Comments on
Euclidean Wormholes and Holography

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arXiv:1903.05658 and arXiv:2110.——

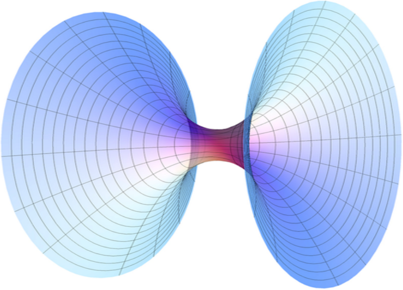
BIRS, Gravitational Emergence in AdS/CFT
27 October 2021

Why to study Euclidean Wormholes

Wormholes are interesting exotic solutions of GR + matter

- Motivation
  - Understand Holography in the presence of multiple boundaries
  - Lorentzian signature $\Rightarrow$ entanglement between two QFTs. What is the analogous property in the Euclidean case?
  - Universal properties of the dual QFT description?
  - Is there a microscopic model that can describe such geometries?
- Clear distinction
  - Lorentzian vs Euclidean
  - Macroscopic vs. Microscopic "gas of wormholes" (α-parameters)
  - Different characteristic scales
    $L_P \ll L_W \sim L_{AdS}$ vs. $L_P \leq L_W \ll L_{AdS}$
- Plan of the talk
  - 1st Part: Bulk Perspective
  - 2nd Part: "Bottom-Up" QFT models
  - 3rd Part: Microscopic models
  - Summary and Future directions

Euclidean Wormholes



- Euclidean Wormholes:
  There is no time, only space
- To have such solutions, one needs locally negative Euclidean Energy to support the throat from collapsing
- Such energy can be provided by axions or "magnetic" fluxes etc...

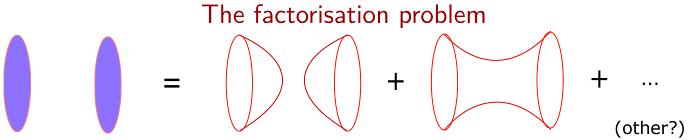
Holographic comments

- No time $\Rightarrow$ No entaglement
- Naively: different QFTs on $\partial\mathcal{M} = \cup_i \partial\mathcal{M}_i$ $\Rightarrow$ Cross-correlations factorise
- Common Bulk dictates otherwise $\Rightarrow$ Some form of interaction?
- Global symmetries for the boundary theories? $\leftrightarrow$ A common Bulk "Gauss Law constraint"

The factorisation problem

[Maldacena - Maoz ...]

The problem that $Z(J_1, J_2) \neq Z_1(J_1) Z_2(J_2)$ is :



Ideas on the market:

- After summing over bulk topologies and other non-perturbative states the correlators factorise ?
  $\Rightarrow$ Need non-perturbative info
- The bulk QGR path integral corresponds to an average over QFT's (JT)
  $\Rightarrow$ Unitarity? higher d (canonical example of $\mathcal{N} = 4$ SYM)?
- Interactions between QFT's $\Rightarrow$ subtle properties (UV softness of the cross-correlator)
- Could the partition function be of the form (S some "sector" ?)

$$Z(J_1, J_2) = \sum_S e^{w(S)} Z_S^{(QFT1)}(J_1) Z_S^{(QFT2)}(J_2)$$

Part I : Bulk perspective

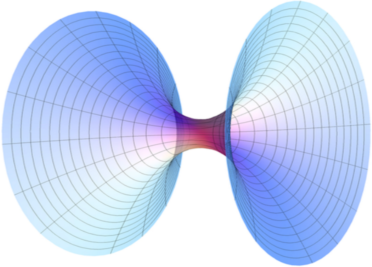
Types of solutions

We study Euclidean solutions with two asympotic AdS boundaries

- Solutions of Einstein - Maxwell - Dilaton in two dimensions, where the throat of the wormhole is supported by the gauge field flux
- Solutions of Einstein - Dilaton theory in three dimensions with hyperbolic slicings, a running dilation and a constant potential
- Solutions of Einstein - Yang - Mills with spherical slices in four dimensions, supported by the non abelian gauge field

- There are many more such solutions
- A recent paper where also stability has been studied is `[Marolf - Santos]`, where they showed that a subset of such solutions are perturbatively stable

Correlators: Two boundaries

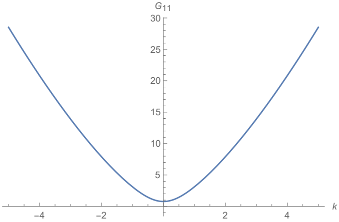


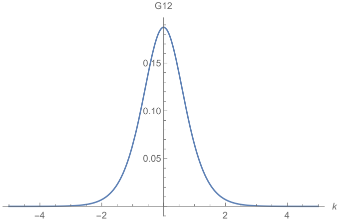
- To study correlators for boundary operators ⇒ Study the (2nd order) bulk fluctuation equation

- We have two boundaries, where the solution can potentially diverge or become a constant
- The extra freedom provides for two types of correlation functions, one on a single boundary which we label by $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ or $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$, and one cross-correlator across the two boundaries $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$

- $EAdS_2$ we can compute the correlators analytically, other cases: numerical results

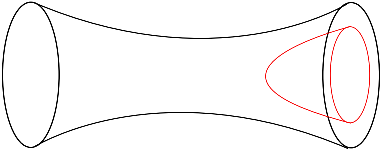
Scalar Correlators: Universal properties

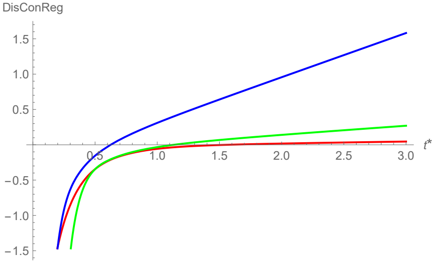




- The $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ have a similar behaviour in the UV as when there is only one boundary (power law divergence)
- In the IR they saturate to a constant positive value
- The cross correlator $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$ goes to zero in the UV and has a finite maximum in the IR
- In position space ($EAdS_2$) they behave as $\sim 1/\sinh^{2\Delta_+}(\tau)$ and $\sim 1/\cosh^{2\Delta_+}(\tau)$ respectively $\Rightarrow$ No short distance singularity for the cross-correlator
- The qualitative behavior of the correlators is the same for all the types of solutions $\Rightarrow$ Universality

Non-local Observables - Wilson Loops

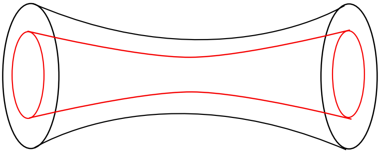


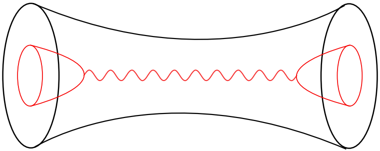


- The disk slices are three spheres and the loop is a circle sitting on the S^3
- Large loops on the boundary penetrate further in the bulk
- In the limit of a large loop we can probe the IR properties of the boundary dual
- The dual bulk string minimal surface, cannot pass through the throat
- Plot of the regularised action S^{reg} as a function of t^*, $L_{throat}/L_{AdS} = 0.6,\ 0.7,\ 0.9$ from red to blue

- $0 < t^* < \pi$ governs the boundary loop size
- For large loop ($t^* \to \pi$) the action scales approximately linearly with t^*
- This scaling is an Area law
- One boundary: indicative of IR confining behaviour only in the infinite volume limit of the S^3 [Witten]
- Here 2 asymptotic boundaries

Wilson Loop correlators





- In the disconnected case the loops can interact only via exchange of perturbative bulk modes

- Study loop - loop correlators $\langle W(C_1)W(C_2)\rangle$, with the two loops residing on different boundaries

- In the regime of large Wilson loops, the leading contribution originates from a single surface connecting the two loops having a cylinder topology $S^1 \times R$

- As we shrink the boundary loops, we find that the leading configuration of lowest action is the one for two disconnected loops

- Large loops $\Rightarrow$ Strong IR cross-coupling!

Part II: "Bottom-Up" QFT models

Universal properties of a putative dual

- Traversable (Lorentzian) Wormholes: to provide for negative energy $\Rightarrow$ Couple the two boundary CFTs by a double trace deformation [Gao - Jafferis - Wall ...]

$$S_{int} = \int dtd^d x\, h(t,x) O_R(t,x) O_L(-t,x)$$

- This is a local interaction term
- Euclidean case: cross interactions are softer at shorter distances, they increase and become strong in the IR
- The two theories interact "mildly" and the mixed correlators do not have short distance singularities
- It is not possible to have this behaviour by introducing a local term
- Initial Bottom up proposal: Take two Euclidean theories S_1 and S_2 and local operators $O_1(x) \in S_1$ and $O_2(x) \in S_2$. Introduce effective (non-local) cross-interactions

$$S = S_1 + S_2 + \lambda \int d^d x \; d^d y \, O_1(x) O_2(y) \, f(x-y)$$

A toy QFT model

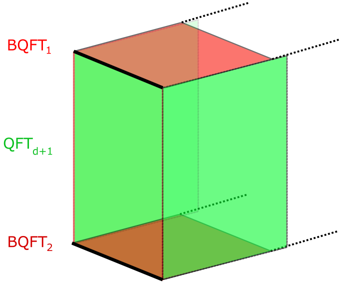
- We can realise such features in a simple model

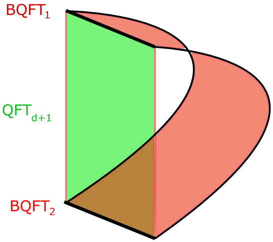
$$S = \int \frac{d^d q}{(2\pi)^d} \left[\phi_1 (q^2 + m^2) \phi_1 + \phi_2 (q^2 + m^2) \phi_2 + \phi_1 \frac{2}{q^2 + \Lambda^2} \phi_2 \right]$$

- Its correlators exhibit the desired universal properties of wormhole correlators
- In the diagonal field basis ($\phi_{\pm} = \phi_1 \pm \phi_2$) one can show that the propagators are positive definite in momentum space for $m^2 \Lambda^2 > 1$
- Fourier transforming to position space we find that we additionally need $\Lambda^2 \geq m^2 + 2$ in order to satisfy reflection positivity
- Nevertheless: We cannot resolve the non-local interaction by integrating in $d-$dimensional fields (propagator not positive definite)

"Sandwich" construction

[van Raamsdonk]





- Another interesting class of models is the following
- Two d-dim (holographic) BQFT's coupled through a $d+1$-dim intermediate (messenger) theory$\Rightarrow$ $d+2$-dim gravitational theory
- The relevant regime for wormholes is $c_{d+1} \ll c_d$
- It was argued that the system should flow to a gapped/confining theory in the IR
- The hope: The dual bulk gravity can localise on $d+1$-dim EOW branes that bend and connect in the IR
- In the final part of the talk we will take the limit where the messenger theory is topological

Toy model (II)

- A simple model of this type ($S_{1,2}$ are separated by an interval $|y_1 - y_2| = L$)

$$S_{1,2} = -\frac{1}{2}\int d^dx\ \phi_{1,2}(x)\left(\Box_d - m^2\right)\phi_{1,2}(x) + \ldots$$

$$S_3 = -\frac{1}{2}\int d^dxdy\ \Phi(x,y)\left(\Box_{d+1} - M^2\right)\Phi(x,y)$$

$$S_{int} = g\int d^dx\ \phi_1(x)\Phi(x,y_1) + g\int d^dx\ \phi_2(x)\Phi(x,y_2)$$

- Integrate out $\Phi(x,y)$ to obtain ($s = p^2$)

$$S = \int \frac{d^dp}{(2\pi)^d}\left[\phi_+(p)D_+(p)\phi_+(-p) + \phi_-(p)D_-(p)\phi_-(-p)\right] + \ldots, \quad \phi_\pm = \frac{\phi_1 \pm \phi_2}{\sqrt{2}}$$

$$D_\pm(s) = s + m^2 + \frac{g^2\ \left(1 \pm e^{-L\sqrt{s+M^2}}\right)}{2\sqrt{s+M^2}},$$

$$D_+^I(s) = s + m^2 + g^2\left(\frac{\coth(L\sqrt{s+M^2}/2)}{2\sqrt{s+M^2}}\right),$$

$$D_-^I(s) = s + m^2 + g^2\left(\frac{\tanh(L\sqrt{s+M^2}/2)}{2\sqrt{s+M^2}}\right)$$

Toy model (II) properties

- There is a regime of parameters for which $D_{\pm}^{-1}(x)$ and $D_{\pm}^{-1\,I}(x)$ are well defined and admit KL spectral rep with positive weight $\Rightarrow$ satisfy reflection positivity
- All the composite operators $O_{1,2}^{m} = \phi_{1,2}^{m}$ have appropriately UV soft cross correlators $\langle O_1^m O_2^m \rangle$
- We can show that this property continues to hold for arbitrary self interactions $V(\phi_1), V(\phi_2)$
- Lorentzian Continuation: Two choices,
- Analytic Continuation of one of the boundary directions ($s = \vec{p}^2 - \omega^2$): well defined only in the case of $D^I(s)$, for $D(s)$ existence of transcendental branch-cut
- In the case of analytic continuation along the messenger direction y, the boundary theories remain Euclidean
- Only the case of $D(x)$ is well defined (reflection positive Euclidean propagator). In the case of $D^I(x)$ the propagators are not reflection positive

Part III: Microscopic models

Microscopic "Sandwich" model $(2d - 1d)$

- Take a $2d$ gYM (τ, z) and couple it to two $1d$ $U(N)$ matrix quantum mechanics theories at the endpoints of an interval I $(z = \pm L)$.
- The action is

$$S_{gYM} = \frac{1}{g_{YM}^2}\int_\Sigma \mathrm{Tr}\, BF + \frac{\theta}{g_{YM}^2}\int_\Sigma \mathrm{Tr}\, B\, d\mu - \frac{1}{2g_{YM}^2}\int_\Sigma \mathrm{Tr}\, \Phi(B)\, d\mu$$

where $F = dA + A \wedge A$ and $A_\tau(\tau, z = \pm L) = A_\tau^{1,2}(\tau)$ is the restriction of the 2d gauge field on the two boundaries

$$S = \int d\tau \mathrm{Tr}\, \left(\tfrac{1}{2}(D_\tau M_{1,2})^2 - V(M_{1,2})\right),\ D_\tau M_{1,2} = \partial_\tau M_{1,2} + i[A_\tau^{1,2}, M_{1,2}]$$

- Take $\Phi(B) = B^2$ (2d YM) and place the system on $I \times S^1$
- The partition function on the cylinder is

$$Z(\beta_1,\, \beta_2) = \sum_R e^{-L\frac{g_{YM}^2}{N}C_R^{(2)} + i\theta C_R^{(1)}} Z_R^{MQM_1}(\beta_1) Z_R^{MQM_2}(\beta_2)\,,$$

$$Z_R^{MQM}(\beta) = \mathrm{Tr}_{\mathcal{H}_R} e^{-\beta \hat{H}_R^{MQM}}$$

with β the S^1 size and R a $U(N)$ representation and $C_R^{1,2}$ its Casimirs
- The two systems are coupled, by carrying common representations

Liberating messenger and MQM gauge groups

- Couple the MQMs with the 2d YM by introducing bifundamental fields $\psi^{1,2}_{\alpha,i}$ on the boundaries that couple to the 2d YM gauge field $A_{\alpha\beta}$
- Latin indices $i = 1, ..., N_{1,2}$ associated to the $U(N_{1,2})$ and Greek indices $\alpha = 1, ..., n$ associated to the $U(n)$

$$Z = \sum_{R,R_1,R_2,R_3} Z^{MQM_1}_{R_1} Z^{MQM_2}_{R_2} e^{-\beta m_1 |R_1| - \beta m_2 |R_2|} C^{R_3}_{RR_1} C^{R_3}_{RR_2} e^{-L \frac{g^2_{YM}}{N} C^{(2)}_R + i\theta |R|}$$

- R is a rep of $U(n)$, while $R_{1,2}$ are reps $U(N_{1,2})$
- The rep R_3 is determined by the fusion rules for the tensor product of $U(N_{1,2}) \times U(n)$ (Littlewood-Richardson Coeff $C^{R_3}_{RR_{1,2}}$)
- $\chi_R(U)\chi_{R_1}(U) = \sum_{R_3} C^{R_3}_{RR_1} \chi_{R_3}(U)$
- For $n \ll N_{1,2}$ we generally have a very small deformation of two decoupled MQM systems (the dual geometries are almost factorised but possibly connected via quantum microscopic wormholes)
- For $n \sim N_{1,2}$ the leading saddle is very different from that of two independent MQMs

Dual geometry

- The singlet sector of one MQM (taking a double scaling limit) is dual to $2d$ linear dilaton background of the $c = 1$-Liouville $\Rightarrow$ One asymptotic region of space
- Non trivial reps with few boxes in their Young diagrams are related to long strings - Large reps deform the background geometry (possibly creating black holes)
  [Maldacena, Kazakov-Kostov-Kutasov, Betzios-O.P ...]
- We took the large representation limit and studied the saddle point equations in order to find the corresponding geometry, $\rightarrow$ technically difficult, hard to read-off the dual geometry (bulk reconstruction ?)
- However, we were able to prove the existence of different saddles some of which we expect to correspond to disconnected and others to connected geometries, it would be interesting to study possible phase transitions

Cross-Correlator

- The n-point cross-correlator takes the form

$$\langle O_{i_1}(\tau_{i_1})\ldots \tilde{O}_{i_2}(\tau_{i_2})\ldots\rangle = \sum_R \langle O_{i_1}(\tau_{i_1})\ldots\rangle_1^R \langle \tilde{O}_{i_2}(\tau_{i_2})\ldots\rangle_2^R \, e^{-L\frac{g_{YM}^2}{n}C_R^{(2)}+i\theta|R|}$$

- where i_1 refers to MQM theory 1 and i_2 to MQM theory 2
- This correlator generically only depends separately on the differences $\tau_{i1}-\tau_{j1}$ and $\tau_{i2}-\tau_{j2}$ and not on time differences that mix the $1,2$ sub-indices or normal with tilde operators
- Cross-communication arises at a leading saddle point level when we take both $N_{1,2}, n \to \infty$
- When $N_{1,2} \to \infty$, but n is finite the leading saddle is factorised and there is a subleading cross-communication from very long thin partitions with $O(N_{1,2})$ boxes
- The two point cross correlators of simple traces of the matrices $M_{1,2}(\tau)$ are simply constants

Higher Dimensional Examples

- Consider a sandwich spacetime, with the topology $\mathcal{M} = \Sigma \times I$
- The 2 boundary holographic QFT's on the interval endpoints $\Sigma_{1,2}$ are coupled via a topological QFT in $\mathcal{M}$ that does not have local propagating degrees of freedom

2D dimensional example

- Two $2d$ BCFT's coupled through a topological field theory like Chern-Simons theory living in $3d$ dimensions
- The partition function is again of the form $Z(J_1, J_2) = \sum_S e^{w(S)} Z_S^{(BCFT1)}(J_1) Z_S^{(BCFT2)}(J_2)$
- The cross correlators are soft in the UV

3D dimensional example

- A natural choice of a messenger theory in $d = 4$, is a BF-type in analogy to the simple 2d gYM theory

Summary and Future

Summary and Future Directions

Summary

- Analysis of various Euclidean wormhole solutions
- Computed correlators of both local and non-local observables
- Found common properties: Cross-correlators do not factorize and they are soft in the UV
- Dual description in terms of a system of interacting QFT's
- Compatibility with geometric dual constrains on the properties of correlation functions
- Possible microscopic description in terms of two matrix quantum mechanics or BCFTs coupled through a topological messenger theory

Future Directions

- Understand better the analytic continuation of these geometries and their holographic duals (cosmologies, bang/crunch universe)
- Case of multiple boundaries
- Find a method to reconstruct the geometry from the microscopic QFT duals
- Top down constructions embeddable in critical string theory

Thank you!