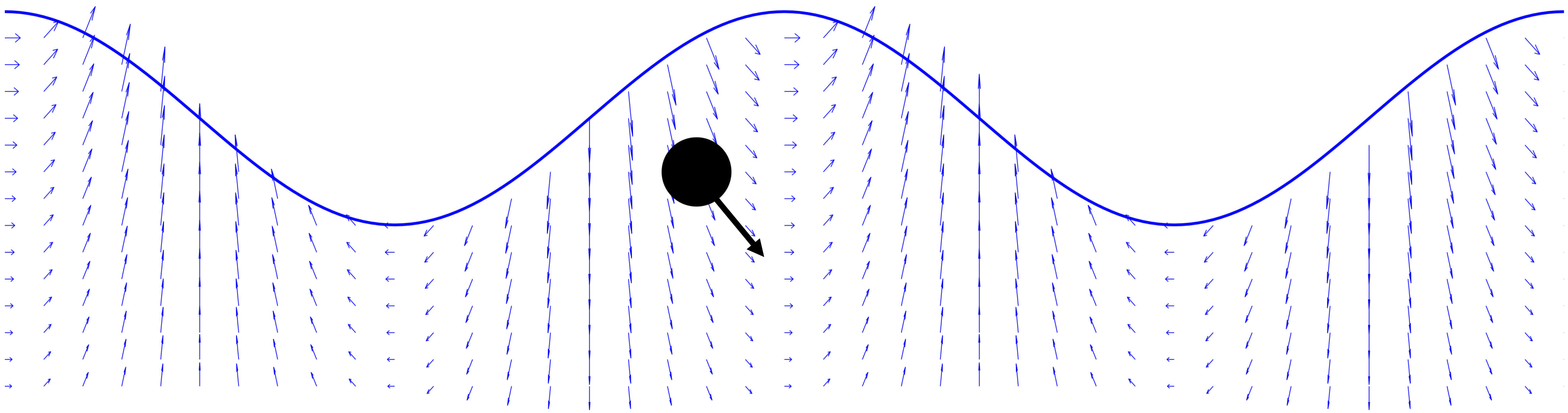
Inertial effects in particle (microplastic) settling through wavy flow



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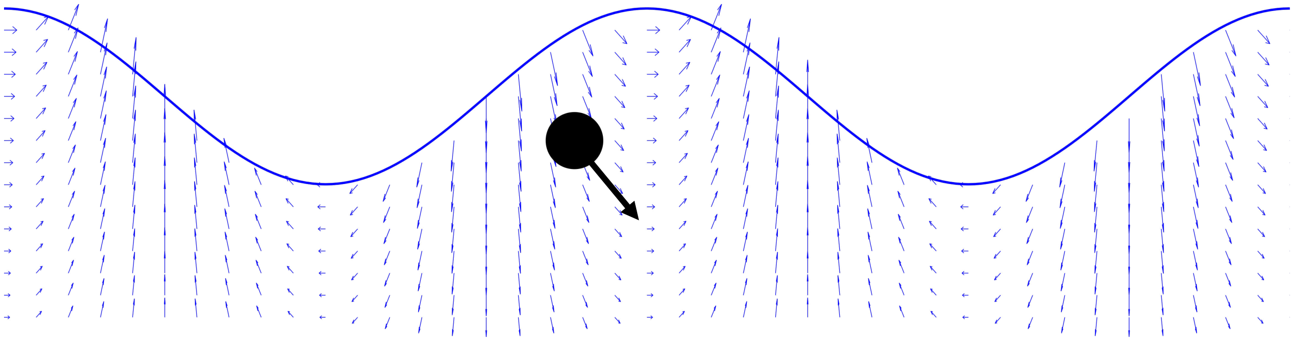
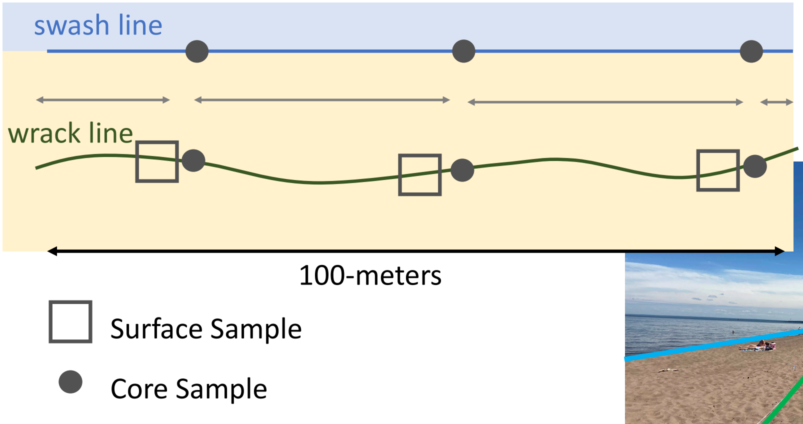
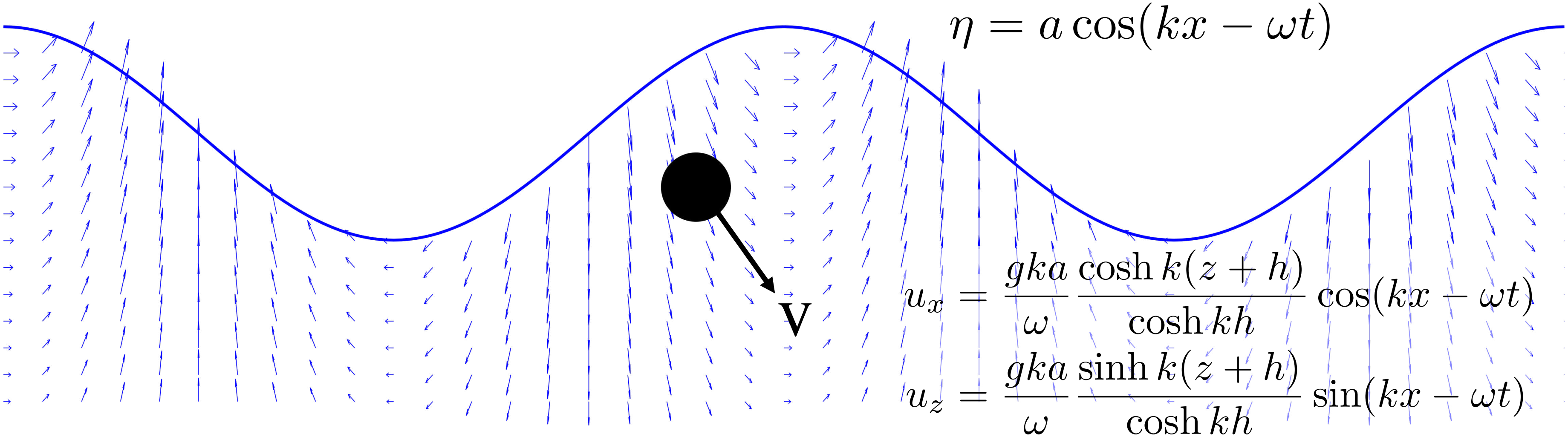
1. Negatively buoyant particles in surface waves



DiBenedetto, M., Clark, L., & Pujara, N. (2022).
Enhanced settling and dispersion of inertial particles in surface waves.
Journal of Fluid Mechanics, 936, A38. doi:10.1017/jfm.2022.95

2. Microplastic beaching and settling in turbulence





$$\eta = a\cos(kx - \omega t)$$

$$u_x = \frac{gka}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\cos(kx - \omega t)$$

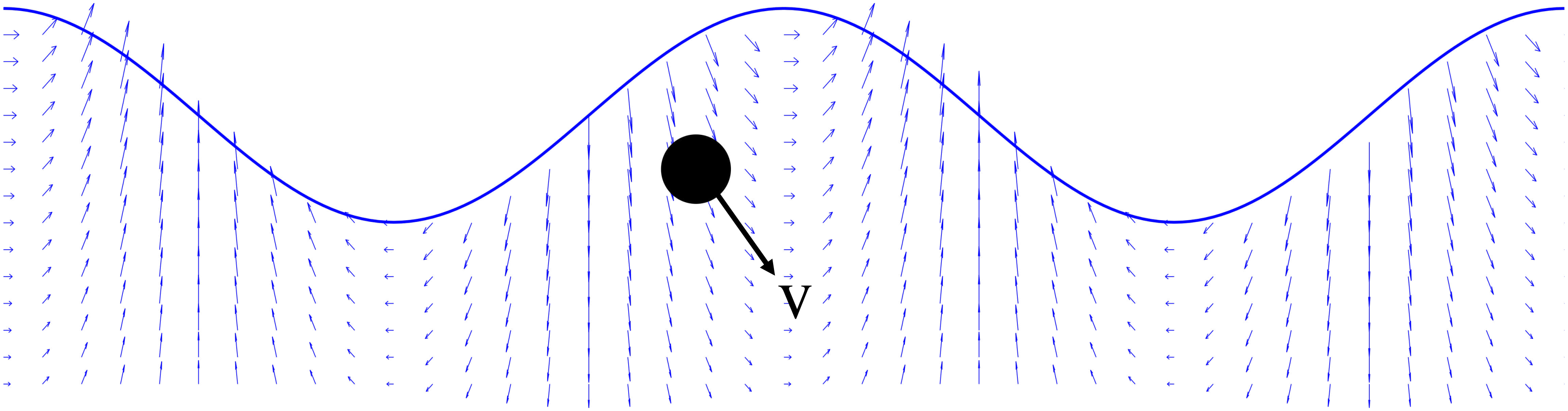
$$u_z = \frac{gka}{\omega}\frac{\sinh k(z+h)}{\cosh kh}\sin(kx - \omega t)$$

$$m\,a = \Sigma\,F$$

$$m_p\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = (m_p - m_f)\boldsymbol{g} + m_f\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} - C_m\left[\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} - \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}\right] - F_{\mathrm{drag}}$$

buoyancy | flow | added mass | drag

Can we predict particle transport without computing numerical solutions of the equation of motion?



Dimensionless equation of motion in linear drag regime ($\mathrm{Re_p} << 1$)

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{(\boldsymbol{u} - \boldsymbol{v})}{\mathrm{St}} + \beta \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} - \frac{v_s}{\mathrm{St}} \boldsymbol{e_z}$$

$\mathrm{St} = \omega \tau_p$ — Stokes number

$\beta = (1 + C_m)/(\gamma + C_m)$ — Fluid forcing coefficient

$\gamma = m_p/m_f$ — Buoyancy ratio

St number expansion

$$\boldsymbol{v} = \boldsymbol{v}_0 + \mathrm{St}\boldsymbol{v}_1 + \mathrm{St}^2\boldsymbol{v}_2 + \dots$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{(\boldsymbol{u}-\boldsymbol{v})}{\mathrm{St}} + \beta\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} - \frac{v_s}{\mathrm{St}}\boldsymbol{e_z}$$

Collect terms at each order

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{v}_s - \mathrm{St}(1-\beta)\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \mathrm{St}^2(1-\beta)\frac{\partial^2\boldsymbol{u}}{\partial t^2} + O(\varepsilon^2\mathrm{St}^2, \varepsilon\mathrm{St}^3)$$

Evaluate using linear wave theory flow field

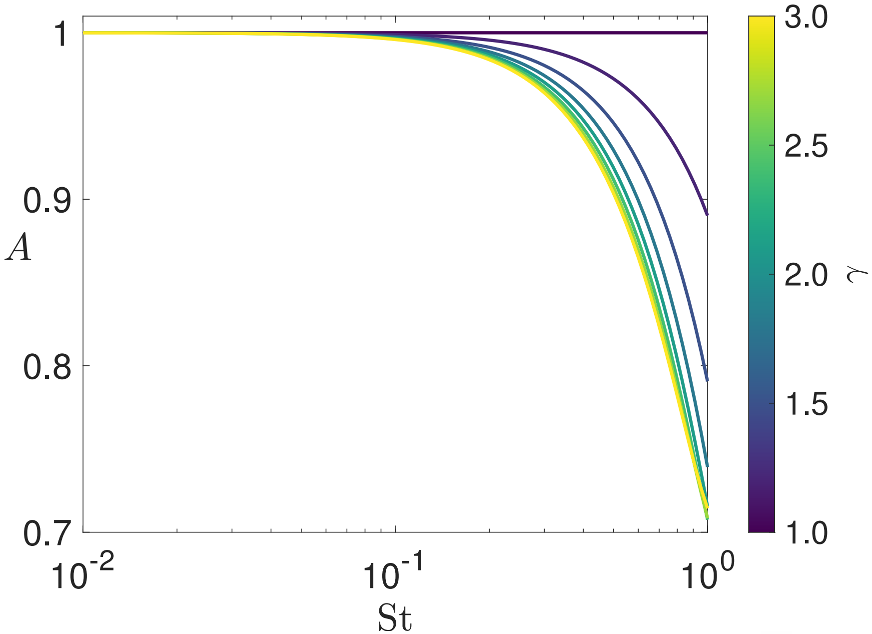
$$v_x = A\,\varepsilon\frac{\cosh z + kh}{\cosh kh}\cos(x - t + \phi) + \varepsilon^2\mathrm{St}(1-\beta)\frac{\sin 2(x-t)}{2\cosh^2 kh}$$

$$v_z = -v_s + A\,\varepsilon\frac{\sinh z + kh}{\cosh kh}\sin(x - t + \phi) - \varepsilon^2\mathrm{St}(1-\beta)\frac{\sinh 2(z+kh)}{2\cosh^2 kh}$$

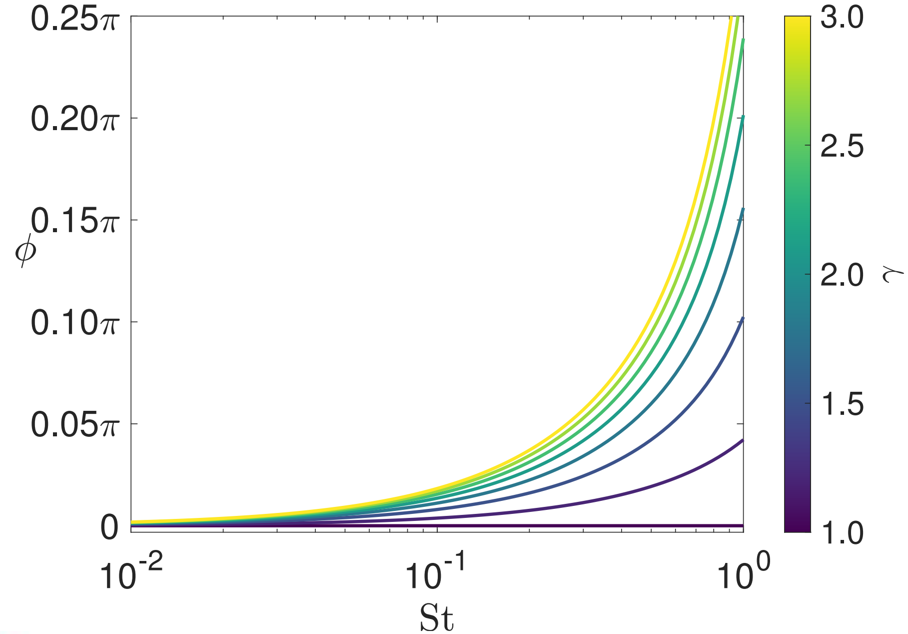
Fluid flow **Non-linear wave effects**

Inertial particle motion

Amplitude



Phase



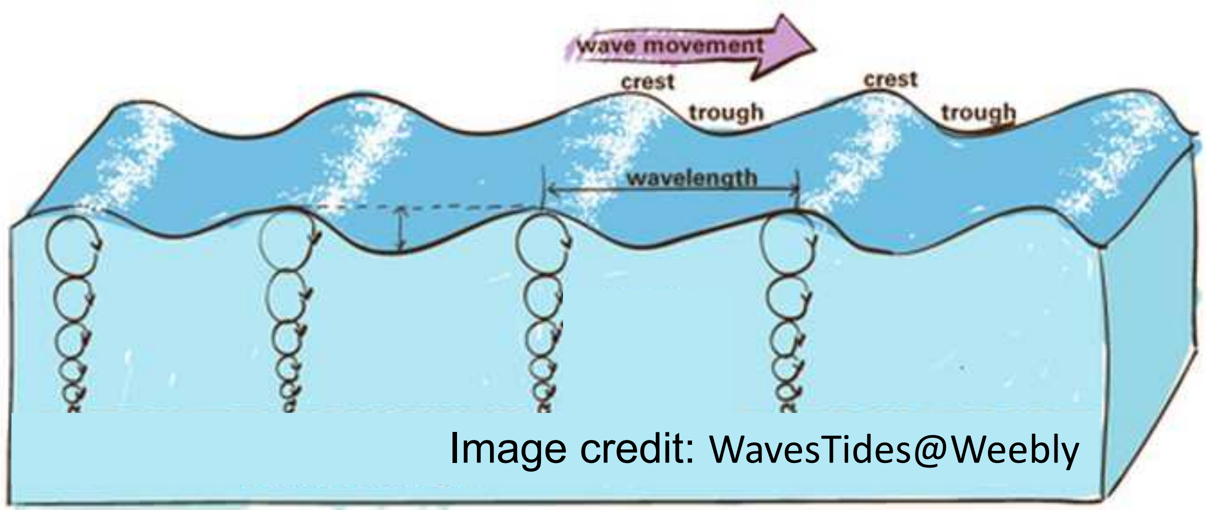
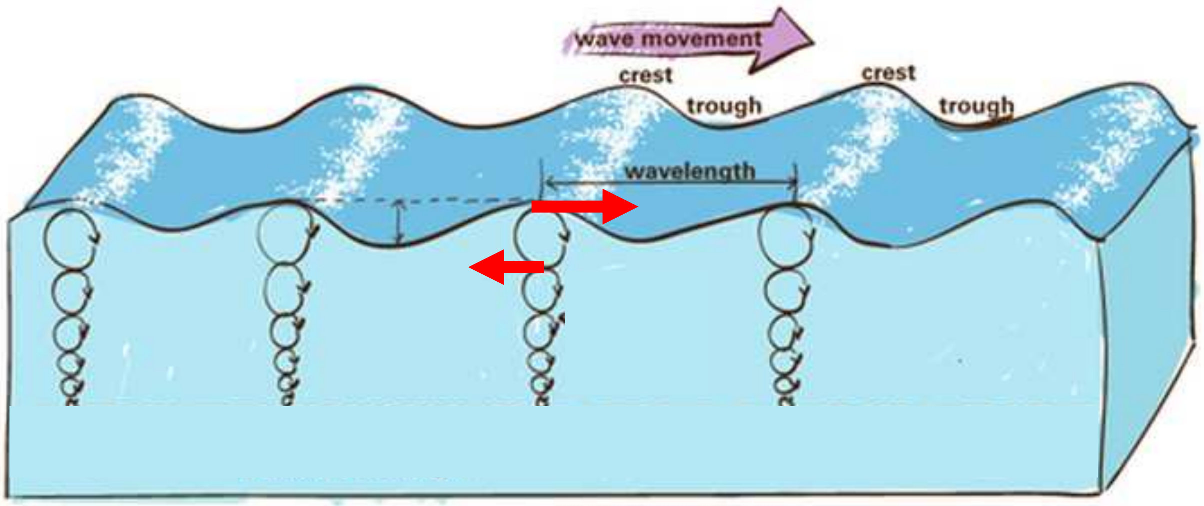
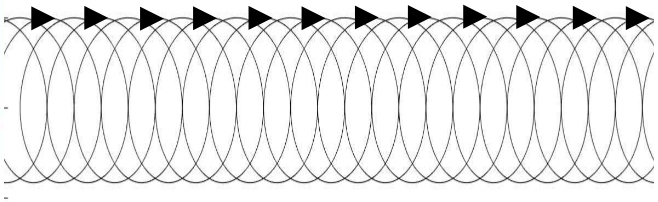


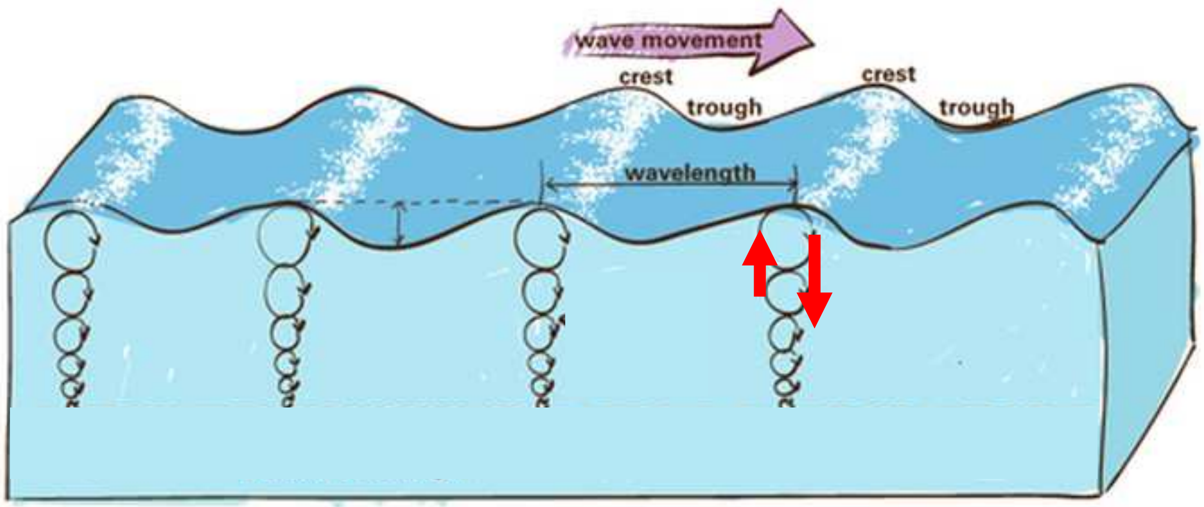
Image credit: WavesTides@Weebly



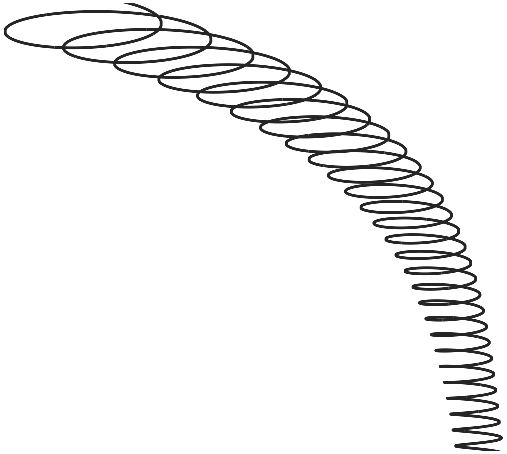
Stokes drift:



Stokes (1847)



Vertical drift:



Eames (2008)
Santamaria et al. (2013)

Multi-timescale expansion

$$\boldsymbol{x}_p(\tau, T) = \boldsymbol{x}_{p0}(\tau, T) + \varepsilon \boldsymbol{x}_{p1}(\tau, T) + \ldots \quad \text{where} \quad \tau = t,\ T = \varepsilon^2 t$$

Substitute into expressions for particle velocity and collect terms at each order. The drifts appear as a solvability condition at second order:

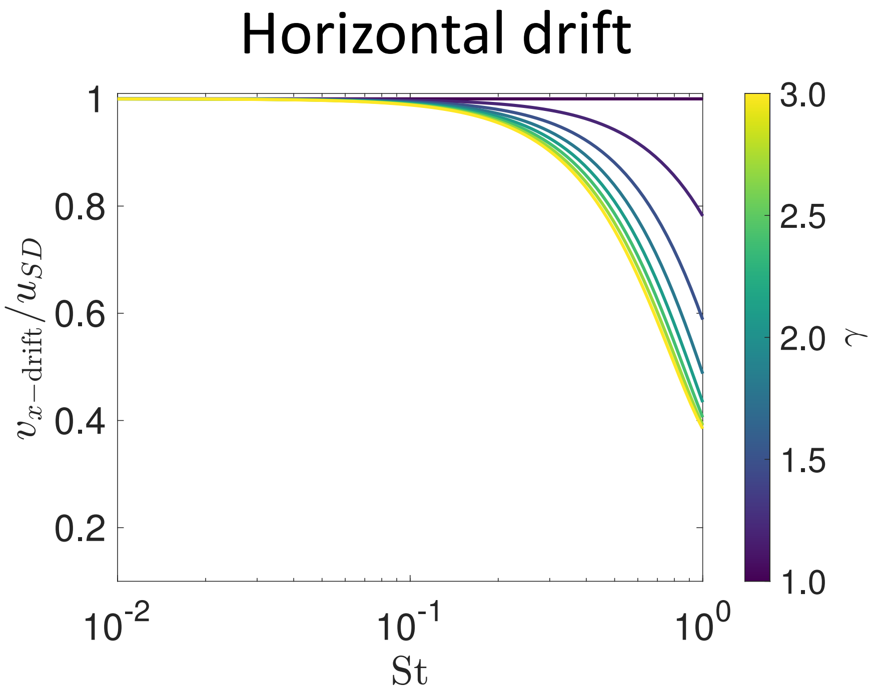
Reduced horizontal drift

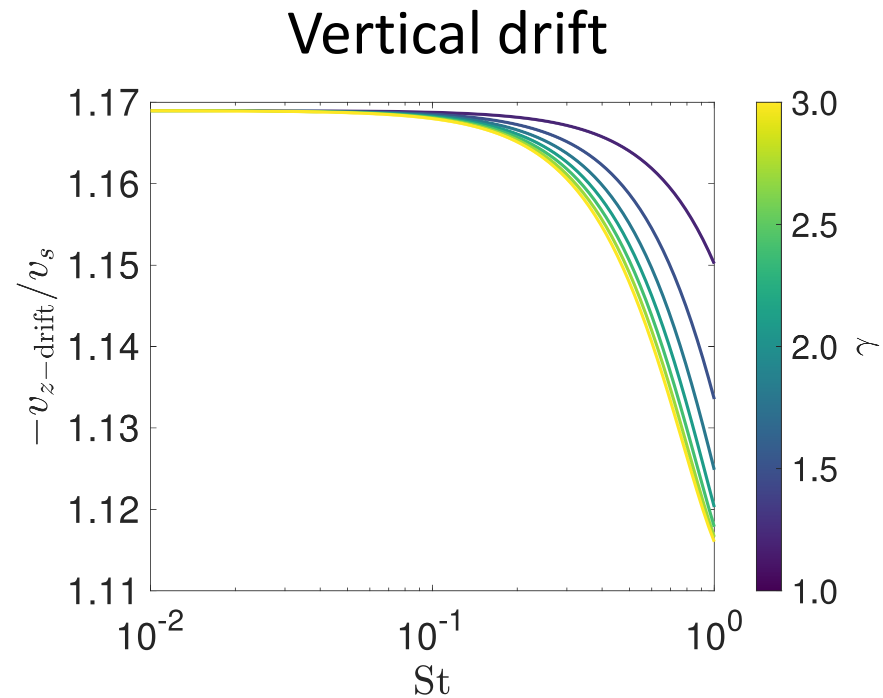
$$v_{x\text{-drift}} = \frac{A^2}{1 + v_s^2} u_{\mathrm{SD}}$$

$$v_{z\text{-drift}} = -v_s \left[1 + \frac{A^2}{1 + v_s^2} u_{\mathrm{SD}} + \frac{1}{2} \tanh kh \frac{\mathrm{d}u_{\mathrm{SD}}}{\mathrm{d}z_{p0}} \right]$$

Enhanced settling

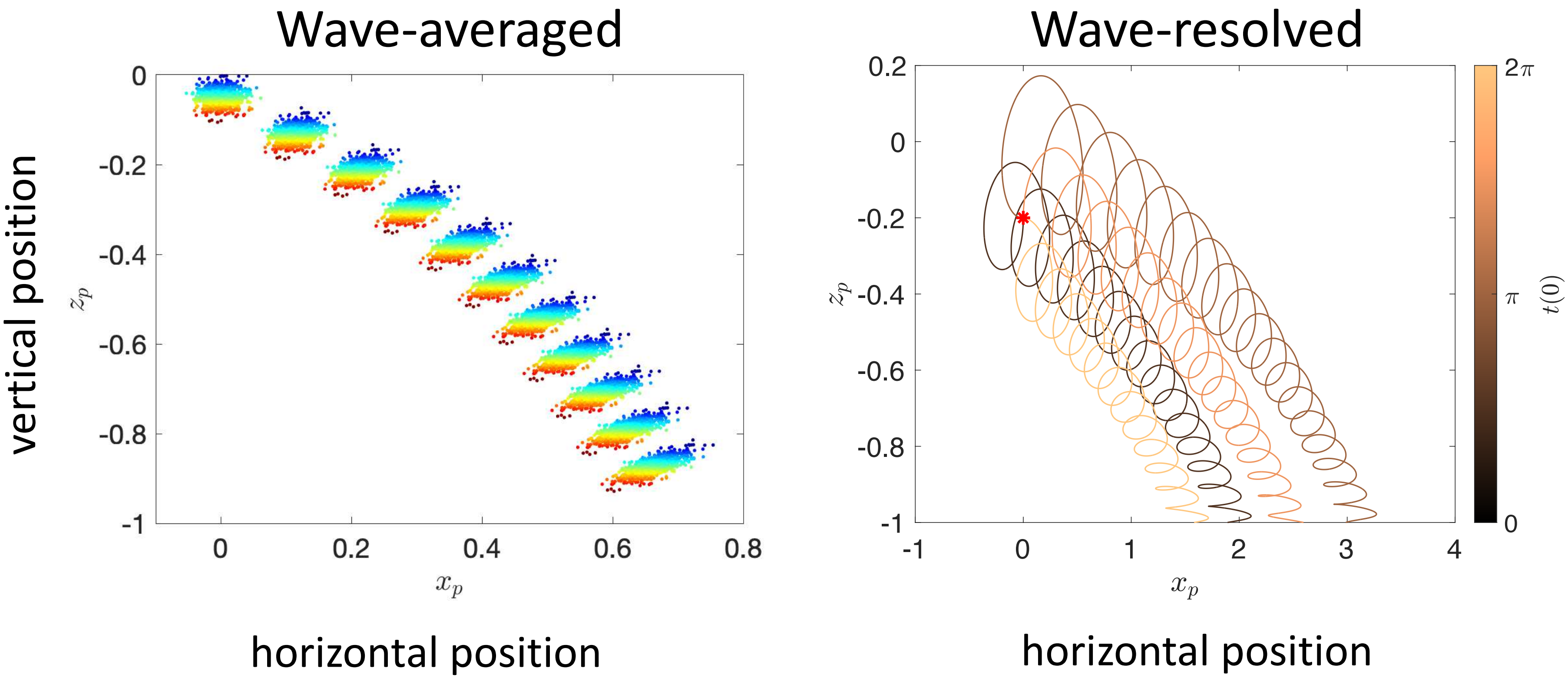
Inertial particle drift





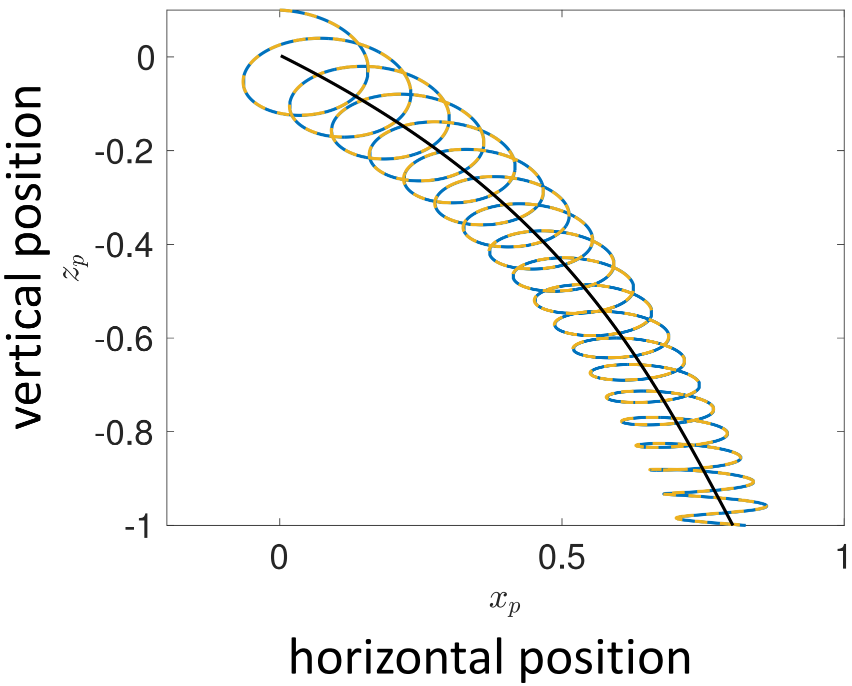
$$\epsilon = 0.33; \quad kh = 1$$

Horizontal dispersion due to vertical shear

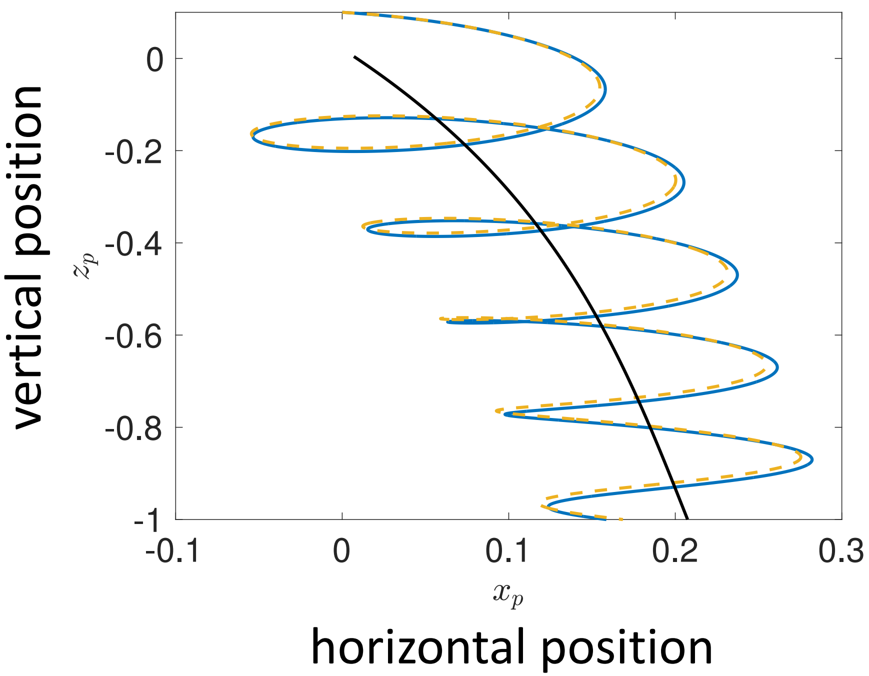


Initial conditions (correctly projected into wave-averaged variables) are important!

Comparison against numerical solutions



$ka = 0.1, kh = 1, \gamma = 1.1, \mathrm{St} = 0.1$



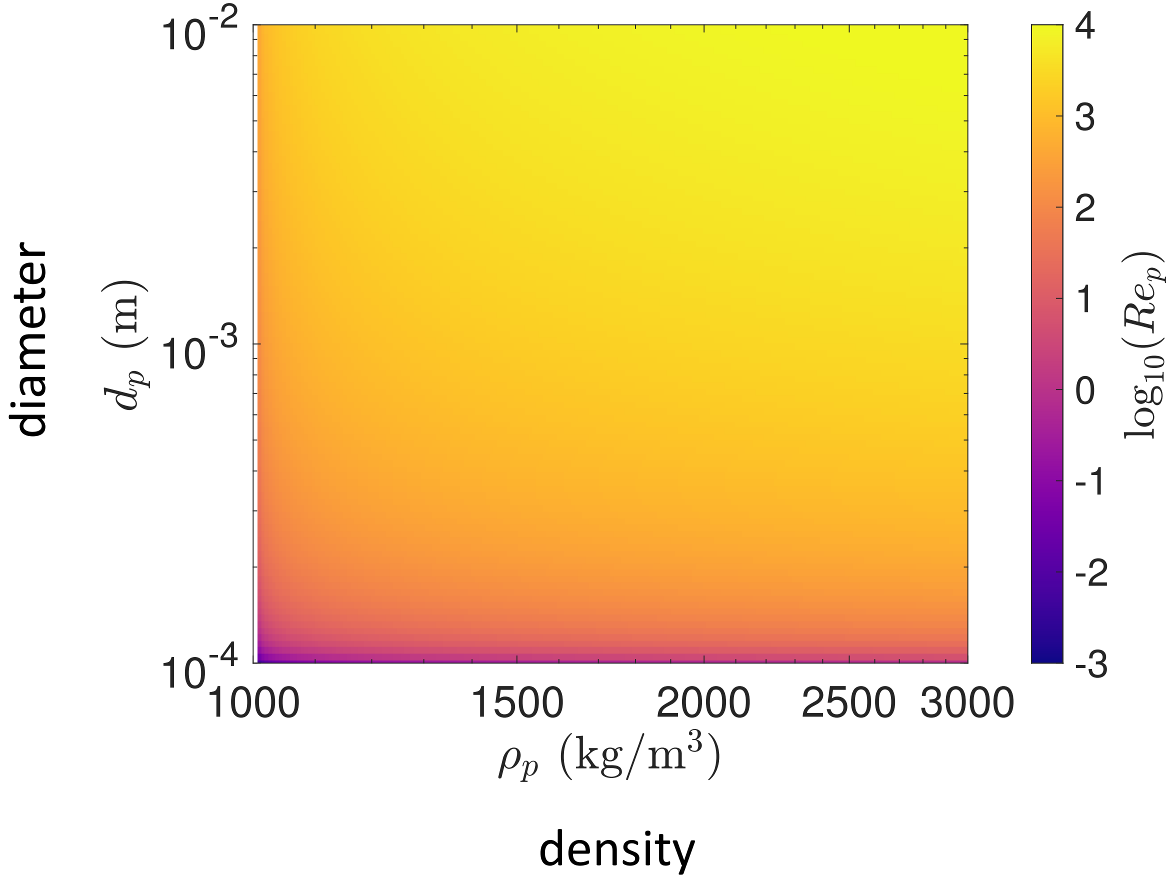
$ka = 0.1, kh = 1, \gamma = 1.05, \mathrm{St} = 0.75$

Numerical solution

Velocity solution

Drift solution

Non-linear drag



Microplastics can fall outside the linear (Stokes) drag regime and into the non-linear (intermediate Reynolds number) regime

Non-linear drag

Schiller-Naumann drag model captures the drag in the intermediate Reynolds number regime

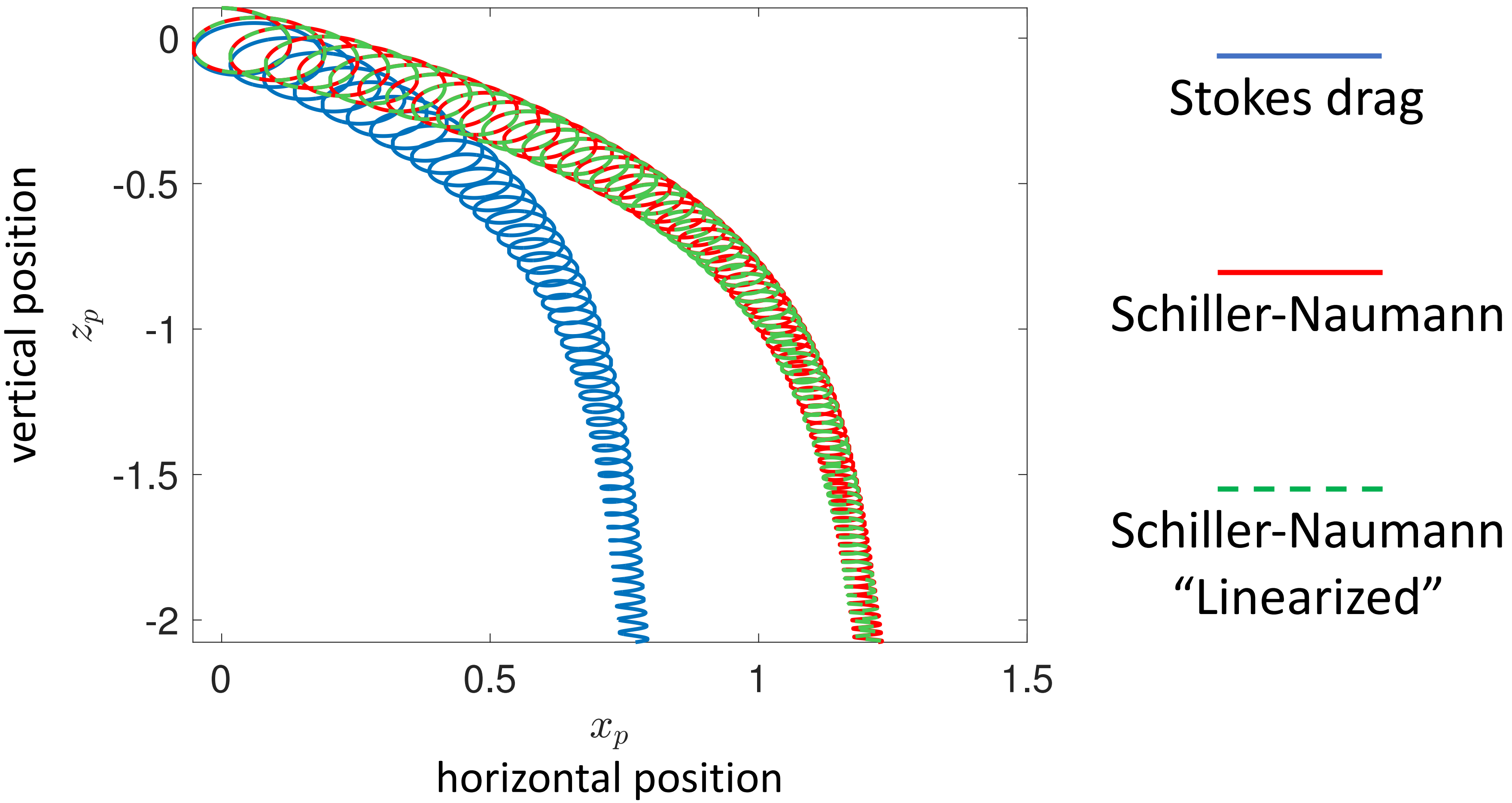
$$C_D = \frac{24}{\mathrm{Re}_p}\left(1 + 0.15\mathrm{Re}_p^{0.687}\right)$$

Linear drag

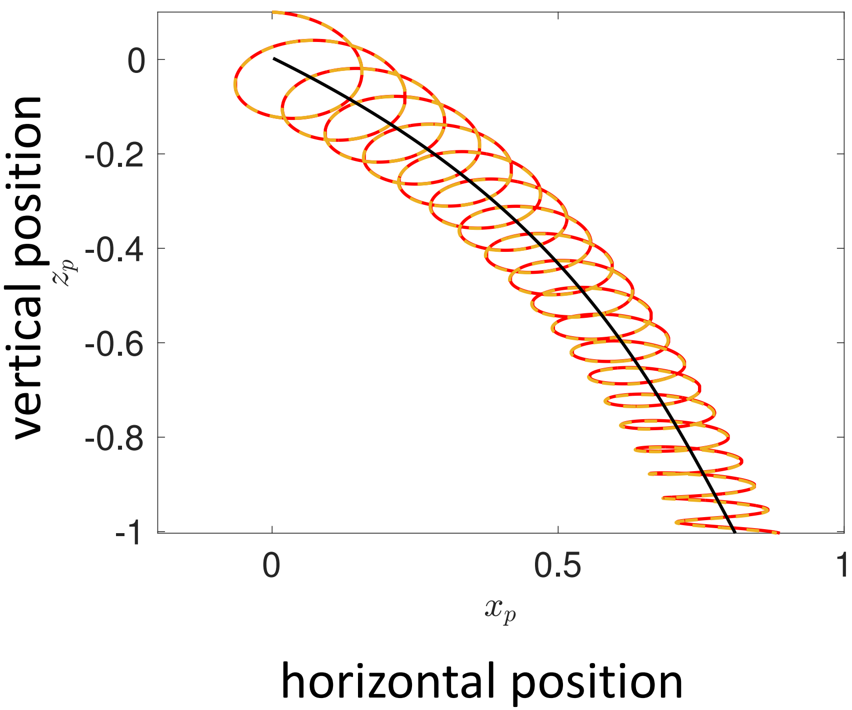
"Linearize" the drag model to allow analysis

$$C_D = \frac{24}{\mathrm{Re}_p}\left(1 + 0.15\mathrm{Re}_{p,t}^{0.687}\right)$$

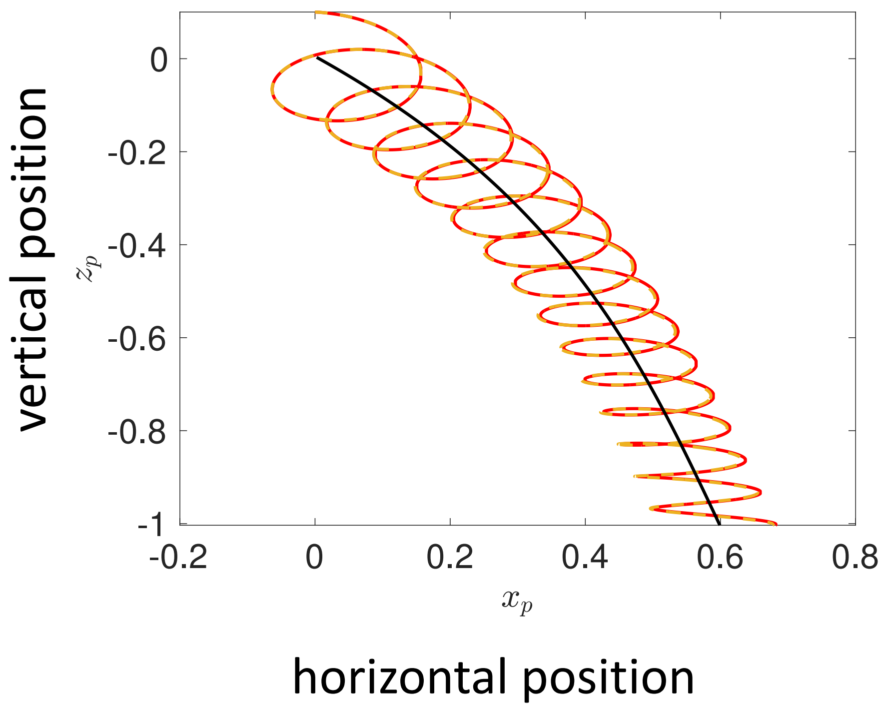
Non-linear drag



Comparison against numerical solutions



$ka = 0.1$, $kh = 1$, $\gamma = 1.1$, $\mathrm{St}_{\mathrm{SN},t} = 0.1$,
$Re_{p,t} = 212$



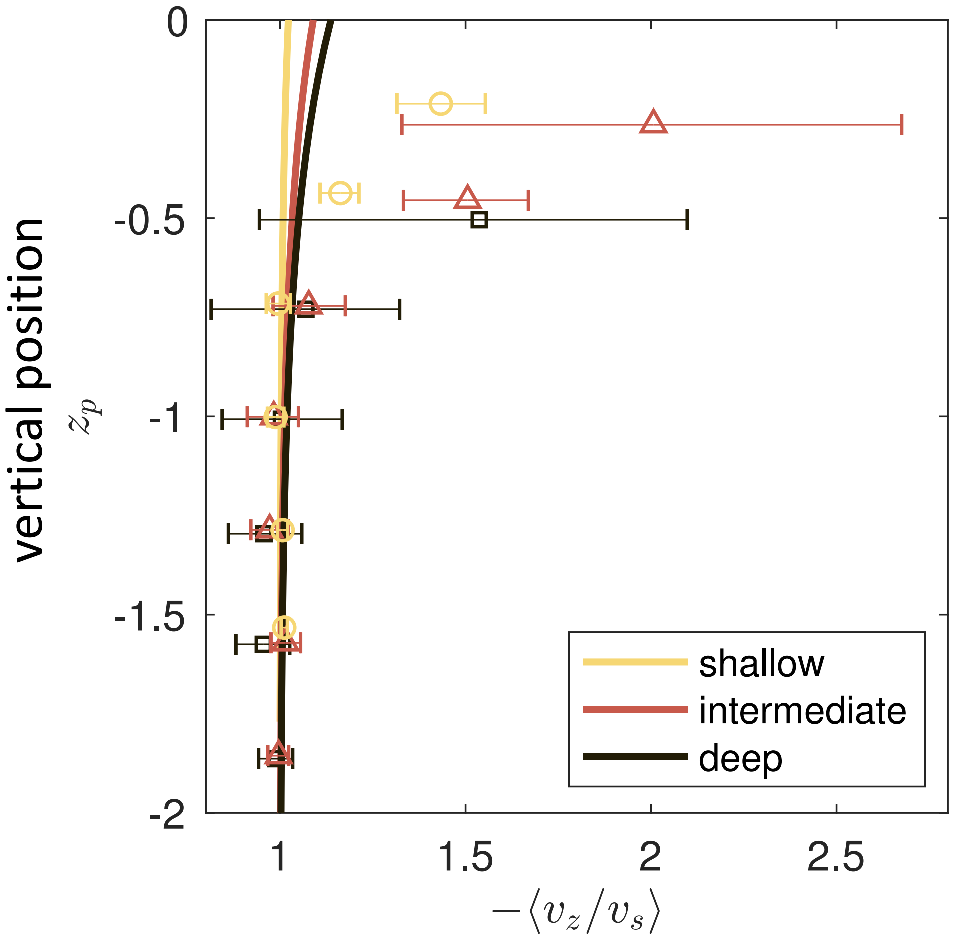
$ka = 0.1$, $kh = 1$, $\gamma = 1.05$, $\mathrm{St}_{\mathrm{SN},t} = 0.5$,
$Re_{p,t} = 1093$

Schiller-Naumann numerical solution

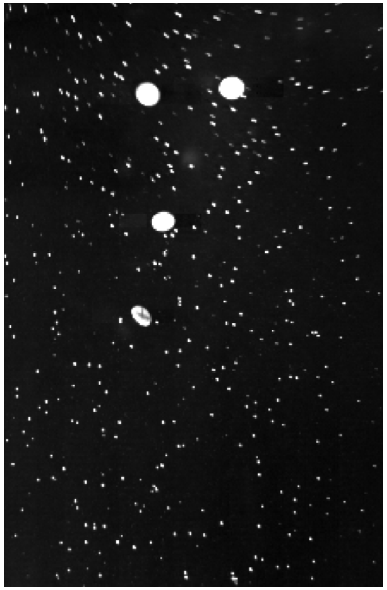
Velocity solution “Linearized” drag

Drift solution “Linearized” drag

Comparison against laboratory experiments



Ensemble-averaged settling velocity



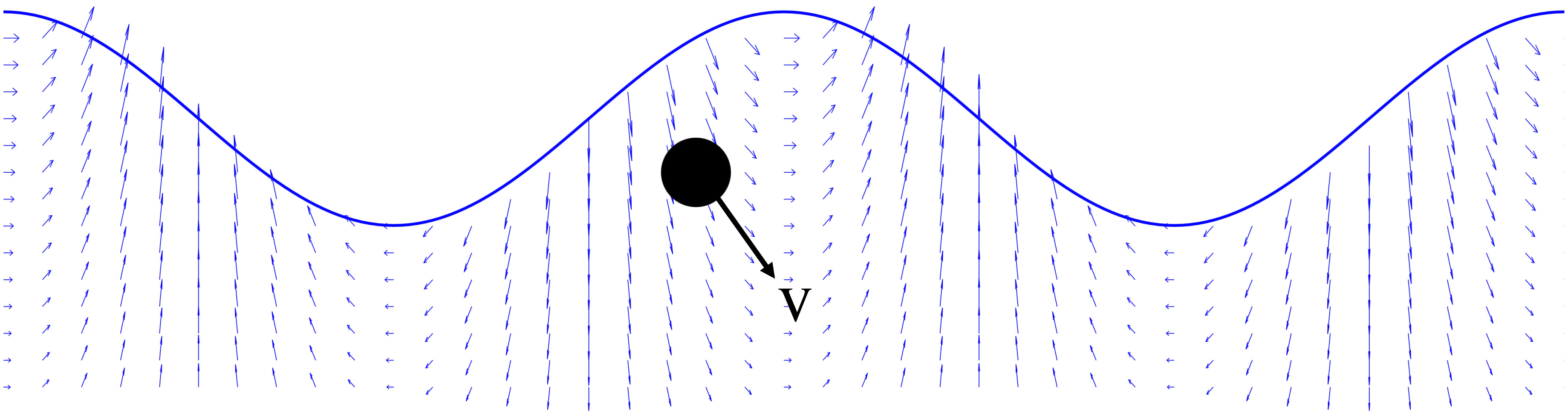
Data from Clark et al. (2020)

Theory captures the trend, if not the full extent of the enhanced settling velocity of inertial particles in surface waves

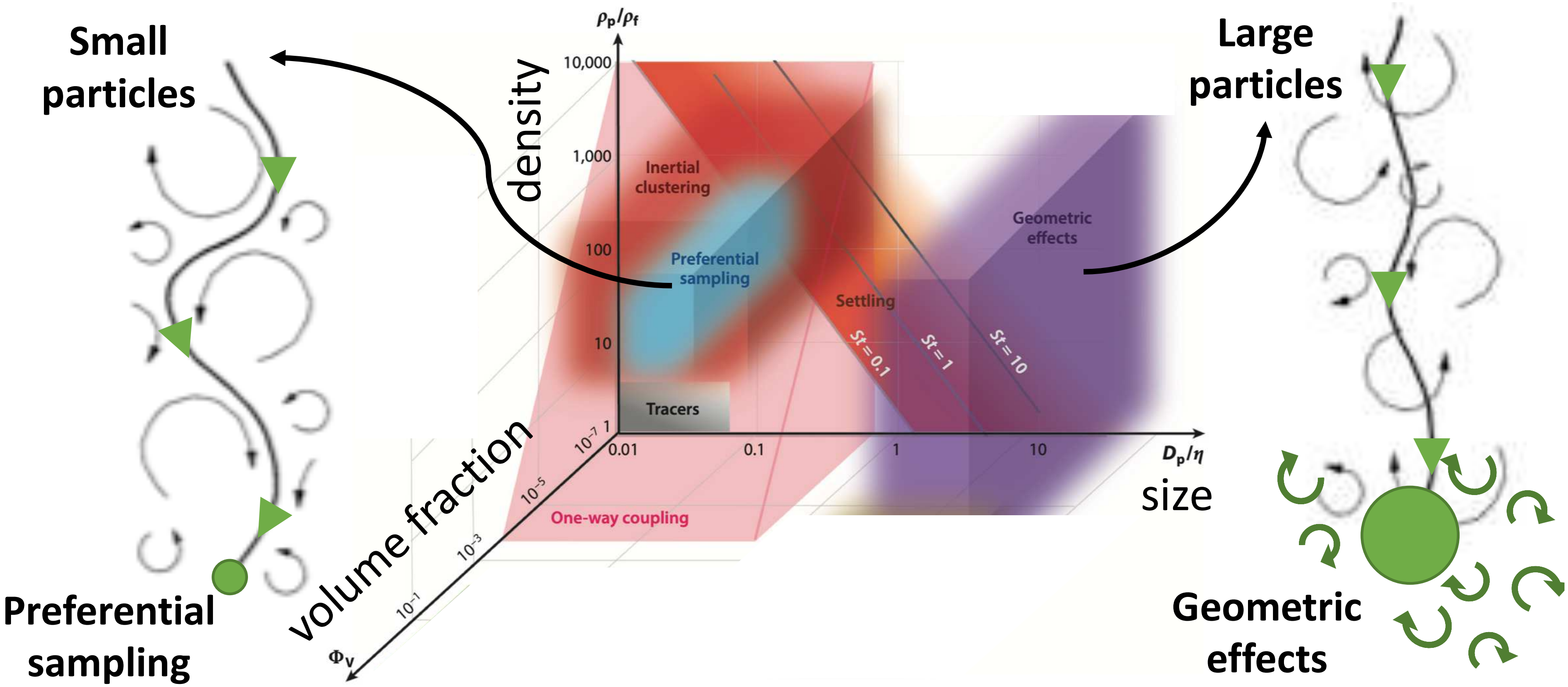
Conclusions

Negatively buoyant particles in surface waves

- Particles trace out small orbitals than fluid particles
- Horizontal drift is reduced relative to fluid particles
- Settling velocity is enhanced relative to terminal settling velocity through dynamical and kinematic (Stokes-drift-like) mechanisms

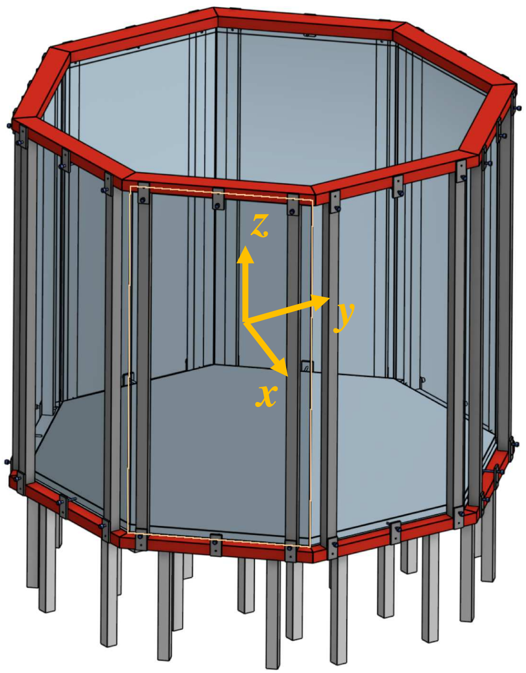


Particle settling at finite-Reynolds numbers



Brandt & Coletti (2022)

Particle settling at finite-Reynolds numbers



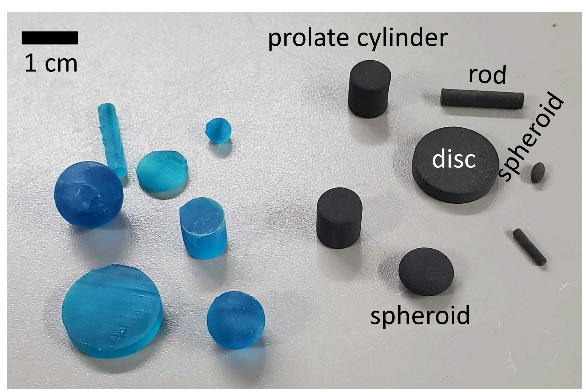
Non-turbulent region for initial particle settling

Large vertical extent of homogenous Isotropic turbulence

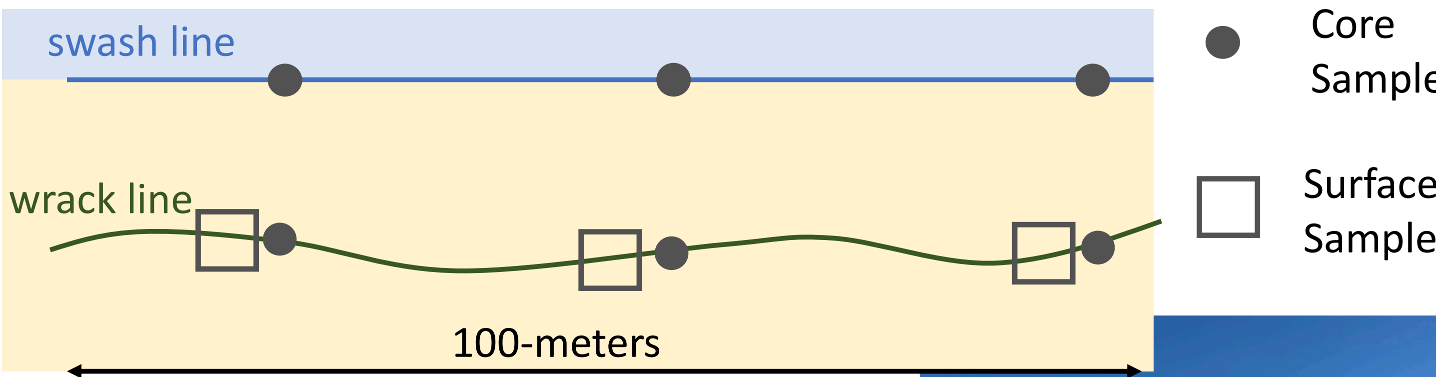
Non-turbulent and wall-affected region where settled particles collect

Turbulence parameters:

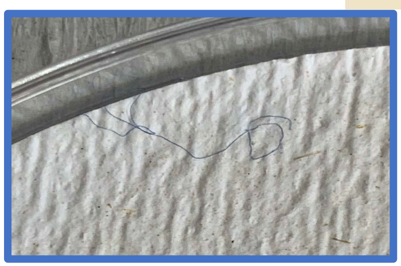
$$\mathrm{Re}_\lambda = 300$$
$$u_{\mathrm{rms}} = 3\ \mathrm{cm/s}$$
$$L_{\mathrm{int}} = 10\ \mathrm{cm}$$
$$T_{\mathrm{int}} = 3\ \mathrm{s}$$
$$\langle \varepsilon \rangle = 10^{-4}\ \mathrm{m}^2/\mathrm{s}^{-3}$$
$$\eta_K = 0.3\ \mathrm{mm}$$
$$\tau_K = 0.1\ \mathrm{s}$$

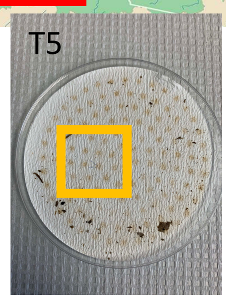


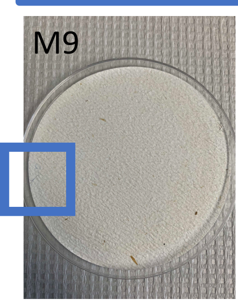
Microplastic beaching and swash burial



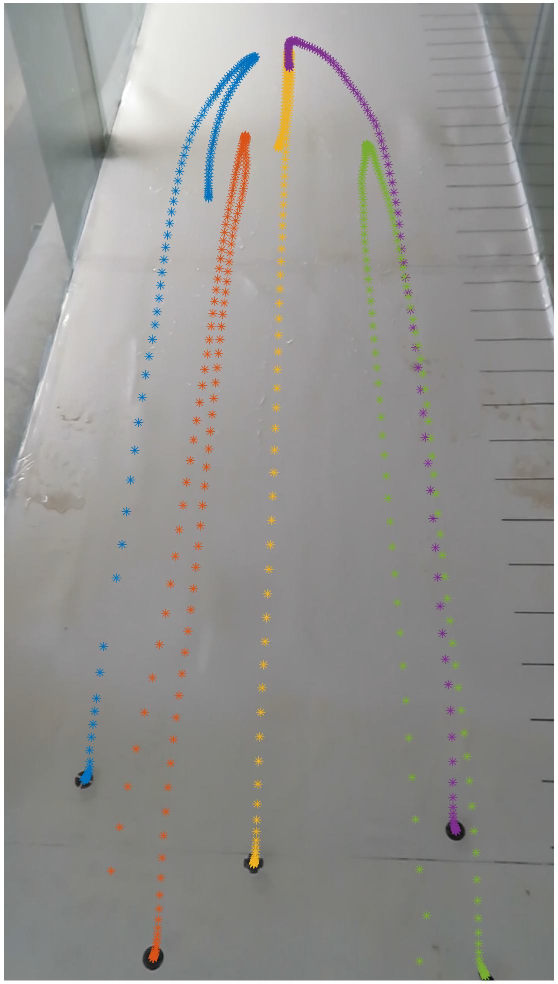
Michigan Sea Grant

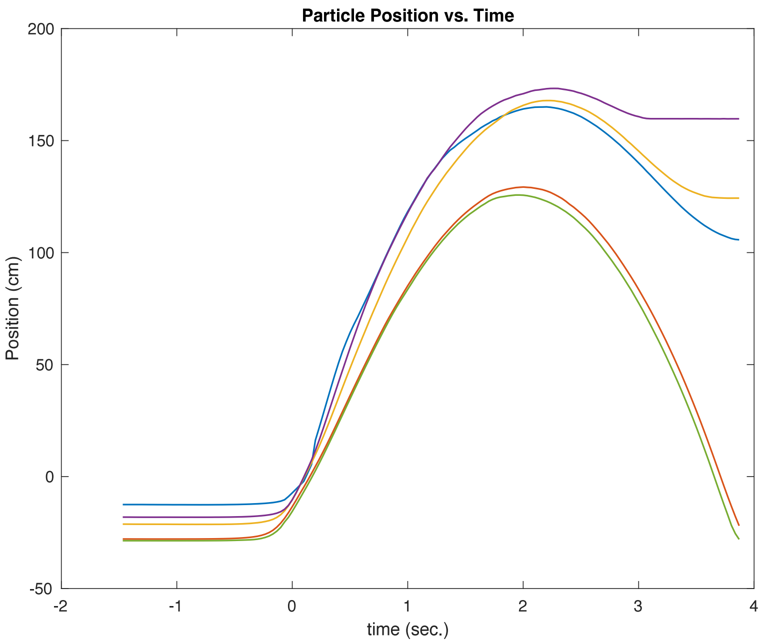






Microplastic beaching and swash burial





Acknowledgements

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Enhanced settling and dispersion of inertial particles in surface waves

Reference

DiBenedetto, Clark, Pujara (2022) *JFM* 936, A38

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Microplastic beaching and swash burial

Joo Young Bang

Particle settling at finite Reynolds numbers

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