BIRS Report

Sarah Reznikoff (Kansas State University)
Maria Grazia Viola (Lakehead University)

1 Overview

The workshop Women in Operator Algebras II was held at the Banff International Research Station from December 6 to December 10, 2021. This was the second event in the Women in Operator Algebras series (the first workshop in the series was held at BIRS in November 2018), and it took place in hybrid form because of the COVID 19 pandemic.

The workshop had forty-two participants from 15 countries (Australia, Canada, China, Czech Republic, France, Germany, India, Italy, Korea, Denmark, Netherlands, New Zealand, Norway, USA, UK). Most of the researchers participated online, while seven researchers from the USA and Canada were on location.

The focus of the workshop was to have women in operator algebras conduct pioneering research in small groups and to build a network of women in the field to drive future research collaborations and offer career support and advice. We believe we have accomplished both goals.

Eight innovative and interesting research projects were proposed by the group leaders for the workshop, dealing with topics related to textiles, topological full groups, the Baum-Connes conjecture, entropy of operator-valued measures, inductive limits of spectral triples, and properties related to the classification of C^*-algebras, such as the dynamic asymptotic dimension of groupoids or the diagonal dimension of C^*-algebras of groupoids.

To avoid inviting only researchers we already knew, and to offer opportunities to graduate students and early researchers to initiate new collaborations, we issued only 17 invitations to group leaders and a few senior researchers, and filled the remaining spots through an application process. A call for applications was sent out using social media posts, an email lists for women in operator algebras, and email lists from major conferences in operator algebras. The application included a series of questions, a ranked list of 2-3 research projects of interest, a brief statement describing how the applicant's expertise fitted with the projects and how they would benefit from participating, and a CV.

Before the workshop, the participants were divided into research groups, according to their expertise and interests. Under the guidance of the group leaders, the members of the group started to read background material and formulate research questions and strategies weeks before the workshop took place.

On the first day of the workshop, the group leaders described the research project and some problems on which their group would be working during the week. During the week the group members met on zoom or/and in person, discussing ideas, background material, approaches to the questions under investigations, methods, and proofs. On the last day of the workshop, the various groups gave a final presentation describing the work done during the week, the results reached, further directions of research, and future plans to carry on the collaborations.

To foster a network of women in operator algebras and offer career support and advice to women in the field, we had a two-hour event in the middle of the week, organized in conjunction with the Women in Operator Algebras Mentor Network. The event was a discussion of difficult situations and issues experienced by the workshop participants or network members in their working environment, and how to find support and solutions when facing similar problems.

The hybrid format worked well in facilitating collaborative work, with all participants actively involved in zoom or in person in discussions and exchange of ideas. Probably the only component that was slightly missing from the workshop was the social exchanges that usually take place at conferences. Although we tried to implement the use of Gathertown to foster interactions between participants in different research groups, the different time zones and the unfamiliarity of many of the participants with Gathertown made its use unpractical.

Offering a brief demonstration on the use of Gathertown to all workshop participants on the first day of the workshop may have encouraged a greater use of the platform.

2 Scientific progress made during the workshop: reports from the research groups

Operator algebras—the study of $*$-algebras of bounded linear operators on Hilbert space—is a major branch of functional analysis that originated with von Neumann in the early part of the 20th century as a mathematical framework for modeling phenomena in quantum mechanics. It has since become a fundamental part of modern theoretical physics, as the natural setting for quantum information theory and quantum computing. In addition, the field has profound connections to several other areas of mathematics such as group theory, knot theory, dynamical systems, ergodic theory, and conformal field theory.

According to the feedback that we have received from the participants, the workshop was very successful and progress was made by each group on some of the research question on which they were working. All groups have expressed their intention of continuing the collaboration and have made concrete plans to meet in person or on zoom in the first few months of 2022.

Details about the progress made and the plans to collaborate further can be found in the following reports submitted by the research groups.

2.1 Project: Finite nuclear dimension of C^*-algebras of étale groupoids

Group Members: Kristin Courtney, Anna Duwenig, Magda Georgescu, Astrid an Huef , Maria Grazia Viola

The classification of unital, separable, simple C^*-algebras of finite nuclear dimension which satisfy the universal coefficient theorem was recently completed by “many hands” [22]. Finite *nuclear dimension* [23], a generalisation to C^*-algebras of covering dimension of a topological space, is a key hypothesis in the classification program. So studying the nuclear dimension of classes of C*-algebras is an interesting problem.

In [11], Guentner, Willett and Yu defined *dynamic asymptotic dimension* for an action of a discrete group on a locally compact space. Motivated by the transformation-group groupoid, they extended their definition to principal étale groupoids. They found a bound on the nuclear dimension of the reduced groupoid C*-algebra involving the dynamic asymptotic dimension of the groupoid and the topological covering dimension of its unit space. During the workshop, we sought to understand the Guentner–Willett–Yu theorem and its proof, with the broad aim of extending their results to non-principal groupoids and/or twisted groupoids.

The proof of the Guentner–Willett–Yu theorem simplifies significantly if the unit space of the groupoid is compact. We felt that there should be a method of passing from the compact to the non-compact case involving only the groupoid and not functions on the groupoid. We now have a promising strategy for this: given an étale groupoid G with non-compact unit space, we have defined a groupoid $\tilde{G}$ with compact unit space such that the reduced/full C*-algebra of $\tilde{G}$ is isomorphic to the smallest unitisation of the reduced/full C*-algebra of $\tilde{G}$. We think that the dynamic asymptotic dimension of $\tilde{G}$ is equal to that of G and we have devised a strategy for proving this. Since the nuclear dimension of a C*-algebra and its smallest unitisation coincide [23], this will be a nicer way of passing from the compact to the non-compact case.

A twist $E \to G$ is a groupoid extension of G by the circle $\mathbf{T}$. Through remarkable representation and rigidity results, for example [3, 14, 20], we know that many C*-algebras, including all classifiable C*-algebras, have a twisted groupoid model. Hence a description of dynamic asymptotic dimension in the twisted groupoid setting promises significant structural insight into this large and well-studied class of C*-algebras. We have developed an approach for showing that for a twist $E \to G$ where G is principal, the nuclear dimension of the reduced C*-algebra of the twist is bounded and our bound is the same as the one found by Guentner, Willett and Yu.

A possible application is to non-principal groupoids H with closed orbits and continuously varying stability subgroups. The full C*-algebras of such groupoids are isomorphic to the full C*-algebras of a certain twist $E \to G$ with G principal [17, 8]. If this isomorphism descends to the reduced C*-algebras and the twist is over an étale groupoid, then our conjecture in the previous paragraph would apply. More generally, we have defined a notion of dynamic asymptotic dimension modulo stability groups which we plan to investigate.

During the week of the workshop, we established a lively and productive collaboration. Our immediate plans are to fill in the proof outlines/strategies we mentioned above. For this, we have allocated subgroups to certain tasks and the whole group meets on Zoom on a monthly basis to report progress and to plan. We have developed a long list of questions about nuclear dimension of C*-algebras and dynamic asymptotic dimension of groupoids on which future work could be based.

2.2 Project: The Baum-Connes conjecture and mapping class groups

Group members: Sara Azzali, Sarah Browne, Indira Chatterji, Maria Paula Gomez-Aparicio, Sanaz Pooya, Hang Wang

Introduction The Baum–Connes conjecture, introduced by Paul Baum and Alain Connes in the 80s, gives a way of computing the K-theory of the reduced C*-algebra of a locally compact group, using the equivariant K-homology of the universal space classifying the proper actions of the group [4, 5].. The conjecture has been proven for a large class of groups that includes all almost-connected Lie groups, amenable group, and hyperbolic groups, those including $\mathrm{SL}(2, \mathbb{Z})$. But the conjecture (in particular the question about surjectivity) remains open for many cases of discrete groups, including $\mathrm{SL}(n, \mathbb{Z})$, for $n > 2$, as well as mapping class groups of higher genus surfaces. The difficulty in many cases involves a rigidity property in representation theory and no real progress has been made in the last 20 years after Lafforgue's spectacular breakthrough in 2001 which remains poorly understood [15].

Mapping class groups are orientation preserving homeomorphisms of surfaces, modulo the ones homotopic to the identity. Despite their several hyperbolic features encoded in the curve complex, the surjectivity of the Baum-Connes map is still open in full generality. The injectivity was proven by Hämenstadt in [12] as a consequence of boundary amenability. Some particular cases of mapping class groups like braid groups or genus one surfaces are known, by work of Oyono-Oyono [19], Schick [21], Chabert-Echteroff [7] (see also the recent [2]).

The project Our project is centered about Vincent Lafforgue's paper *K-théeorie bivariante pour les algèbres de Banach et conjecture de Baum-Connes* [15] and its relation with mapping class groups.

Lafforgue's works represented a very important progress. He showed the Baum-Connes conjecture for large classes of locally compact groups with property (T): all semi-simple real or reductive p-adic Lie groups, and for discrete cocompact subgroups of a rank 1 Lie groups.

The main tool is Lafforgue's bivariant KK-functor for Banach algebras, KK^{ban} and the main point is the equality of classes $\gamma = 1$ in KK^{ban}, which is realised by an homotopy.

Mapping class groups and the Baum–Connes assembly map The injectivity of the Baum-Connes assembly map for mapping class groups was proven by Hämenstadt in [12] as a consequence of boundary amenability. Some particular cases of mapping class groups like braid groups or genus one surfaces are known, by work of Oyono-Oyono [19], Schick [21], Chabert-Echteroff [7] (see also the recent [2]). The surjectivity of the Baum-Connes map is still open in full generality.

The WOA 2 week During the WOA 2 meeting, we have learned about

1. the Rips complex, which is used in showing that the γ element acts as identity on the left hand side of Baum–Connes.

2. Banach KK-theory, in which one can perform the homotopy from γ element to the trivial representation.

3. mapping class group reading Brendle–Childers's introduction to mapping class groups written for "Office Hours with a geometric group theorist."

4. the construction of γ element in Kasparov–Skandalis in the case of group acting properly on bolic spaces [13].

We have identified a sequence of steps and related questions. We plan to develop Lafforgue's methods and approach MCG groups.

Plans for the near future As short term goal, we plan to write a survey about Chapter 2 in Lafforgue's paper. Our long term goal is to investigate the strong bolicity property of Lafforgue in order to know if there is a way of relaxing it so that mapping class groups satisfy it.

We applied to a Research in Pairs program at the IHP in Paris and our application got accepted hence, the part of our group that is based in Europe is going to meet in person for two weeks in May 2022, and we will meet with the others online to continue working on our project.

2.3 Project: The f-divergence, entropy, and related measures of two operator-valued measures:

Group members: Sarah Plosker, Hui Tan, Kateryna Tatarko, Elisabeth Werner, Runlian Xia

Main Goal Our main goal is to generalize f-divergence and related concepts used to compare two classical (scalar-valued) measures to the setting of operator-valued measures.

Positive Operator-valued Measures and Quantum Random Variables: Positive, operator-valued measures (POVMs) arise as natural objects of study in quantum mechanics. Let $\mathcal{H}$ be a finite dimensional or separable Hilbert space, $\mathcal{B}(\mathcal{H})$ the algebra of all bounded operators on $\mathcal{H}$, $\mathcal{T}(\mathcal{H})$ the Banach space of all trace-class operators (all operators in $\mathcal{B}(\mathcal{H})$ that have a finite trace under any orthonormal basis), and $\mathcal{S}(\mathcal{H}) \subset \mathcal{T}(\mathcal{H})$ the convex subset of all positive, trace-one trace-class operators ρ (called *states* or density operators). We use X to denote a locally compact Hausdorff space and $\mathcal{O}(X)$ to denote the σ-algebra of Borel sets on X.

Then a *positive operator-valued measure (POVM)* $\nu : \mathcal{O}(X) \to \mathcal{B}(\mathcal{H})_+$ is an ultraweakly countably additive function. That is, for every countable collection $\{E_k\}_{k\in\mathbb{N}} \subset \mathcal{O}(X)$ of disjoint Borel sets one has

$$\nu\left(\bigcup_{k\in\mathbb{N}} E_k\right) = \sum_{k\in\mathbb{N}} \nu(E_k),$$

with the sum converging in the ultraweak topology.

f-divergence in the Classical Setting: An f-divergence is a function that measures the difference between two probability distributions. Let (X, μ) be a measure space and let $P = p\mu$ and $Q = q\mu$ be probability measures on X that are absolutely continuous with respect to the measure μ. Let $f : (0,\infty) \to \mathbb{R}$ be a convex or a concave function and define $f^* : (0,\infty) \to \mathbb{R}$ of f by

$$f^*(t) = tf(1/t), \;\; t \in (0,\infty). \tag{1}$$

The f-divergence $D_f(P,Q)$ of the measures P and Q is then defined by

$$\begin{aligned} D_f(P,Q) \;\; &= \;\; \int_{\{pq>0\}} f\left(\frac{p}{q}\right) q d\mu + f(0)\, Q\left(\{x \in X : p(x) = 0\}\right) \\ &+ \;\; f^*(0)\, P\left(\{x \in X : q(x) = 0\}\right), \end{aligned} \tag{2}$$

provided the expressions exist, where $f(0) = \lim_{t\to 0^+} f(t)$ and $f^*(0) = \lim_{t\to 0^+} f^*(t)$. We write

$$D_f(P,Q) = \int_X f\left(\frac{p}{q}\right) q d\mu. \tag{3}$$

An important example of an f-divergence is the relative entropy $D_{KL}(P\|Q)$: For $f(t) = t\ln t$ (with *-adjoint function $f^*(t) = -\ln t$), the f-divergence is *Kullback-Leibler divergence* or *relative entropy* from P to Q

$$D_{KL}(P\|Q) = \int_X p \ln \frac{p}{q} d\mu. \tag{4}$$

Outcome and further research During the event at BIRS, we came up with two possible extensions of the concept of f-divergence to the setting of POVM. It is now our goal to compare these two definitions, investigate in more detail their properties and in particular establish inequalities related to these notions. In the literature there already exist notions of f-divergence for quantum channels. We want to explore how our notions fit in this context.

2.4 Project: (Inductive) Limits of spectral triples

Group members: Carla Farsi, Therese-Marie Landry, Nadia Larsen and Judith Packer

Start with an inductive limit of unital C^*-algebras

$$\mathcal{A} \;=\; \left(\lim A_j\right) = A_1 \to A_2 \to A_2 \ldots$$

with embeddings $\varphi_j : A_j \to A_{j+1}$. In addition assume that each A_j is endowed with a 'compatible' group action of a group $G_j \cong G$ (with each G_j isomorphic to a fixed finite abelian group G, $|G| = |\hat{G}| = n$), such that the fixed point subalgebra $(A_j)^{G_j} \cong A_{j-1}$.

- If the above cover is 'regular' in the sense of Aiello et. al [1]. Then:

$$\mathcal{A} \;=\; \left(\lim A_j\right) \hookrightarrow A_0 \otimes \mathrm{UHF}(n^\infty);$$

- If in addition to regularity we assume that $A_j \cong A_{j+1}$, for all j, then Aiello et. al prove that there exists an inductive limit spectral triple on

$$\mathcal{A} \;=\; \lim A_j.$$

We want to investigate if the second of the above two results of Aiello et al. generalize when we remove the hypotheses $A_j \cong A_{j+1}$. The first one is already stated and proved in [1, Section 1]. But for completeness we will start by reviewing it below.

Given an inductive limit of unital C^*-algebras

$$\mathcal{A} \;=\; \left(\lim A_j\right) = A_0 \to A_1 \to A_2 \to A_2 \ldots$$

with embeddings $\varphi_j : A_j \to A_{j+1}$, assume that each A_j is endowed with a 'compatible' group action of a finite abelian group $G_j \cong G$ (isomorphic to a fixed group G, $|G| = n$), such that the fixed point subalgebra $(A_j)^{G_j} \cong A_{j-1}$.

Also assume that the above cover is 'regular' in the sense of Aiello et. al [1], i.e., there exists a section σ that brings each eigenspace of the G-action on B to a unitary $\sigma : \hat{G} \to B_k \cap U(B)$.

Theorem 2.1. *(Aiello et al. [1, Theorem 1.5]) In the above situation we have:*

$$\mathcal{A} \;=\; \left(\lim A_j\right) \hookrightarrow A_0 \otimes UHF(n^\infty), \text{ where } n = |\hat{G}|.$$

Next, resume looking at the situation of 2 algebras in the chain, that is, $\varphi : A \to B$. Now note that there is an expectation: $E_{B,A} : B \to A$, given by $E_{B,A}(b) := \sum_{g \in G} gb$. Endow A and B with the traces τ_A and τ_B such that $\tau_B = \tau_A \circ E_{B,A}$. By the GNS construction, τ_J induces a GNS representation on $\mathcal{H}_J$, for $J = A, B$.

Proposition 2.2. *(See [1, Propositions 1.8 and 2.4]) With A, B, $E_{B,a}$, τ_A, and τ_B as above,*

1. *The GNS representation associated to τ_B is unitariy equivalent to the representation $\widetilde{\pi_A}$ obtained from the representation π_A associated to τ_A by [1, Proposition 1.8]:*

$$\widetilde{\pi_A}(b) := [\pi_A(M(b)_{hk})]_{h,k \in \hat{G}} \in M_{\hat{G}}(\mathcal{H}_A) \cong \mathcal{B}(\mathcal{H}_A \otimes \mathbb{C}^n).$$

2. *The above map extends to an isomorphism of the above Hilbert spaces implemented by the unitary operator:*

$$v_i : \mathcal{H}_B \to \mathcal{H}_A \otimes \mathbb{C}^n$$

given by:

$$\xi \to \sum_{k \in \hat{G}}^{\oplus} \sigma(k)^{-1} \xi_k.$$

Now consider a spectral triple over B

$$\left(\mathcal{L}_B, \pi_B : B \to \mathcal{B}(\mathcal{H}_B), D_B\right), \tag{5}$$

where $\mathcal{L}_B$ is a dense subalgebra of B, and $\pi_B : B \to \mathcal{B}(\mathcal{H}_B)$ is as in Proposition 2.2.

We then want to use Proposition 2.2 to transport the spectral triple in Equation (2.2) to a spectral triple over the Hilbert space $\mathcal{H}_A \otimes \mathbb{C}^n$.

Moreover, we also want to, whenever possible, translate the spectral triple in Equation (5) to be a spectral triple of type

$$\left(\mathcal{L}_A, \widetilde{\pi_A} : A \to \mathcal{B}(\mathcal{H}_A \times \mathbb{C}^n), D_A\right). \tag{6}$$

In [1], Aiello, Guido and Isola concentrate on the case where each $A_j \cong A_\theta$, for some rational $\theta \in [0,1]$. We call this the *stable* case. Our aim is to generalize the results of [1] where $A_j \cong A_{\theta_j}$ to the *non-stable case*, i.e. the case of varying $\{\theta_j\} \subset [0,1]$ which need not be rational.

2.5 Project: "Dubious textiles"

Group Members: Samantha J. Brooker, Priyanga Ganesan, Elizabeth Gillaspy, Ying-Fen Lin, Julia Plavnik

The research project and its connections with other branches of mathematics The overall goal of this research project is to leverage C^*-algebraic techniques and invariants to enhance our understanding of 2-dimensional shifts of finite type (2D SFTs). We hope to build on the successful connections that have been established between 1-dimensional shifts of finite type and Cuntz–Krieger algebras (equivalently, graph C^*-algebras) by a variety of researchers (notably Matsumoto-Matui and Carlsen-Rout, among many others). It is now well established that important equivalence relations among 1D SFTs (flow equivalence, two-sided conjugacy, one-sided conjugacy) are mirrored by equivalence relations among the associated C^*-algebras (stable isomorphism, diagonal-preserving stable isomorphism, gauge-intertwining stable isomorphism).

In the symbolic dynamics community, 2-dimensional shift spaces are generally understood to be much more diverse and complicated than 1D SFTs, and most researchers have focused their attention on specific subclasses of 2D SFTs. Johnson and Madden established in 1999, however, that every 2D SFT is conjugate to one which arises from a textile system. Subsequently, Kang, Pask and Webster established that higher-rank graphs of rank 2 (or *2-graphs*) give rise to textile systems. However, not all textile systems arise from 2-graphs – only those which are LR (in the notation of Nasu, who introduced the concept of textile systems in 1995). While we know (thanks to the 2000 paper of Kumjian and Pask) how to compute the C^*-algebras of higher-rank graphs, it is not clear how to associate a C^*-algebra to a 2D SFT which is not LR.

A first focus of our investigation is to understand how conjugacy for 2D SFTs manifests at the C^*-algebra level. In their 1999 paper, Johnson and Madden established that any conjugacy for textile systems (and hence for any 2D SFTs) can be written as a sequence of in- and out-splittings, amalgamations (the inverse of in/outsplitting), and inversion (switching horizontal and vertical directions). Eckhardt et al (2021) established that insplitting for 2-graphs gives an isomorphism of C^*-algebras. However, we quickly discovered at BIRS that textile system insplitting and 2-graph insplitting are not the same. Worse, the result of a textile-system in- or out-split will never be an LR textile system. In other words, many 2-graph SFTs are conjugate to SFTs which do not arise from 2-graphs.

Progress made during the week Our main focus during the week at BIRS was to understand the relevant constructions: textile systems, 2D SFTs, 2-graphs, and in/outsplitting.

After realizing the incompatibility of 2-graph insplitting and textile system insplitting, we began work on a proof that 2-graph in- and out-splitting does yield a conjugate 2D SFT.

Future research In addition to finalizing the proof referenced above, we also plan to pursue the following lines of research:

- Find more classes of 2D SFTs which have C^*-algebraic models. We hope that Spielberg's "categories of paths" will be useful here.
- Identify which conjugacy classes of 2D SFTs have 2-graph representatives. For these classes, we have a C^*-algebra model which we hope will shed light on the dynamics.

Plans to continue the research We are planning to meet in person at the University of Montana in March 2022 to continue working on this project. We are also planning to maintain the momentum by regular Zoom meetings. Due to geographical constraints, we cannot all meet at the same time (even over Zoom) so we are planning to have two standing biweekly meetings which everyone can attend at least one of.

2.6 Project: Connections between ample groupoid C*-algebras, topological full groups and inverse semigroups

Group members: Lisa Orloff Clark, Becky Armstrong, Eun Ji Kang, Mahya Ghandehari, Dilian Yang

Overview of Project: Given an action of the integers on the Cantor set, Giordano, Putnam and Skau define a corresponding full group in [10]. Building on the results of [10], Matui considers a more general setting in [16]: given any Hausdorff effective étale groupoid with compact unit space, he defines a corresponding *topological full group*. He shows that this group is a groupoid invariant for certain classes of groupoids. The construction was generalised to non-effective groupoids by Nekrashevych in [18].

Let G be an ample groupoid with compact Hausdorff unit space $G^{(0)}$. Then the topologoical full group $[[G]]$ is the collection of all compact open bisections $B \subseteq G$ such that $s(B) = r(B) = G^{(0)}$. The set $[[G]]$ is a group with identity $G^0 \in [[G]]$. The product and inverse in $[[G]]$ are given by:

$$BD := \{bd : b \in B, d \in D \text{ such that } (b,d) \in G^{(2)}\} \quad \text{and} \quad B^{-1} := \{b^{-1} : b \in B\}.$$

. We consider the following two questions:

1. The group $[[G]]$ sits inside of the inverse semigroup S_G of all compact open bisections of G. Bice shows that the groupoid of ultrafilters of S_G is isomorphic to G itself giving a kind of duality [6, Proposition 1.2] (see also [9]). Can we view the set of ultrafilters of $[[G]]$ as a subset of the ultrafilters of S_G?
2. From the inverse semigroup point of view, we also see that there is a representation of the group algebra of $[[G]]$ into the Steinberg algebra of G. Similarly for the C*-algebras? What are the properties of these representations?

Progress made during the week: We considered the second question first and learned that the natural representation of the group algebra of $[[G]]$ is typically not injective. We are in the process of developing a combinatorial proof that considers the number of orbits in the unit space. In terms of the first question, we conducted a detailed investigation of some examples. We learned that singletons $\{g\}$ where $g \in [[G]]$ are in fact filters themselves but are almost never ultrafilters. Work is ongoing.

Future plans: We have set aside the week of 20 February to meet daily via zoom to continue working on this project.

2.7 Project: Diagonal dimension of C*-algebras

Group Members: Anshu Nirbhay, Dawn Archey, Camila F. Sehnem, Marzieh Forough, Ja A Jeong, Karen Strung

This project set out to study the so-called "diagonal dimension" of certain C*-algebras containing C*-diagonals. The diagonal dimension is a refinement of the completely positive approximation property that is designed to keep track of a prescribed C*-diagonal subalgebra. It was introduced by Li, Liao and Winter.

A canonical example of a C*-algebra with a C*-diagonal is given by the inclusion of $C(X) \subset C(X) \rtimes_\alpha G$, where $C(X) \rtimes_\alpha G$ is the crossed product associated to a minimal free action of a discrete group G on a compact metric space X. In the case that $G = \mathbb{Z}$, Li, Liao and Winter showed that the diagonal dimension of $C(X) \rtimes_\alpha \mathbb{Z}$ can be estimated using the dimension of X and the Rokhlin dimension of the homeomorphism α.

The original plan of the project was to extend this result to the case of a crossed product by a Hilbert $C(X)$-bimodule. When the Hilbert $C(X)$-bimodule $\mathcal{E}$ is left and right full, it has the following form: As a right Hilbert $C(X)$-bimodule, it has the structure of a module of sections $\Gamma(\mathscr{V})$ of a line bundle $\mathscr{V}$ over X. A left module structure is given by defining $f\xi = \xi f \circ \alpha$, where α is a homeomorphism. Using the notion of Rokhlin dimension for a Hilbert C*-correspondence, we were quickly able to obtain the result. It also became clear that both the diagonal dimension and the nuclear dimension of $C(X) \rtimes_{\mathcal{E}} \mathbb{Z}$ depends only on the dimension of X and the Rokhlin dimension of α.

The crossed product of $C(X)$ by a Hilbert bimodule can also be realised as a twisted groupoid C*-algebra for some twist over the transformation groupoid $X \times \mathbb{Z}$. This led us to conjecture that neither the nuclear dimension nor the diagonal dimension can distinguish between a principal étale groupoid and a twist over that groupoid. By the end of the workshop, the group decided to focus on proving this conjecture.

The group are continuing to collaborate and have had two zoom meetings since the end of the conference. Our research continues to focus on how the nuclear and diagonal dimensions relates the C*-algebras of groupoids and their twists. We hope to meet as a group, or some subset of our group, in person in the summer of 2022.

3 Acknowledgement

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