New Perspectives in Colouring and Structure (Online)
20w5143

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1 Organization of the meeting

The workshop was planned for 42 participants. However, with the date of the workshop approaching, many cancellations were received due to the emerging Covid-19 pandemic that made international travel impossible. In the week preceeding the workshop, less than half of the original participants were expected (the remaining participants were either from Canada, or international visitors that were already in Canada). Just two days before the meeting, the province of Alberta announced ban on international gatherings, and the workshop was cancelled.

However, in a couple of days the organizers together with the technical help from BIRS, were able to organize the online event featuring most of the intended lectures. The event was a great success as the audience was about 60 participants for most of the talks.

2 Presentations

In the end, almost all the originally scheduled presentations took place. The final schedule was as follows:

Monday

9:00-10:00 Sergey Norin 10:30-11:30 Luke Postle
2:30-3:00 Andrew Thomason 3.30-4:00 Vaidy Sivaraman (remote) 4:00 problem session

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Tuesday

9:00-10:30 Maria Chudnovsky (survey+talk)
11:00-11:30 Nicolas Trotignon
2:00-2:30 Carla Groenland 2:30-3:30 David Conlon
3:30-4:00 Bhargav Narayanan

Wednesday

9:00-10:00 David Wood (survey) 10:30-11:00 Vida Dujmovic 11:00-11:30 Chun-Hung Liu

Thursday

9:00-9:30 Bartosz Walczak 9:30-10:00 Penny Haxell 10:30-11:00 Robert Samal
2:00-2:30 Jon Noel 2:30-3:00 Tom Trotter

3 Overview of the subject area of the workshop

The study of graph colouring is a central theme in combinatorics. Calculating the chromatic number of a graph is well-known to be NP-hard (indeed it is NP-hard even to approximate the chromatic number), and so it is perhaps not surprising that graph colouring has a rich theory, with many important open problems. The colouring of graphs, and more generally of directed graphs and hypergraphs, also has connections and applications in many other areas, including algorithm design, scheduling and resource allocation, statistical physics, and social choice theory.

A common theme in colouring problems is the relationship between chromatic number and graph structure. For instance, one of the oldest problems in graph theory was the celebrated Four Colour Conjecture on colouring planar graphs, which was raised in 1852 and inspired a huge body of work (it was only proved in 1976). It was shown by Wagner in 1937 that planar graphs can be characterized in structural terms: a graph is planar if and only if it does not have two specific subgraphs as minors. Thus the Four Colour Theorem provides a connection between structure and chromatic number.

A vastly more general conjecture was made by Hadwiger in 1943: the conjecture asserts that if a graph cannot be properly coloured with $k-1$ colours then it must contain the complete graph on k vertices as a minor. The conjecture has been proved when k is at most 6 (Wagner showed already that the result for $k = 5$ is equivalent to the Four Colour Theorem; the proof of the $k = 6$ case by Robertson, Seymour and Thomas won the 1994 Fulkerson Prize). For $k \geq 7$, the problem is still open, although there are some promising partial results. In the last few years, there has been a lot of interesting work work looking at defective forms of Hadwigers Conjecture, where colour classes do not need to be stable sets. This has led to a broader and rapidly advancing theory of defective colouring.

Another important structural question is to understand what induced subgraphs must be contained in graphs of large chromatic number. For instance, it has long been known from work of Tutte and then Erdos that graphs of large chromatic number need not contain large complete subgraphs (or even short cycles). A celebrated result in this area is the Strong Perfect Graph Theorem, conjectured by Berge in 1961 and proved by Chudnovsky, Robertson, Seymour and Thomas in 2006 (and subsequently awarded the 2009 Fulkerson Prize): every graph with chromatic number larger than clique number contains either an odd hole or an odd antihole. The Strong Perfect Graph Theorem gives a precise structural characterization, but it is natural to ask what more can be said when the chromatic number is *much* larger than the clique number. Gyarfas made a sequence of beautiful conjectures concerning such graphs, and there has been major progress on these in the last three years, and a new structural theory is beginning to emerge. More generally, there has been a large burst of research on chi-bounded classes of graphs, and on the algorithmic problem of colouring graphs with specific forbidden induced subgraphs.

Another important collection of problems concerns flows (which arise as a dual problem to colouring). Tuttes 3-Flow and 5-Flow Conjectures have been open for decades, and there are many interesting related problems. Recent progress by Thomassen and his coauthors has introduced some new techniques, and it seems there is potential for further progress.

This is only a small sample of the many open questions and theoretical advances on colouring. Many questions have been open now for decades with seemingly little progress. However, some powerful new techniques have been developed in the last few years. These have led to significant breakthroughs, and their full potential is yet to be ascertained. We believe that the new methods are robust and powerful enough to be used to resolve other important coloring questions and can be extended beyond their original application to attack new areas. By bringing the originators of these new tools, other respected researchers in graph coloring and bright young minds together, we hope that collaboration at this conference will spur the development of these new techniques to attack some of the remaining important open questions in graph coloring.

4 Workshop press release

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