spectroscopy at less symmetric point



$\longleftrightarrow$



$$\text{KK Ansatz: } \underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$$

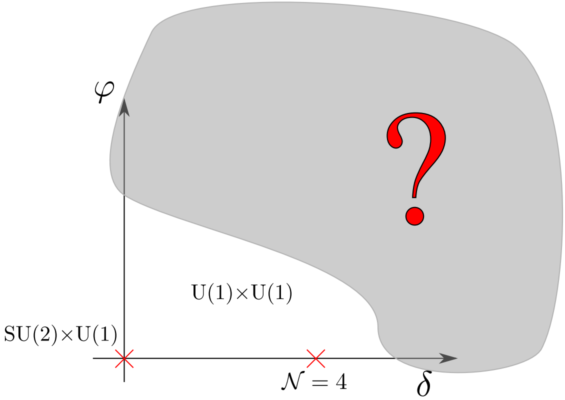
Multiplication by $E_{7(7)}$ matrix, $M_{AB}(x)$!

Use same harmonics as for max. symmetric point

$\mathcal{N} = 2$ AdS$_4$ family

$[\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$ supergravity

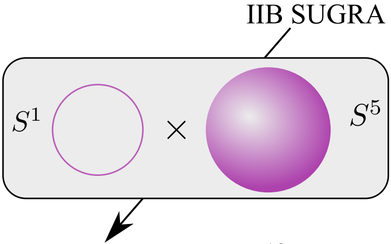
2 moduli $(\varphi, \delta) \in \mathbb{R}^2_{\geq 0}$ in 4-d theory $\Leftrightarrow$ $\mathcal{N} = 2$ conformal manifold
[Guarino, Sterck, Trigiante '2020]



Expected to be compact e.g. [Perlmutter, Rasteli, Vafa, Valenzuela, '20]

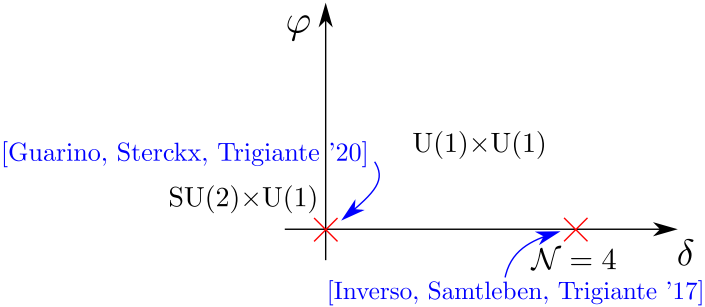
Uplift to IIB string theory

[Inverso, Samtleben, Trigiante '16]



4-D $[\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$ SUGRA

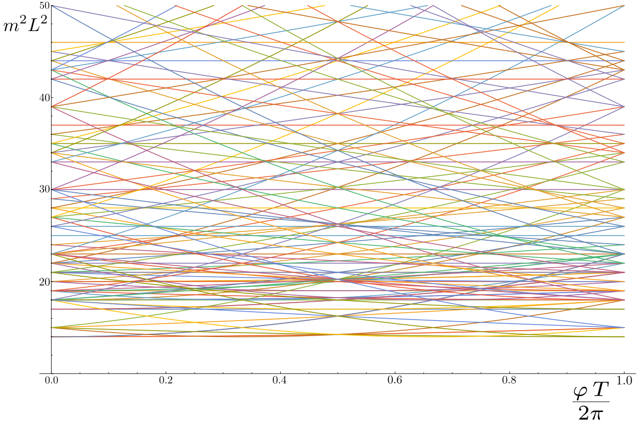
$\mathrm{AdS}_4 \times S^5 \times S^1$ "S-fold" of IIB



Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

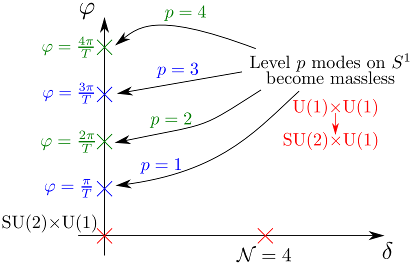
[Giambrone, EM, Samtleben, Trigiante '21]



$\varphi \sim \varphi + \frac{2\pi}{T}$, T radius of S^1

Space invaders

Higher KK modes become massless when $\varphi = \frac{p\,\pi}{T}$, $p \in \mathbb{Z}$
[Giambrone, EM, Samtleben, Trigiante '21]



Spectrum identical for $\varphi = \frac{2\,p\,\pi}{T}$, $p \in \mathbb{Z}$
Spectrum differs for $\varphi = \frac{(2\,p+1)\,\pi}{T}$, $p \in \mathbb{Z}$

Compactness of $\mathcal{N} = 2$ moduli space

[Giambrone, EM, Samtleben, Trigiante '21]

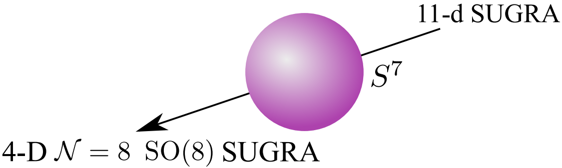
$\varphi \in \mathbb{R}^+$ is a 4-d artefact

$\varphi \in [0, \frac{2\pi}{T})$ in 10 dimensions

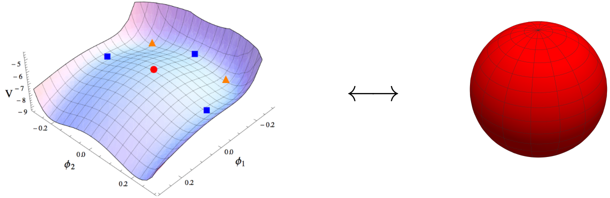
$\varphi \to \mathbb{C}$-structure modulus on $S^5 \times S^1$

$\varphi \to$ locally coordinate transformation

Non-SUSY SO(3)× SO(3) AdS$_4$ vacua

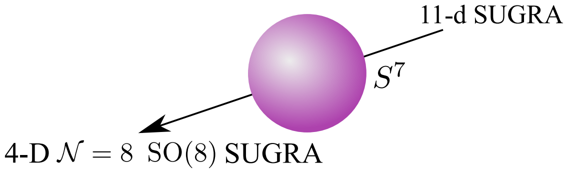


- Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
- Non-SUSY SO(3) × SO(3) AdS$_4$ vacuum [Warner '83]

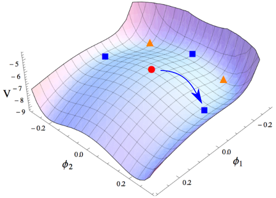


- Instability?

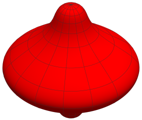
Non-SUSY SO(3)$\times$ SO(3) AdS$_4$ vacua



- Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
- Non-SUSY SO(3) $\times$ SO(3) AdS$_4$ vacuum [Warner '83]



$\longleftrightarrow$



- Instability?

Perturbative stability?

4-d "zero-mode" stability enough for 11-d perturbative stability?

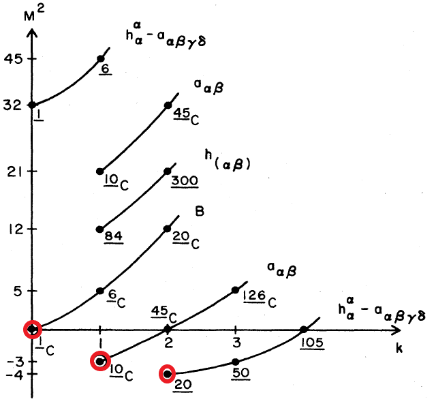
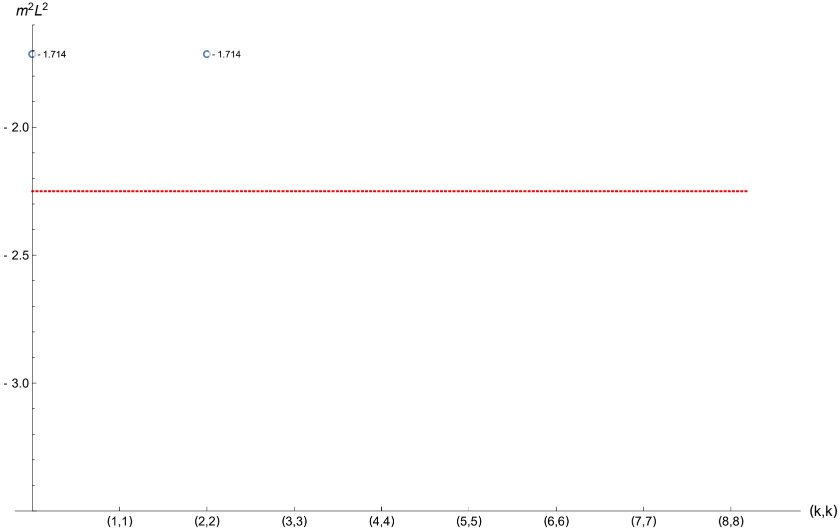


FIG. 2. Mass spectrum of scalars.

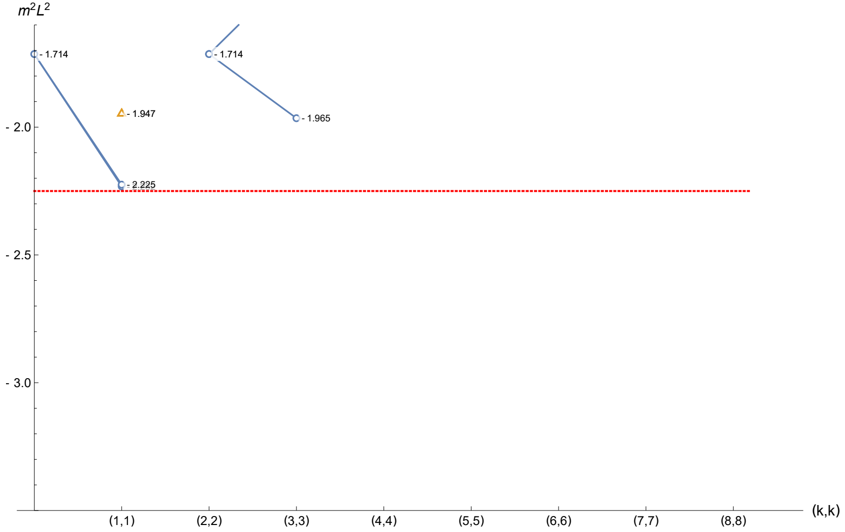
Tachyonic KK modes

Modes $\ell = 0$: $\mathcal{N} = 8$ supergravity multiplet [Fischbacher, Pilch, Warner '10]



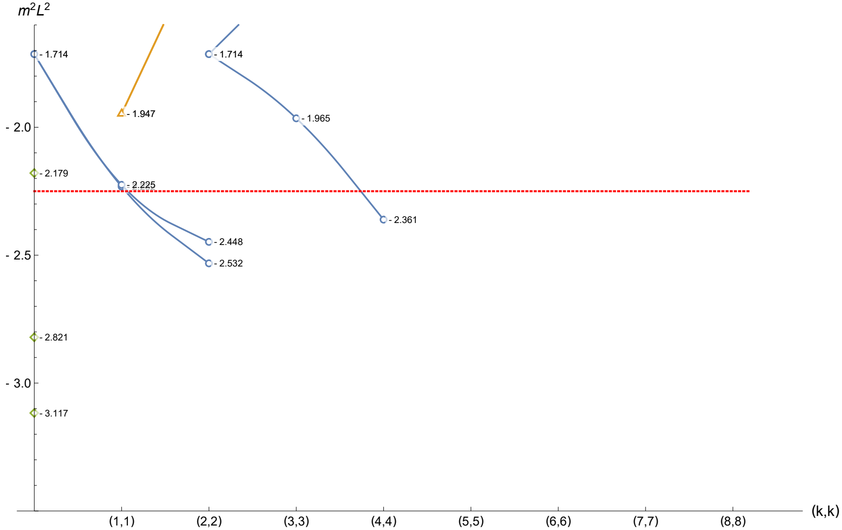
Tachyonic KK modes

Modes $\ell \leq 1$: still stable! [EM, Nicolai, Samtleben '20]



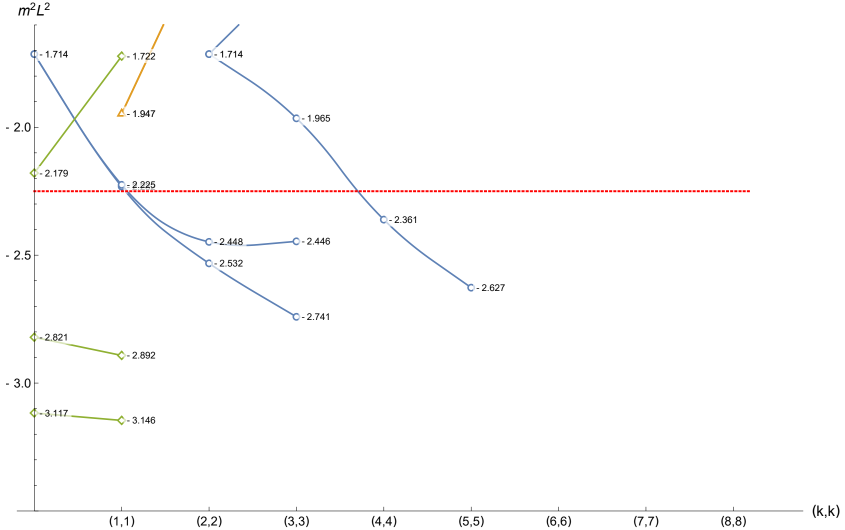
Tachyonic KK modes

Modes $\ell \leq 2$: **tachyons!** [EM, Nicolai, Samtleben '20]



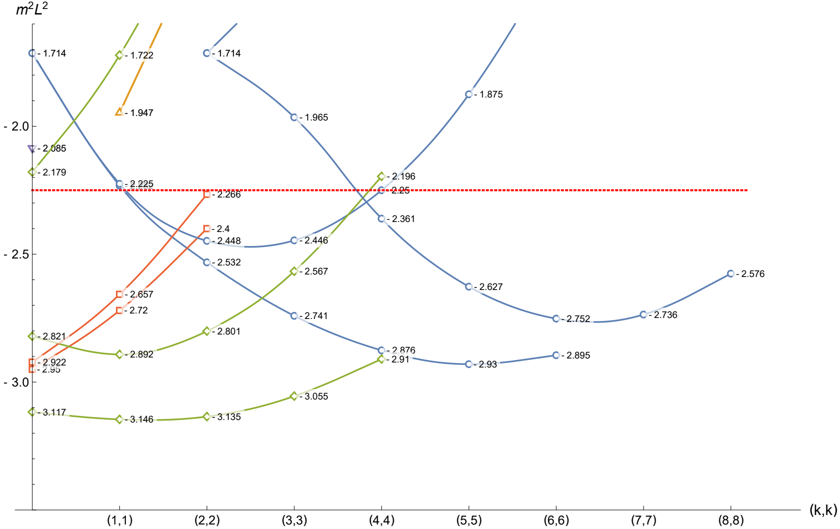
Tachyonic KK modes

Modes $\ell \leq 3$ [EM, Nicolai, Samtleben '20]



Tachyonic KK modes

Modes $\ell \leq 6$ [EM, Nicolai, Samtleben '20]



Kaluza-Klein instability

Higher KK modes are tachyonic!
[EM, Nicolai, Samtleben '20]

- Non-SUSY SO(3) × SO(3) AdS_4 [Warner '83] is perturbatively unstable
- "Zero-mode" stability does not guarantee perturbative stability in higher dimensions
- Related to brane-jet instability [Bena, Pilch, Warner '20]?

- Examples of perturbatively stable non-SUSY AdS_4 vacua in 10-d
[Guarino, EM, Samtleben '20]
[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

Conclusions

GG: construct consistent truncations
& compute full KK spectrum

- Scale separation: Many gSUGRA in "Swampland"?
- Higher KK modes crucial for physics, e.g. compactness, stability
- AdS/CFT: KK spectrum $\Leftrightarrow$ Anomalous dimensions
  [Bobev, EM, Robinson, Samtleben, van Muiden '20]

Thank you!