Observational signatures for extremal black holes

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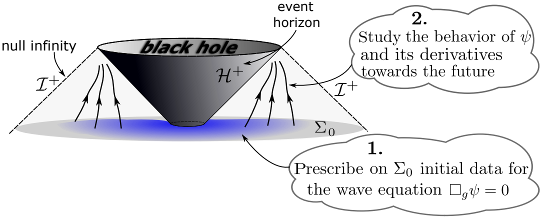
Scalar perturbations

- Scalar fields: Investigate the evolution of solutions to the wave equation

$$\Box_g \psi = 0$$

on Reissner–Nordström or Kerr backgrounds.



- Motivation: In harmonic gauge $\Box_g x^\mu = 0$ the vacuum equations take the form

$$\Box_g g_{\mu\nu} = N_{\mu\nu}(g, \partial g).$$

- Observational signatures at null infinity

Scalar perturbations

- Asymptotics: Schematically, we are looking for estimates of the form:

$$\psi(\tau, r, \theta, \phi) = Q(r, \theta, \phi) \cdot \frac{1}{\tau^p} + O\left(\frac{1}{\tau^{p+\epsilon}}\right)$$

  Similar estimates for the projections ψ_ℓ of the angular decomposition

$$\psi = \sum_{\ell \geq 0} \psi_\ell.$$

- Goal: To show the relevance of conservation laws

- Applications:
  - Upper bounds for stability considerations (black hole exterior)
  - Lower bounds for strong cosmic censorship (black hole interior)

- Results and methods in physical space.

  Schwarzschild $\rightarrow$ sub-extremal Reissner–Nordström (RN) $\rightarrow$ extremal RN $\rightarrow$ sub-extremal Kerr $\rightarrow$ extremal Kerr

Learning from Minkowski

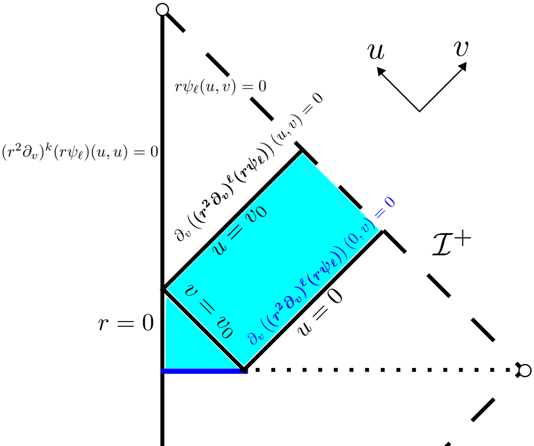
The wave equation on Minkowski gives

$$\partial_u \left[r^{-2\ell} \partial_v \left((r^2 \partial_v)^\ell (r\psi_\ell) \right) \right] = 0$$

and hence

$$\partial_v \left((r^2 \partial_v)^\ell (r\psi_\ell) \right)$$

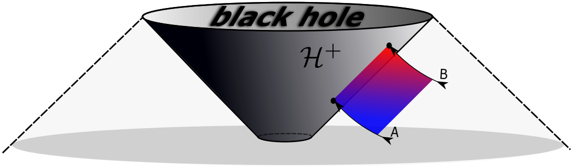
is conserved in the u direction. So $Q = 0$ (SHP).



What about black holes?

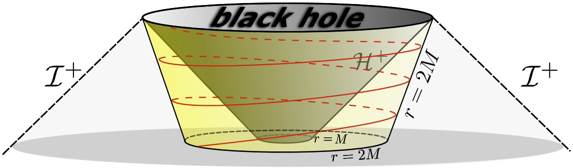
Positivity of ADM mass makes a big difference.

Additional issues include the redshift effect at the horizon



What about black holes?

and the trapping effect at the photon sphere.



What about black holes?

- Contributors: Dafermos, Rodnianski, Andersson, Tataru, Moschidis, Blue, Holzegel, Shlapentokh-Rothman, Sbierski, Fournodavlos, Dyatlov, Häfner, Bony, Smulevici, Klainerman, Ionescu, Tohaneanu, Sterbenz, Soffer, Schlue, Luk, Oh, Finster, Kamran, Smoller, Yau, Donninger, Schlag, Vasy, Hintz, Metcalfe, Wald, Franzen, Teixeira da Costa, ...
- Decay for all $|a| < M$ (Dafermos–Rodnianski–Shlapentokh-Rothman)

Late-time asymptotics

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a Schwarzschild space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-2I_0^{(1)}[\psi]\cdot\tau^{-2}+8MI_0^{(1)}[\psi]\log\tau\cdot\tau^{-3}$

Comments:

- Generically $I_0^{(1)}[\psi]\neq 0$
- Correlated asymptotics along $\mathcal{H}^+$ ($\psi\sim 8I^{(1)}[\psi]\cdot\tau^{-3}$) and $\mathcal{I}^+$ ($r\psi\sim -2I^{(1)}[\psi]\cdot\tau^{-2}$).
- Asymptotics recover semi-analytical work of Leaver.
- Independently obtained by Hintz.

$I_0^{(1)}[\psi]$ in terms of the initial data on $t = 0$

For initial data on the hypersurface $t = 0$, with non-trivial support on the bifurcation sphere, we have

$$I_0^{(1)}[\psi] = \frac{M}{4\pi}\int_{\{t=0\}\cap S_{\mathsf{BF}}} \psi \, d\Omega + \frac{M}{4\pi}\int_{\{t=0\}} \frac{1}{1-\frac{2M}{r}} \partial_t \psi \, r^2 dr d\Omega.$$

$I_0^{(1)}[\psi]$ in terms of the radiation field on $\mathcal{I}^+$

$$I_0^{(1)} = \frac{M}{4\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi \, d\Omega d\tau$$

- The late time tails are dictated by the weak-field dynamics, namely by dynamics at very large r.

$I_0^{(1)}[\psi]$ in terms of conservation laws

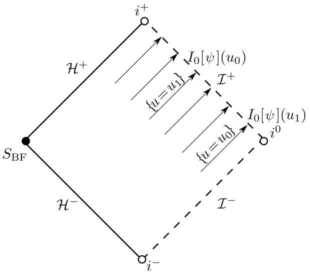
It turns out that the function

$$I_0[\psi](u) = \lim_{r\to\infty} v^2\partial_v(r\psi_0)$$

is constant, that is independent of u. This yields a **conservation law** along $\mathcal{I}^+$. The associated constant

$$I_0[\psi] := I_0[\psi](u) \tag{1}$$

is called the Newman–Penrose constant of ψ.



Now,

$$\boxed{I_0^{(1)}[\psi] = I_0[T^{-1}\psi]}$$

Price's law

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a Schwarzschild space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi_\ell\|_{\mathcal{H}}$	$\psi_\ell\|_{r=R}$	$r\psi_\ell\|_{\mathcal{I}}$
$A_\ell (2M)^\ell I_\ell^{(1)}[\psi] \cdot \tau^{-(2\ell+3)}$	$A_\ell R^\ell I_\ell^{(1)}[\psi] \cdot \tau^{-(2\ell+3)}$	$B_\ell I_\ell^{(1)}[\psi] \cdot \tau^{-(\ell+2)}$

- $I_\ell[\psi](\theta,\phi) = \lim_{r\to\infty} r^2 \partial_v (q_\ell \partial_v (q_{\ell-1} \partial_v (\dots (q_1 \partial_v (r\psi_\ell)\dots))))$ with $q_\ell \sim r^2$.
- Almost sharp decay rates by Donninger, Schlag and Soffer.
- Sharp decay rates by Hintz.

Schwarzschild asymptotics

Theorem

If ψ is a solution to the wave equation on a Schwarzschild space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-2I_0^{(1)}[\psi]\cdot\tau^{-2}+8MI_0^{(1)}[\psi]\log\tau\cdot\tau^{-3}$

R-N asymptotics

Theorem

If ψ is a solution to the wave equation on a sub-extremal R-N space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-2I_0^{(1)}[\psi]\cdot\tau^{-2}+8MI_0^{(1)}[\psi]\log\tau\cdot\tau^{-3}$

- The charge does not seem to affect the asymptotics. Then what about the extremal case?

Extremal R-N asymptotics

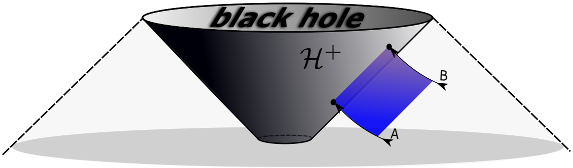
Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a extremal R-N space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$2M^{-1}H[\psi]\cdot\tau^{-1}$	$\frac{4M}{R-M}H[\psi]\cdot\tau^{-2}$	$\left(4MH[\psi]-2I_0^{(1)}[\psi]\right)\cdot\tau^{-2}$

- Horizon asymptotics significantly slower.
- What about the constant $H[\psi]$?

Degeneracy of the redshift effect at extremal horizons



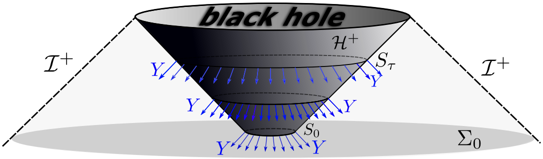
…due to the vanishing of the surface gravity.

Proposition (A.)

If ψ satisfies the wave equation on extremal Reissner–Nordström then the integral

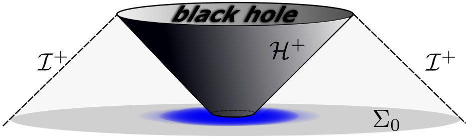
$$H[\psi] = -\int_{S_\tau} \left(Y\psi + \frac{1}{2M}\psi \right) dvol$$

is independent of τ. Here Y is transversal to the horizon.



“Outgoing radiation”

Solutions ψ with $H[\psi] \neq 0$ and compactly supported initial data



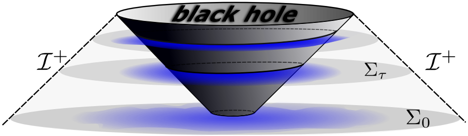
Horizon instability of ERN

- Outgoing perturbations and perturbations with an initially static moment ($H[\psi] \neq 0$) satisfy along the event horizon:

  1) **Non-decay**: $Y\psi \to -\frac{1}{M}H[\psi]$
  2) **Blow-up**: $YY\psi \to \frac{1}{M^3}H[\psi] \cdot \tau$

- $H[\psi]$: "horizon" "hair" since

  1) Energy density measured by incoming observers: $\boldsymbol{T}_{rr}[\psi] \sim H[\psi]$ where $\boldsymbol{T}$ is the E-M tensor,
  2) $Y^k\psi, \boldsymbol{T}_{rr}[\psi] \to 0$ away from the horizon.



- Later extensions/applications by: Reall, Murata, Casals, Zimmerman, Gralla, Tanahashi, Bizon, Lucietti, Angelopoulos, Gajic, Ori, Sela, Tsukamoto, Kimura, Harada, Hadar, Dain, Dotti, Godazgar, Burko, Khanna, Bhattacharjee, Cvetic, Pope, Chow, Berti et al, Cardoso et al,...

Measuring the horizon hair H from afar

- Can we observe/measure the horizon instability from afar?

  Yes.

A signature of extremality at null infinity

Let ψ be a scalar perturbation of Reissner–Nordström (RN) (with mass M, charge e) supported initially near the event horizon.
Let's define:

$$\boxed{s[\psi] := \frac{1}{4M}\lim_{\tau\to\infty}\tau^2\cdot(r\psi)|_{\mathcal{I}^+} + \frac{1}{8\pi}\int_{\mathcal{I}^+\cap\{u\geq 0\}} r\psi|_{\mathcal{I}^+}}$$

For all scalar perturbations on sub-extremal RN we have

$$\boxed{\text{If } |e| < M \text{ then } s[\psi] = 0}$$

Moreover,

$$\boxed{\text{If } s[\psi] \neq 0 \text{ then } |e| = M \text{ (ERN) and } s[\psi] = H[\psi]}$$

- **Extremal black holes admit classical externally measurable hair.**
- **The horizon hair $H[\psi]$ could potentially serve as an observational signature.**
- **For extremal black holes information "leaks" from the event horizon to null infinity.**

Transient signature

The behavior of nearly extreme black hole hair and its measurement at future null infinity as a transient phenomenon by Burko, Khanna and Sabharwal.

Schwarzschild asymptotics

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a Schwarzschild space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-8I_0^{(1)}[\psi]\cdot\tau^{-3}$	$-2I_0^{(1)}[\psi]\cdot\tau^{-2}$

- What about Kerr asympototics?

Kerr asymptotics

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a sub-extremal Kerr space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$-8I_0^{(1)}[\psi] \cdot \tau^{-3}$	$-8I_0^{(1)}[\psi] \cdot \tau^{-3}$	$-2I_0^{(1)}[\psi] \cdot \tau^{-2}$

- Spherical mean wrt BL spheres.
- NP constants, $T-$invertibility need some care.
- Explicit expressions of all constants.
- Asympotics derived by Hintz using a different approach.

Kerr asymptotics for modes $\ell = 1, 2$

Decompose

$$\psi = \sum_{\ell \geq 0} \psi_\ell$$

using the Boyer-Lindquist spheres in Kerr.

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a sub-extremal Kerr space-time with smooth compactly supported initial data then

Asymptotics in the exterior region

mode	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$\ell = 1$	$-\frac{32R}{3} I_1^{(1)}(\theta, \varphi_*) \cdot \tau^{-5}$	$-\frac{4}{3} I_1^{(1)}(\theta, \varphi_*) \cdot \tau^{-3}$
$\ell = 2$	$-\frac{16}{3}\sqrt{\frac{\pi}{5}} a^2 I_0^{(1)} \cdot Y_{20}(\theta) \cdot \tau^{-5}$	$[-\frac{1}{10} I_2^{(1)}(\theta, \varphi_*) + \frac{8}{3}\sqrt{\frac{\pi}{5}} a^2 I_0^{(1)} \cdot Y_{20}(\theta)] \cdot \tau^{-4}$

- Horizon oscillations for $\ell = 1$ (Barack–Ori)

$$\psi_1 \sim_{\tau \to \infty} -\frac{32 r_+}{3} \cdot \sum_{m=-1}^{1} I_{1m}^{(1)} \cdot Y_{1m}(\theta, \varphi_{\mathcal{H}^+}) \cdot \frac{e^{im\omega_+ \tau}}{\tau^5}$$

- Slower decay compared to Schwarzschild (mode coupling).

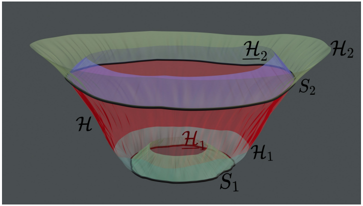
Extremal Kerr

Decompose in azimuthal frequency $\psi = \sum_{m \geq 0} \psi_m$.

- ψ_0: Similar behavior as in ERN. Numerical confirmation by Khanna et al
- ψ_m: Amplified instability at the horizon: $\tau^{-\frac{1}{2}}$ decay for ψ, $\tau^{\frac{1}{2}}$ growth for $Y\psi$ (Gralla, Zimmerman, Casals).
- $\sum_{m \geq m_0} \psi_m$: open
- Non-linear perturbations: formation of naked singularities from smooth data?

Addendum: Characteristic gluing constructions

Conservation laws are obstructions to characteristic gluings.



- Linear wave equation: Necessary and sufficient conditions (A.)
- Einstein equations: Small data (A., Czimek, Rodnianski). Charges related to mass, linear and angular momentum, center of mass.

Thank you!