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A review of generalised geometry.

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1. Introduction

What is generalized geometry? Minimalistically

formalism or set of tools naturally adapted to flux backgrounds in supergravity.

such that

- geometrises all bosonic degrees of freedom
- unifies symmetries
- plays nicely with supersymmetry.

"geometry of supergravity" ⇝ explore landscape

Outline:

- basic structure + formalism
- generalize, generalize, ...

  extend conventional geometrical constructions

  CY, Lie group, ... "gen. G-structure"
- example 1: $d=4$ flux backgrounds

  eg moduli
- example 2: consistent truncations landscape.
- beyond supergravity.

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2. Generalised geometry and supergravity : type II

[Siegel '93] [Hitchin / Gualtieri '02, '04] ... As simplest example:

- NSNS sector of type II : g , B , φ

  metric B-field dilaton.

- compactify : M_{10} = (non-compact) $\times$ M

  d-dimensional

2.1 Symmetries

- diffeos $\xi \in \Gamma(TM)$ + gauge $\lambda \in \Gamma(T^*M)$

- $\delta g = \mathcal{L}_\xi g \qquad \delta B = \mathcal{L}_\xi B + d\lambda \qquad \delta\varphi = \mathcal{L}_\xi \varphi$

Get algebra:

$$\xi'' = [\xi, \xi'] \qquad\qquad d\lambda'' = \mathcal{L}_\xi d\lambda' - \mathcal{L}_{\xi'} d\lambda$$

Lie bracket

- Combine into a "generalized vector"

$$V^M = \begin{pmatrix} \xi^m \\ \lambda_m \end{pmatrix} \in \Gamma(TM \oplus T^*M) \qquad\qquad M = 1, \dots, 2d$$

use notation

$$V = \xi + \lambda \qquad\qquad E = TM \oplus T^*M$$ gen tang. space

comes with natural SO(d,d) metric

$$\eta(V,V) = i_\xi \lambda = \xi^m \lambda_m = \frac{1}{2} V^M \begin{pmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{pmatrix}_{MN} V^N$$

- integrate $d\lambda''$ and choose: $(\mathcal{L}_\xi \alpha = d(i_\xi \alpha) + i_\xi d\alpha)$

$$\xi'' + \lambda'' = [\xi, \xi'] + \mathcal{L}_\xi \lambda' - i_{\xi'} d\lambda$$

$$:= L_V V'$$

"generalized Lie derivative"

or Dorfman derivative.

key property: preserves metric for $V = \xi + \lambda$

$$\eta(L_V U, W) + \eta(U, L_V W) = \mathcal{L}_\xi(\eta(U, W))$$

mathematically

- $L_V W$ defines Courant algebroid structure on $\Gamma(E)$
- <u>not</u> Lie algebroid since $L_V W \neq - L_W V$

- can consider generalised tensors = other reps of $SO(d,d)$ will decompose into ordinary tensors under $GL(d,\mathbb{R}) \subset SO(d,d)$

adjoint: $\operatorname{ad} F \simeq \underbrace{TM \otimes T^*M}_{gl(d,\mathbb{R})} \oplus \Lambda^2 TM \oplus \underbrace{\Lambda^2 T^*M}_{B}$

spinors: $S^+ \simeq (\det T^*M)^{-1/2} \otimes \Lambda^+ T^*M$ even forms

$S^- \simeq (\det T^*M)^{-1/2} \otimes \Lambda^- T^*M$ odd forms

(assume orientable).

to deal with $(\det T^*M)^{1/2}$ naturally consider:

reps of $SO(d,d) \times \mathbb{R}^+$ $\qquad S^\pm_{1/2} \simeq \Lambda^\pm T^*M$

$\uparrow$ weight of tensor density

- because L_v preserves η one can extend (by Leibniz) to any generalized tensor

$$\mathcal{L}_\xi \varphi^\pm + d\lambda \wedge \varphi^\mp$$

eg $L_v \varphi^\pm = i_\xi \varphi^\pm + \lambda \wedge \varphi^\pm$ for $\varphi^\pm \in \Gamma(S^\pm_{1/2})$

2.2 Gen. Riemannian geom.

- In ordinary geometry:

metric g invariant under $O(d) \subset GL(d, \mathbb{R})$

max. compact subgp.

analogue:

gen. metric G invariant under $SO(d) \times SO(d) \subset SO(d,d) \times \mathbb{R}^+$

explicitly $G \in S^2(E^*)$

$$G_{MN} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}_{MN}$$

to define $SO(d) \times SO(d) \subset SO(d,d) \times \mathbb{R}^+$ also need

$$|\mathrm{vol}_G| := e^{-2\varphi} \sqrt{g} \in \Gamma(\det T^* M)$$

generalized metric encodes NSNS fields.

- analogue of Levi-Civita connection?

• Generalised connection:

$$D_M V^N = \partial_M V^N - \Gamma_M{}^N{}_P V^P \qquad D_M \eta_{NP} = 0$$

$$\partial_M = \begin{pmatrix} \partial_m \\ 0 \end{pmatrix} \begin{matrix} \leftarrow T^*M \\ \leftarrow TM \end{matrix}$$

• metric compatible: $D_M G_{NP} = D_M |\mathrm{vol}|_G = 0$

• torsion-free: define gen. torsion using LvW

$$T \in \Gamma(\Lambda^3 E \oplus E) \qquad T = 0$$

NB more general than conventional $SO(d)\times SO(d)$ connection:

$V^M D_M$ derivative along ξ^m and λ_m

- then define gen. Ricci tensor R_{MN} + gen. Ricci scalar R

$$\boxed{\begin{aligned} &\int \mathrm{vol}_G\, R = \int \sqrt{g}\, e^{-2\varphi}\left(R + 4(\partial\varphi)^2 - \tfrac{1}{12}H^2 \right) \\ &\text{equations of motion :} \qquad R_{MN} = 0 \end{aligned}}$$

NSNS sugra = $SO(d,d)\times\mathbb{R}^+$ gen. Einstein

- structure group of E = $SO(d,d)\times\mathbb{R}^+ \supset GL(d,\mathbb{R})$
- local symmetry = $SO(d)_+\times SO(d)_- \supset SO(d)$

"doubled Lorentz"

2.3 Full type II

- RR fields

$F^{\pm} = dA^{\mp} \in \Gamma(S^{\pm}_{1/2})$ generalized spinors

IIA / IIB "democratic formulation" $F^{\pm} = \ast F^{\pm}$

- fermions: representations of $SO(d)_+ \times SO(d_-)$

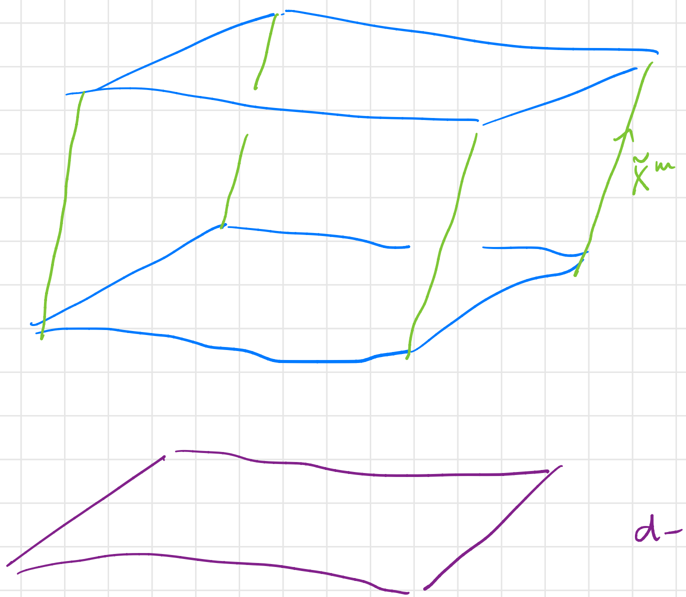
$\psi^{\pm}_{m} \in \Gamma(\text{vector}_{\mp} \times \text{spinor}_{\pm})$ gravitini

$\lambda^{\pm} \in \Gamma(\text{spinor}_{\pm})$ dilatini

- $d=10$ w/ Lorentzian signature.

gen metric : $SO(9,1)_+ \times SO(1,9)_- \subset SO(10,10) \times \mathbb{R}^+$

- double spacetime ?? **DOUBLE FIELD THEORY** [Hohm, Hull, Zwiebach]
  - interpretation? (see later)
  - "double then constrain" $\rightsquigarrow$ same equations.



2d-dim $\qquad \partial_{\tilde{x}^m}(\text{anything}) = 0$

"strong constraint"

d-dim slice/projection

3. Generalised geometry: $E_{d(d)} \times \mathbb{R}^+$

Can one geometrize RR fields? Gen. geom. for M-theory?

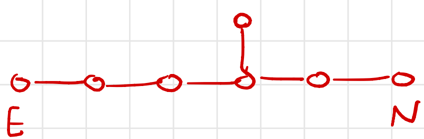
3.1 Exceptional generalized geom

For $SO(d,d)$:

$$\eta : \quad S^2 E \longrightarrow N = \mathbb{R}$$

now consider for $E_{d(d)} \times \mathbb{R}^+$ (split form of exceptional group)

$$\gamma : \quad S E^2 \longrightarrow N$$



- maximal subspace $V \subset E$ such that $\gamma(V,V) = 0$:

(i) $\dim V = d$ invariant under $GL(d,\mathbb{R}) \subset E_{d(d)} \times \mathbb{R}^+$

(ii) $\dim V = d-1$ invariant under $GL(d-1,\mathbb{R}) \subset E_{d(d)} \times \mathbb{R}^+$

defines two gen. tangent spaces $d \leq 7$

$$E \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^7 T^*M)$$

M-theory $\dim M = d$

$$E = TM' \oplus 2T^*M' \oplus \Lambda^3 T^*M' \oplus 2\Lambda^5 T^*M' \oplus (T^*M \otimes \Lambda^6 T^*M)$$

type IIB $\dim M' = d-1$

$$\supset TM' \oplus T^*M'$$

- For definiteness take M-theory on M_7 with fields

metric g, potential $A \in \Gamma(\Lambda^3 T^*M)$, dual pot $\tilde{A} \in \Gamma(\Lambda^6 T^*M)$

where for $*_{11}\tilde{F}$ = flux on non-compact 4d

$$F = dA, \qquad \tilde{F} = d\tilde{A} + \tfrac{1}{2} A \wedge dA$$

then symmetry (56-dimensional)

$$E \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^7 T^*M)$$

$$V = \underbrace{\xi}_{\text{diffeo}} + \underbrace{\omega}_{\text{gauge}} + \underbrace{\sigma}_{\text{dual gauge}} + \underbrace{\tau}_{\text{"dual diffeos"}}$$

L_V : gen. Lie derivative. (indep. of τ; preserves $E_{7(7)} \times \mathbb{R}^+$)

3.2 Exceptional gen. Riemannian geom.

Just follow as before:

max. compact.

gen. metric G invariant under $SU(8) \subset E_{7(7)} \times \mathbb{R}^+$

$$\Leftrightarrow \quad \{ g, A, \tilde{A}, \Delta \}$$

where Δ is warp factor

$$ds_{11}^2 = e^{2\Delta} \left(\text{4d external} \right) + \left(\text{internal} \right).$$

then

generalized connection D_M

- preserves $E_{7(7)} \times \mathbb{R}^+$ structure
- torsion $\in \Gamma(K \oplus E^*)$

$K = 912$

- Leri-Civita = metric compatible + torsion-free then gives

$$S = \int \mathrm{vol}_G\, R = \int \sqrt{g}\, e^{4\Delta} \left(R + 12 (\partial\Delta)^2 - \tfrac{1}{2}|F|^2 - \tfrac{1}{2}|\tilde{F}|^2 \right)$$

$R_{MN} = 0$ equations of motion

gives M-theory restricted to M_7

- fermions : SU(8) representations.

- for type IIB – get full bosonic theory NSNS + RR + fermions.

- Full $d=11$ theory? [Hohm, Samtleben]... EXCEPTIONAL FIELD THEORY

$TM_{11} = T_{3,1} \oplus T_7$ $\qquad E = T_7 \oplus \Lambda^2 T_7^* \oplus \cdots$

decompose 4d geom + 7d gen geom.

$g_{\mu\nu}$, $g_{\mu m}$, g_{mn} etc.

reformulate as "4d" theory + ∞ # of fields [de Wit + Nicolai]

- $E_{8(8)}$ – needs more structure (dual diffeos), $E_{9(9)}$, ... ??

- "ADE" split forms: $SL(d,\mathbb{R}) \times \mathbb{R}^+$ $\qquad$ others? yes, some

$SO(d,d) \times \mathbb{R}^+$ $\qquad$ eg $SO(d,d+n) \times \mathbb{R}^+$

$E_{d(d)} \times \mathbb{R}^+$ $\qquad$ heterotic theory

4. <u>Generalized G-structures</u>

So far, just reformulation. New tools for

- supersymmetric flux bkgrounds? exploring landscape?
  AdS-cft? dualities? topological theories? stability?
- mathematics?

Mantra: conventional geom $\underset{\text{to}}{\overset{\text{extend}}{\rightsquigarrow}}$ gen. geom.

$\rightsquigarrow$ generalized CY manifolds, G_2-manifolds, complex structures, Lie groups, etc, etc.

4.1 Conventional G-structures

- Useful formalism for describing many different geometries:

FM = frame bundle for TM = principal $GL(d,\mathbb{R})$ bundle.

$\{\hat{e}_a\}$ $\qquad \hat{e}'_a = \underbrace{M_a{}^b}_{GL(d,\mathbb{R})} \hat{e}_b$

then: for $G_s \subset GL(d,\mathbb{R})$ reduction of structure group

$FM \supset P$ = principal G_s sub-bundle $\qquad$ G_s-structure.

for example $G_s = O(d)$

P = set of orthonormal frames $\Leftrightarrow$ metric g

invariant tensor

- other examples: $d = 2n$
  - $G_s = GL(n, \mathbb{C}) \Leftrightarrow$ almost complex structure, I

    $\Leftrightarrow T_{\mathbb{C}}M = T^{1,0} \oplus T^{0,1}$

    $+i$ eigenbundle — $-i$ eigenbundle

  - $G_s = Sp(d, \mathbb{R}) \Leftrightarrow$ almost symplectic structure, ω

    (no requirement $d\omega = 0$)

  - $G_s = SU(n) \Leftrightarrow$ almost Calabi-Yau structure (ω, Ω)

    (no requirement $d\omega = 0$, $d\Omega = 0$)

- integrability (or "torsion free")

  $\exists$ torsion-free compatible connection. (may not be unique)

  - $GL(n, \mathbb{C})$: Nijenhuis tensor $= 0$ "involutive"

    $\Leftrightarrow$ $[v, w] \in \Gamma(T^{1,0})$ $\forall$ $v, w \in \Gamma(T^{1,0})$

  - $Sp(d, \mathbb{R})$: $d\omega = 0$

  - $SU(n)$: $d\omega = d\Omega = 0$

  etc.

- obstruction = "intrinsic torsion" eg vanishes identically for $G_s = O(d)$

4.2 Generalised G-structures

- Extend same ideas to gen. geom:
  - FE = frame bundle for E = principal $\begin{matrix} SO(d,d)\times\mathbb{R}^+ \\ E_{d(d)}\times\mathbb{R}^+ \end{matrix}$ bundle
  - $\tilde{P}$ = principal G_s subbundle gen. G_s structure.

    $\Leftrightarrow$ invariant gen. tensor(s)
  - integrable : $\exists$ compatible, torsion-free gen. connection D

- Classic example: [Hitchin, Gualtieri] generalised complex structure:

  - $G_S = U(n,n) \subset SO(2n,2n)$

    $\Leftrightarrow$ invariant tensor $J \in \Gamma(E \otimes E^*)$

    $\Leftrightarrow$ $E_{\mathbb{C}} = L_1 \oplus L_{-1}$ (eigenspaces)

  interpolates between cplx & sympl.

  - torsion-free $\Leftrightarrow$ L_1 is involutive

    $L_V W \in \Gamma(L_1)$ $\forall V, W \in \Gamma(L_1)$

extend to generalized CY structure:

  - $G_S = SU(n,n)$ $\Leftrightarrow$ $\varphi_\pm \in \Gamma(S^\pm_{\mathbb{C}}) = \Gamma(\Lambda^\pm T^*M)$

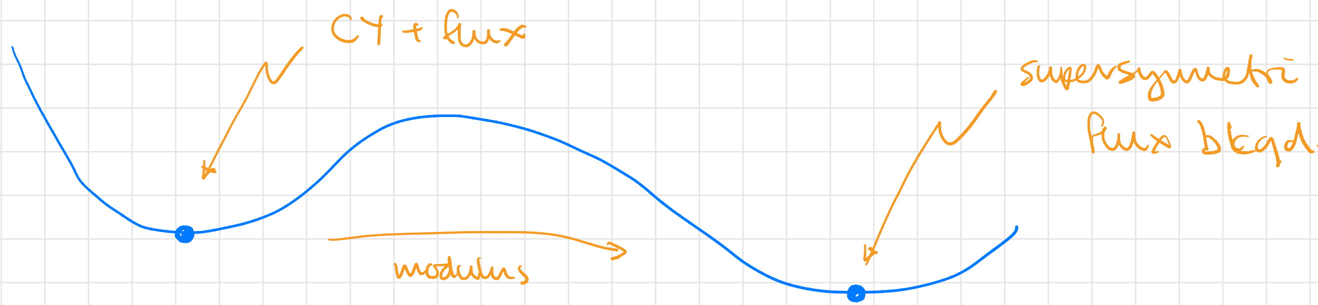
  - torsion-free $\Leftrightarrow$ $d\varphi^\pm = 0$

- more generally for generic flux backgrounds questions:

reframe supergravity problem as gen. G-structure

5. Example 1: $d=4$ flux backgrounds

Landscape questions:



- how characterize generic susy bkgd? • stability?
- moduli at CY + flux = moduli at actual vacuum?

Reminder: no go theorems $\Rightarrow$ need flux sources

5.1 Type II blegds [Graña, Minasian, Petrini, Tomasiello 05]

- Generalizes Calabi-Yau compactification; warp factor $e^{2\Delta}$
  - $G_S = SU(3)\times SU(3) \subset SO(6,6)\times \mathbb{R}^+$ structure

    $\varphi_+ \in \Gamma(S^+_{\mathbb{C}})$ $\qquad\qquad$ $\varphi_- \in \Gamma(S^-_{\mathbb{C}})$

  - non-vanishing intrinsic torsion = RR flux F

    $d\varphi_- = 0$ $\qquad\qquad$ $d(e^{-\Delta}\,\mathrm{Re}\,\varphi_+) = 0$ $\qquad\qquad$ IIB

    $d^{J_-}(e^{-3\Delta}\,\mathrm{Im}\,\varphi_+) = -\frac{1}{8}F$

  where $d^{J_-} = [J_-, d]$ (cf. d^c in cplx geom)

- Can use formulation in many ways [GMPT, Martucci, Koerber, Lüst, Louis, Tsimpis, Grimm, Blumenhagen, Andriot, Lukas, ...]

  - look for new vacua
  - extended Gukov-Vafa-Witten superpotential, effective theories
  - characterize supersymmetric branes (calibrations)
  - analyse generic moduli
  - non-susy.
  - stability of vacua to tunnelling
  - σ-models ...

5.2 Moduli [Ashmore, Strickland-Constable, Tennyson, DW 2019] [+ Smith '22]

Lift problem to $E_{7(7)} \times \mathbb{R}^+$ gen. geom.

- supersymmetry parameter $\varepsilon \in \Gamma(S)$ $\underline{8}$ of $SU(8)$
- $N=1$: $\rightsquigarrow$ torsion-free, $SU(7)$ structure.

Differential conditions: "nicely arranged"

- "F-term"

$$E_{\mathbb{C}} = E_{+3} \oplus E_{+1} \oplus E_{-1} \oplus E_{-3}$$

involutivity : $L_V W \in \Gamma(E_{+3}) \quad \forall V, W \in \Gamma(E_{+3})$

- "D-term" : moment map

- Geometrically: Z = ∞-dim. space of $SU(7)$ structures
    - natural Kähler metric for $E_{7(7)} \times \mathbb{R}^+ / SU(7)$ coset.
    - moment map for action of GDiff = Diff + gauge.

Involutivity defines Lie algebroid on L_3

cohomology $\Lambda^p E_3^* \xrightarrow{d_L} \Lambda^{p+1} E_3^*$ counts moduli

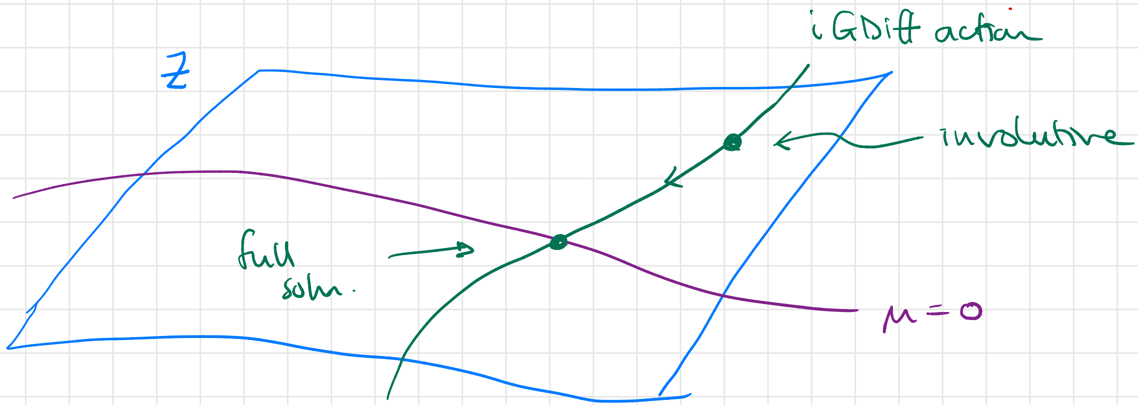
$$\Updownarrow$$

cohomology defined by J - [cf Gualtieri ; Cavalcanti]

twisted by flux

$\to$ flux lifts moduli

- As for Calabi-Yau: don't need full solution only involutive moduli can be calculated away from moment map soln.



- Same calculation in M-theory (also heterotic)

  reduces to de Rham or Dolbeault cohom w/ flux

- CAVEAT: no perturbation of sources.

Two asides

- same ideas apply to AdS solutions
  - new AdS-cft duals. different susy / dimensional [eg Tomasiello]
  - analysis of finite marginal deformations [Ashmore, Petrini, Tasker, Aw]
    cohomology counts chiral ops

- new perspective in mathematics
  - framework of stability and geometrical susy. theory
    (cf. HYM eqns, KE eqns etc.)
  - analyse existence of solutions.? G_2?

6. Example 2: Consistent truncations

solution of truncated theory = solution of full theory

example:

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\lambda)^2 - \frac{1}{2}m^2\lambda^2 - \frac{1}{2}g\lambda\varphi^2$$

$$\partial^2\varphi = g\lambda\varphi \quad , \qquad \partial^2\lambda = m^2\lambda + \frac{1}{2}g\varphi^2$$

$\lambda = 0$ $\qquad\qquad$ $\varphi = 0$

inconsistent : λ sourced by φ

(ok when $E \ll m$)

consistent

· keep singlets under $\mathbb{Z}_2$

$\varphi \to -\varphi \quad \lambda \to \lambda$

- Consistent dimensional reduction: no requirement of scale separation

$$S_D = \int_{X \times M} \sqrt{-g}\,(R + \dots)$$

$$\Big\downarrow \text{ reduce on } M$$

$$S_{D-d} = \int_X \sqrt{-\tilde{g}}\,(\tilde{R} + \dots)$$

keeping finite number of Kaluza-Klein modes

- Natural question:

what theories on X arise as consistent truncations in string theory / supergravity?

- General class: Scherk-Schwarz reduction.

M = Lie group G (or G/Γ) keep left-invariant modes.

in particular $\{\hat{e}_i\}$ left-invariant vectors with $[\hat{e}_i, \hat{e}_j] = f_{ij}{}^k \hat{e}_k$

Singlets can't source charged modes

S_{D-n} = gauge theory w/ gauge group G (right-action)

- Extend to gen. geom?

$\{K_A\} \in \Gamma(E)$ $\qquad L_{K_A} K_B = X_{AB}{}^C K_C$

"Leibniz parallelization" $\qquad G_S = \mathbb{1}$, X = intrinsic torsion

- expanding gen.-metric G etc. in K_A gives consistent truncation of type IIA/B + M-theory . .

S_{D-n} = gauged maximal supergravity.

- examples: mysterious sphere cases [deWit + Nicolai ; –]
  - M-theory on S^7, S^4
  - IIB on S^5

- Poisson-Lie U-duality : [Hassler ; Malek, Sacchetti Thompson ; Bulycheva, Hulik, Valach, Ru]

two Leibniz parallelizations w/ same algebra.

- [Dall'Agata, Inverso, Trigiante 2012] found family of $SO(8)$ gauged $d=4$, $N=8$ supergravities. $0 \leq \omega < \frac{1}{8}\pi$

  $\omega = 0$: S^7 Leibniz parallelism

  $\omega \neq 0$: prove no sugra construction

  [Lee, S-Constable, Waldram]

- More generally: [Cassani, Josse, Petrini, DW]

  G structure w/ singlet intrinsic torsion $\overset{\Rightarrow}{\Leftarrow}$? consistent trunc.

  can use to start classifying landscape of theories. [Emanuel's talk]

  [du Bosque, Hassler, Lüst; Inverso; Bugden, Hulik, Valach, DW]

7. Beyond supergravity

- Very broad question

why generalised geometry?

minimal data for extending diffeo. symmetries [Bugden, Hulik, Valach, DW]

- vector spaces (E, N)
- "non-generate" map $\gamma : S^2 E \to N$

$\rightsquigarrow$ $\left\{ \begin{array}{l} \text{gen. tang. space } E \\ \text{structure group} \\ \text{gen. Lie derivative } L_V \end{array} \right.$

Somehow intrinsic to string theory? supersymmetry? q. gravity???

- How extend to higher string modes?

7.1 DFT and EFT

- Consider string theory on T^n and keep winding and momentum

  doubled space $= T^{2n}$ [Hull, Zwiebach; Hohm, Hull, Zwiebach]

  - "strong constraint" $\xrightarrow{\text{project}}$ T^n gives gen. geom.

  - "weak constraint": "consistent truncation" integrate out osc. modes from SFT [Sen]

    formally describe using L_∞ alg. [Arvanitakis, Hohm, Hull, Lekeu]

agree to cubic order

- Local string constraint: "non-geometrical vacua"

   patched by T-duality [Hellerman, McGreevy, Williams; Hull; ...]

 local gen. geom description.

- extend to other geometries?

   - group manifolds: [Blumenhagen, Du Bosque, Hassler, Lüst]

      modifies gen. geom. constructions.

   - gauge enhancement on T^n in heterotic

      dependence on extra coordinates, extend E

      [Aldazabal, Graña, Iguri, Mayo, Nuñez, Rosabal; ...]

7.2 Curvatures and higher-orders

- In both $SO(d,d)$ & $E_{d(d)}$ (cf. cplx structure)

  gen. Levi-Civita connection D is not unique.

Furthermore:

- can define gen. Riemann tensor for $SO(d,d)$ but depends on choice of D

- no gen. Riemann tensor for $E_{d(d)}$

but

- can always define gen. Ricci tensor and indep. of choice of D

- We know supergravity has corrections:

$$S = S^{\text{type II}}_{\text{sugra}} + \int \sqrt{-g}\, (\text{Riem}^4 + \dots)$$

- Very suggestive for restricting higher order corrections

can one construct new unambiguous invariants from D ?

so preserves $SO(d,d) \times \mathbb{R}^+$ structure & local $SO(d) \times SO(d)$ symmetry

- Actually seems v. difficult (rule out R^2 in bosonic theory / heterotic) without modifying L_V [Hohm-Zwiebach; Bedoya, Marques, Nunez]

- Progress using weaker conditions inspired by gen. geom.

[Coimbra + Minasian]

- Finally can consider topological sectors

eg cplx stre $\Omega \in \Gamma(\Lambda^3 T^*_{\mathbb{C}} M)$ $\rightsquigarrow$ gen. CY $\varphi^- \in \Gamma(\Lambda^- T^*_{\mathbb{C}} M)$

SU(3) structure SU(3,3) structure

(includes B-field)

string topological B-model

action = Hitchin functional = $\begin{cases} \int_M i\Omega \wedge \bar{\Omega} \\ \int_M i \bar{\varphi}^- \varphi^- \end{cases}$

SO(6,6) spinor pairing

[Pestun & Witten 2005] quantize + 1-loop

φ^- - functional agrees w/ B-model.

- More generally: $G_S \subset E_{d(d)} \times \mathbb{R}^+$ structure [Ashmore, S-Const., Tennyson, Dw]

involutive $L \subset E_{\mathbb{C}}$ $\longleftrightarrow$ gen. Hitchin functional = Kähler pot. on space of structures Z

gives new examples of possible topological theories (including RR fields or in M-theory) also heterotic [Ashmore et al]

(Includes flux extension of G_2 functional. [Dijkgraaf, Gukov, Neitzke, Vafa])

8. Conclusions

To reiterate:

- generalised geometry as a tool for investigating landscape
  - flux bkgd moduli
  - consistent truncations
  - metric on space of structures
- extends conventional notions of geometry.
  - inherent to strings, q-gravity ??
  - interesting mathematics / AdS-CFT.
- some potential to go beyond supergravity.