

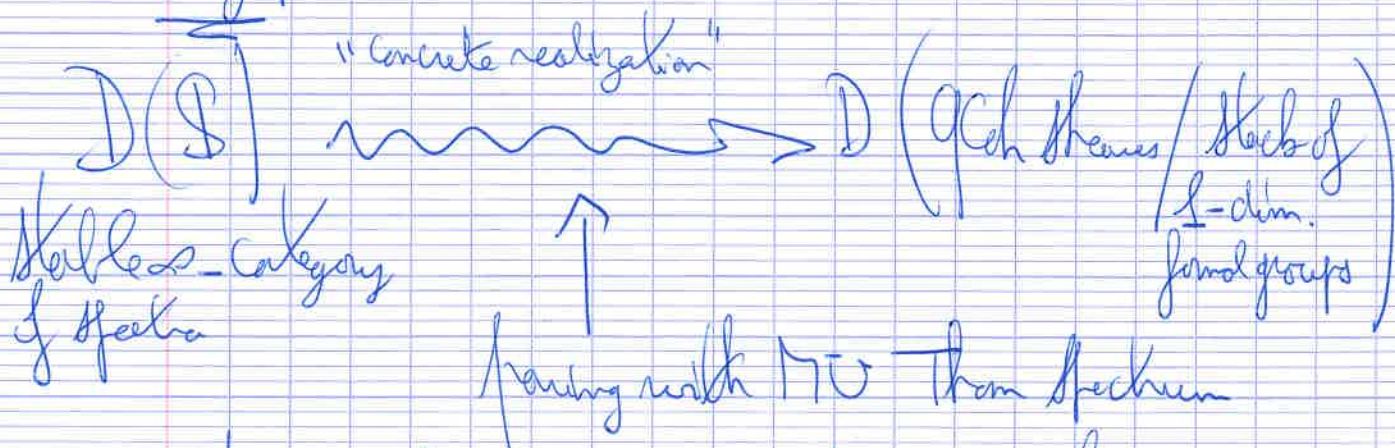
Some ideas about chromatic homotopy theory

Hofmann talks Harvard 2007

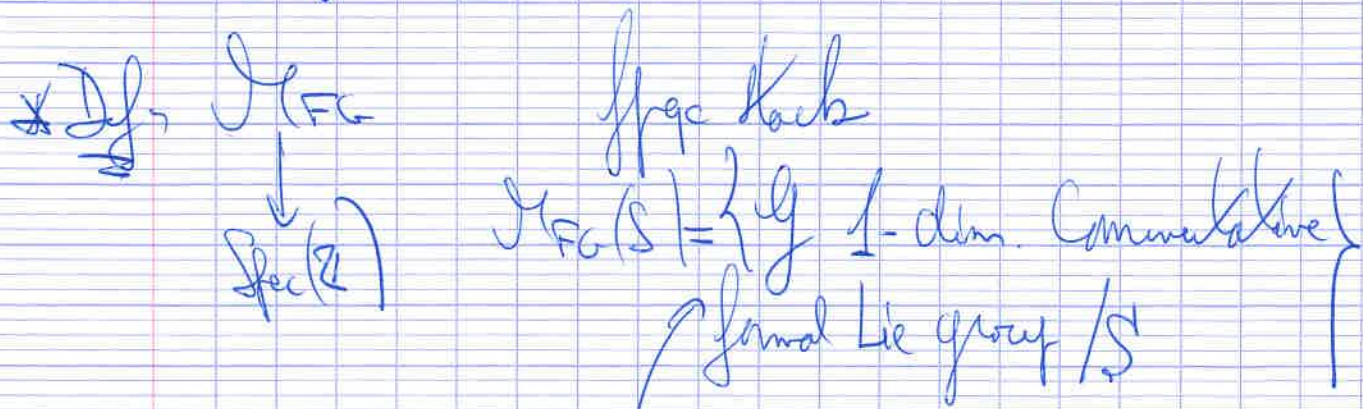
Jacob Lurie (saw in Chicago, biology) + discusses

Lars Hesselholt

Starting point: Thom lives in books ~ 2006.



→ Study \mathcal{M}_{FG} moduli of 1-dim. formal groups.



formal group scheme locally $\hat{\mathbb{A}}^1_S$

Look for other one when

Local Presentation:

$\tilde{\mathcal{M}}_{FG}$ = moduli scheme of 1-dim.
 Zariski presentation fam of gp. laws

\mathcal{M}_{FG} fin a coordinate

$$\tilde{\mathcal{M}}_{FG}(S) = \left\{ (g, \iota) \mid \iota: \hat{A}_S^1 \xrightarrow{\sim} g \right\}$$

$\circ \leftrightarrow$ neutral section

$$\tilde{\mathcal{M}}_{FG} = \text{Spec } (\Lambda)$$

$\Lambda =$ Lazard ring

$$\cong \mathbb{Z}[\langle T_g \rangle]_{g \geq 1}$$

\uparrow non-commutative Witt vectors

Localization at p :

$$\mathcal{M}_{FG} \otimes_{\mathbb{Z}} \mathbb{Z}_p \text{ simpler presentation}$$

by moduli of p -typical formal group laws

$$\iota: \hat{A}_S^1 \xrightarrow{\sim} g \text{ of } \mathbb{Z}_p \text{ seen as a curve}$$

in the Cartier Moduli
 is p -typical

i.e. $\text{Fm. } \nu = 0 \quad \text{if } (n, p) = 1$

$$\simeq \text{Spec}(\mathbb{Z}_p[V_b]_{b \geq 0})$$

(2)

Universal formal gp. law given by the Cohen
 module with V-base γ and relation

$$F\gamma = \sum_{b \geq 0} V^b [V_b] \gamma$$

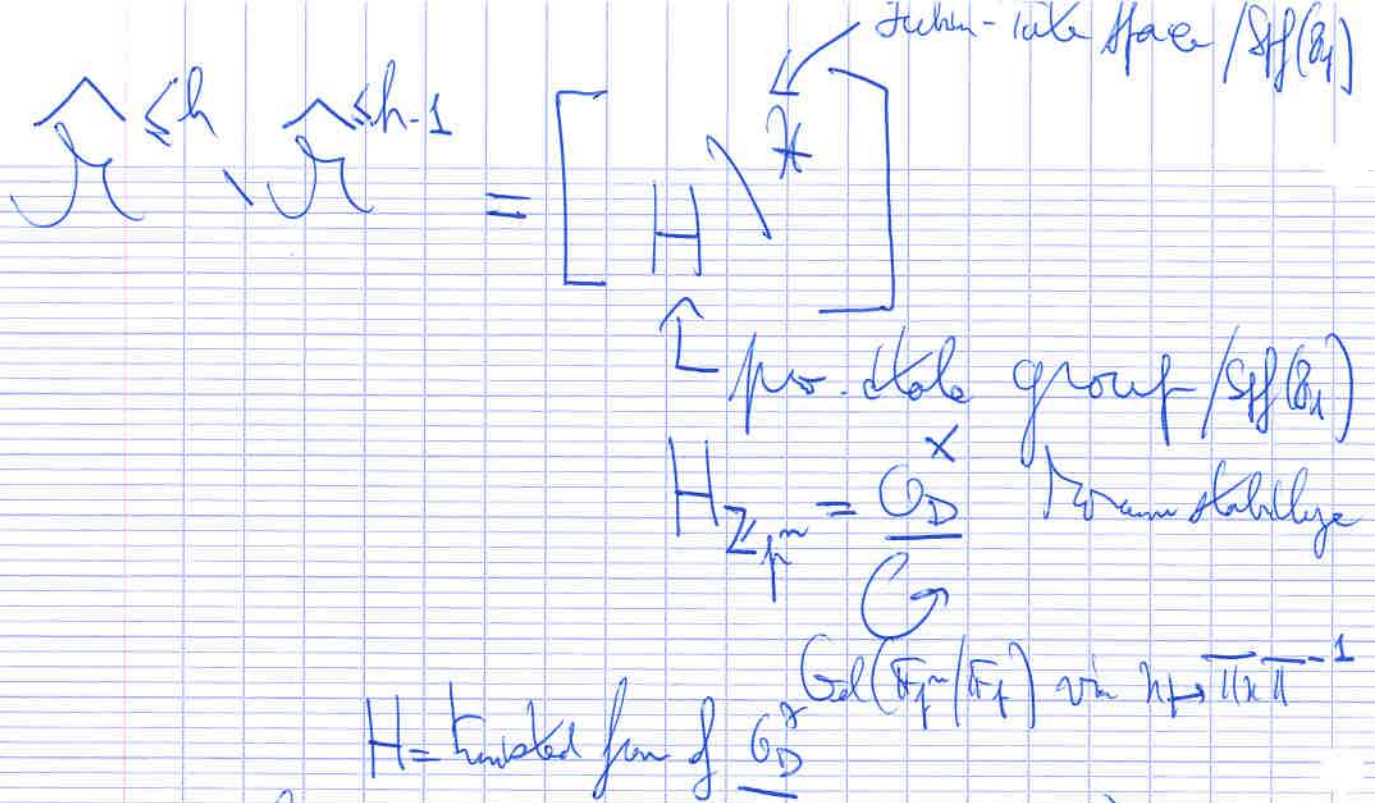
Height stratification $(\mathcal{I}_{FG} \otimes \mathbb{Z}_p)^{\leq h}$ open subset ~~of~~
 where height $\leq h$, given by $V_h \neq 0$
 \rightsquigarrow plenty of computations.

* Completion at p : $\widehat{\mathcal{I}_{FG}}$ p -adic completion

\hookrightarrow Change the test
 Category

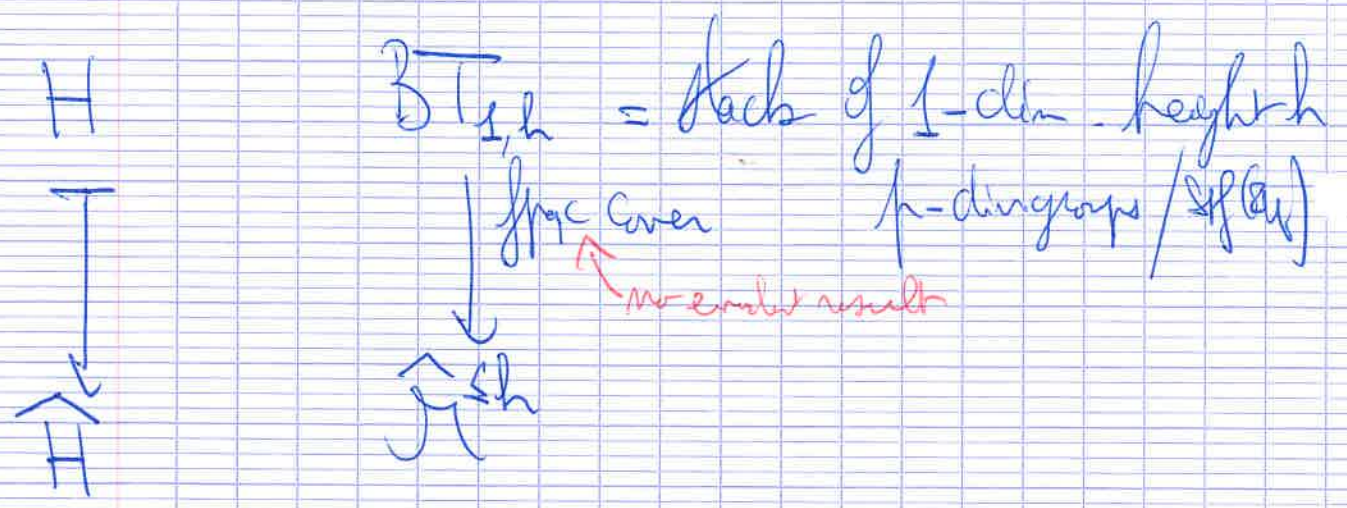
Help = Schemes on which
 $\hookrightarrow p$ is loc. nilpotent

$S \mapsto \{1\}$ -dim. formal gp. / S



$H = \text{twisted form of } \mathcal{O}_D^\times$
 \leadsto plenty of Computations (see Gauss liftings)

Another presentation of $\widehat{K}^{\leq h}$



$\text{BT}_{1,h}$ is a complicated object - Simplify it.

(3)

The real thing. Game fiber as a demand knots

$(\widehat{\mathcal{J}}_{FG})_n = \mathcal{V}$ -knots on $\text{Perf}_{\mathbb{Q}_r}$
 \uparrow Scholze's \mathcal{V} -Topology

Knots associated to the prestacks

$$(R, R^+) \mapsto \{ \text{1-Mod. } \mathcal{O}_R / R^+ \}$$

\mathcal{P} of $\text{perf}/\mathcal{O}_R$

$$(BT_{1,h})_n$$

\mathcal{V} -Cover

$$(\widehat{\mathcal{J}}_{FG})_n \leq h$$

given by Hodge-Tate periods

th:

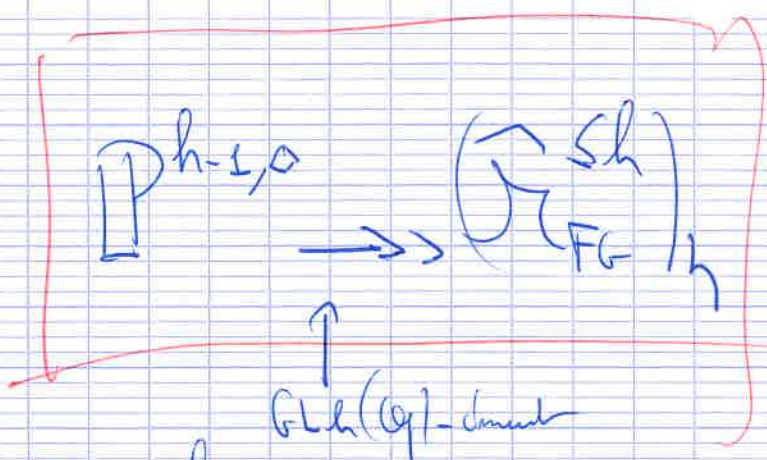
$$(BT_{1,h})_n$$

$$\left[\underline{GL}_h(\mathbb{Q}_r) \mid \mathbb{P}_{\mathbb{Q}_r}^{h-1, \diamond} \right]$$

Simplified description

Explicit

\Rightarrow Cartan chart



+ explicit description of the height stratification (chromatic layers)

$\Omega \subset P^{h-1,0}$ open stratum

Second theorem: $(BT_{1,h})_s$ on $Proj_{\mathbb{F}_q}$ special fiber

Th: $(BT_{1,h})_s \xrightarrow{\sim} Bun_h^1$

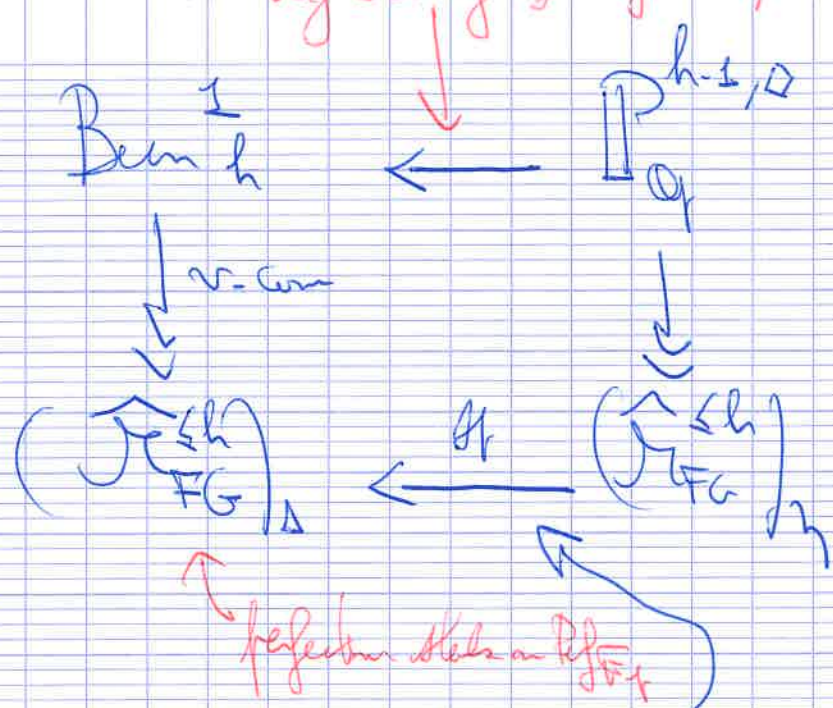
height stratification

stack of degree 1
sh. h. v. h. on curve
H.V. stratification

modification of deg. 1 of \mathcal{O}^h /curve

(4)

has



+ Spezialisierung



Lemma: $(BT_{1,h})_m =$ Subs of \mathcal{O} -modules

over $A_{\text{ang}}(\mathbb{A}^1, \varphi)$ of rank h s.t.
 Coh of \mathcal{O} killed by ξ
 $\text{ker } \theta = (\xi)$

→ Maybe should study ~~the~~
p-adic sheaves on $\mathbb{P}_q^{h, \text{ét}}$?

\triangleleft : Locally free \mathcal{O} -modules / v.l. $\mathbb{P}_q^{h, \text{ét}} \hookrightarrow (-) / \mathbb{P}_q^{h, \text{ét}}$

$\sum_{\text{v.l.}} \mathcal{O} / \text{Spa}(k)^\diamond = \text{Rep}_{\mathbb{G}_m}(\text{Gal}(\bar{k}/k))$
(See theory)