# Pre-scaling, hydrodynamic attractors and entropy production in heavy ion collisions

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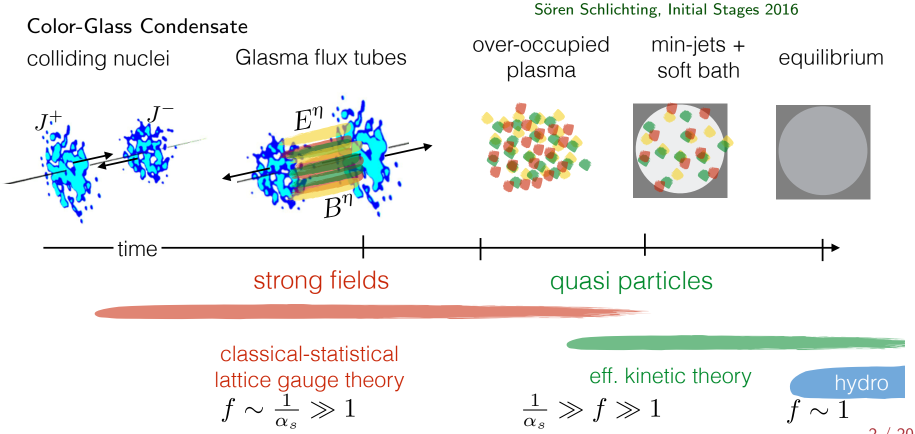
AM and Jürgen Berges, PRL 122 (2019) [arXiv:1810.10554]
G. Giacalone, AM, S. Schlichting, [arXiv:1908.02866]

*Isolated quantum systems and universality in extreme conditions*

# Non-equilibrium QCD descriptions at weak coupling $\alpha_s \to 0$

## At high energies mid-rapidity is dominated by small Bjorken-$x$ gluons

- $p \sim Q_s$ saturation scale $\gg \Lambda_{QCD}$, strong gluon fields $A_\mu \sim \frac{1}{\alpha_s} \gg 1$
  $\Longrightarrow$ classical-statistical simulations
- decoherence of classical fields at $\tau Q_s \gg 1$
  $\Longrightarrow$ kinetic evolution of gluon phase space distribution $f$



Sören Schlichting, Initial Stages 2016

**Color-Glass Condensate**

colliding nuclei

Glasma flux tubes

over-occupied plasma

min-jets + soft bath

equilibrium

strong fields

quasi particles

classical-statistical lattice gauge theory

$f \sim \frac{1}{\alpha_s} \gg 1$

eff. kinetic theory

$\frac{1}{\alpha_s} \gg f \gg 1$

hydro

$f \sim 1$

# High temperature gauge kinetic theory

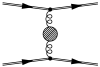
Boltzmann equation for distribution $f$ of quark and gluon quasi-particles.

Arnold, Moore, Yaffe (2003)[1]

$$\partial_\tau f_{g,q} - \frac{p_z}{\tau}\partial_{p_z} f_{g,q} = -\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]$$

Leading order processes in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

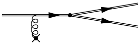
1. $2 \leftrightarrow 2$ elastic scatterings: $gg \leftrightarrow gg$, $qq \leftrightarrow qq$, $qg \leftrightarrow gq$, $gg \leftrightarrow q\bar{q}$



$$= |\mathcal{M}^{gg}_{qq}|^2 = \lambda^2\,16\frac{d_F C_F}{C_A^2}\left[C_F\left(\frac{u}{t}+\frac{t}{u}\right) - C_A\left(\frac{t^2+u^2}{s^2}\right)\right]$$

   Hard Thermal Loop resumed propagators, screening mass $m_D \sim gT$

2. $1 \leftrightarrow 2$ medium induced collinear radiation: $g \leftrightarrow gg$, $q \leftrightarrow qg$, $g \leftrightarrow q\bar{q}$



$$= |\mathcal{M}^{g}_{qq}|^2 = \frac{k'^2+p'^2}{k'^2 p'^2 p^3}\underbrace{\mathcal{F}_q(k';-p',p)}_{\text{splitting rate}}$$

   Resummed multiple scatterings with the medium (LPM suppression).

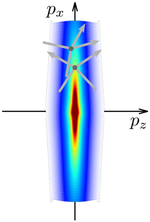
*QFT ⇒ transport theory ⇒ hydrodynamics*

# "Bottom-up" thermalization scenario

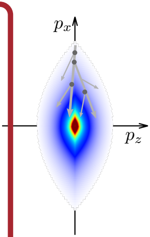
Baier, Mueller, Schiff, and Son (2001)[10]

## Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$

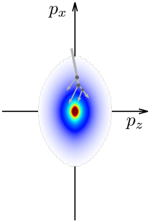
| | | |
|---|---|---|
| I) over-occupied | $p_z \sim \frac{Q_s}{(Q_s\tau)^{1/3}}$ | $1 \ll Q_s\tau \ll \alpha_s^{-3/2}$ |
| II) under-occupied | $p_z \sim \sqrt{\alpha_s} Q_s$ | $\alpha_s^{-3/2} \ll Q_s\tau \ll \alpha_s^{-5/2}$ |
| III) mini-jet quenching | $p_z \sim \alpha_s^3 Q_s (Q_s\tau)$ | $\alpha_s^{-5/2} \ll Q_s\tau \ll \alpha_s^{-13/5}$ |



$2 \leftrightarrow 2$ broadening

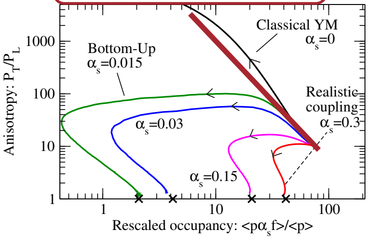


collinear cascade



mini-jet quench

nonthermal attractor



Berges, Boguslavski, Schlichting, Venugopalan (2014) [9]

Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018) [2, 3, 5, 4]

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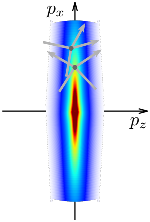

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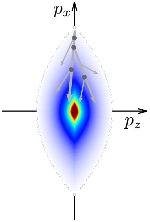

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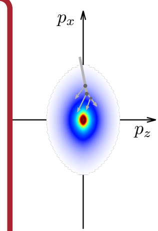
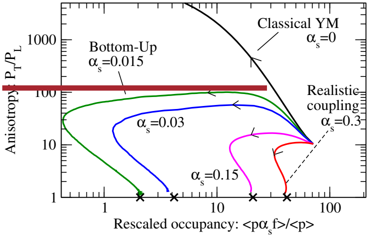


mini-jet quench



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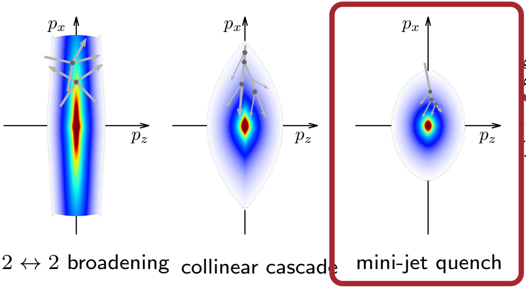
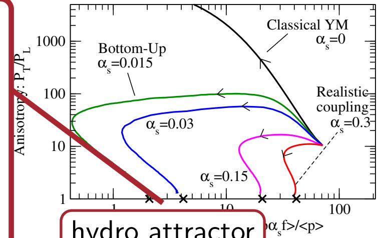


$2 \leftrightarrow 2$ broadening collinear cascade mini-jet quench



hydro attractor

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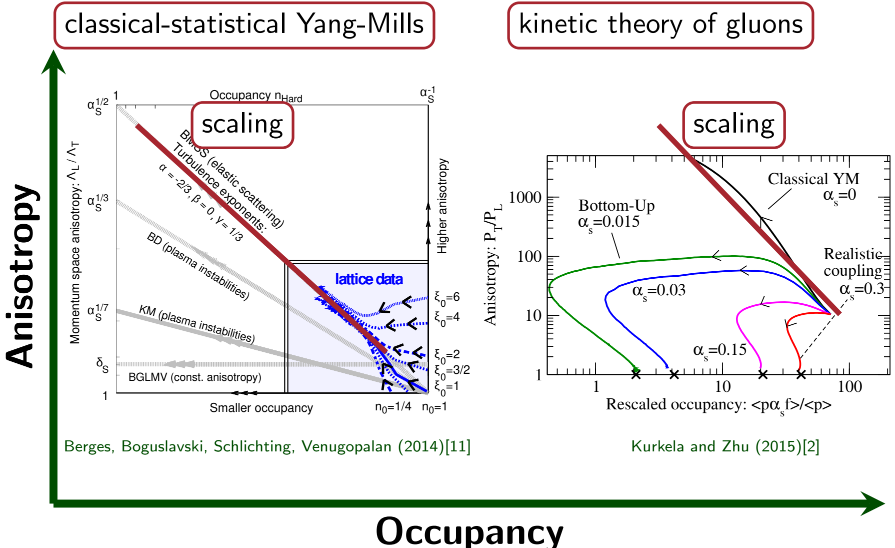
# Part I: Self-similar evolution at weak couplings

# From classical simulations to kinetic theory



Berges, Boguslavski, Schlichting, Venugopalan (2014)[11]

Kurkela and Zhu (2015)[2]

*Self-similar evolution of distribution function*

$$f_g(p_\perp, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_\perp, \tau^\gamma p_z), \quad \alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$$

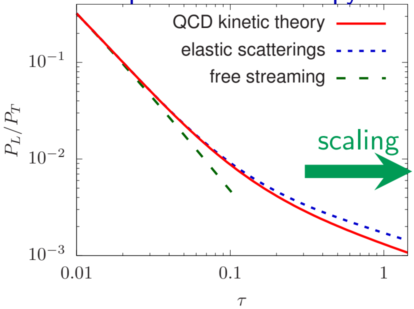
# Scaling in leading order QCD kinetic theory

Initial conditions $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}, \quad \sigma_0 = 0.1, \quad g = 10^{-3}, \quad \xi = 2$

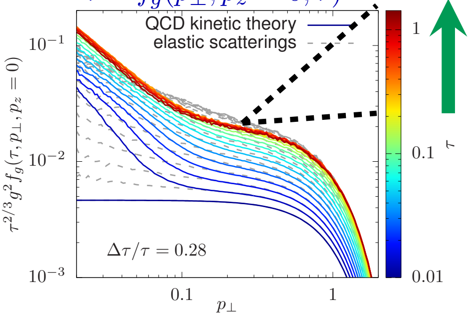
Scaling regime is reached at late times

$$f_g(p_\perp, p_z, \tau) = \tau^{-2/3} f_S(p_\perp, \tau^{1/3} p_z), \quad \tau \to \tau/\tau_{\text{ref}}$$

pressure anisotropy



$\tau^{2/3} f_g(p_\perp, p_z = 0, \tau)$



*Approach to a non-thermal fixed point in full QCD kinetic evolution.*

# Pre-scaling regime in QCD kinetic theory

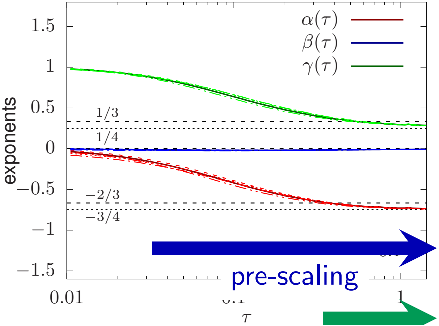
## Non-equilibrium dynamics undone by self-similar renormalization

$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

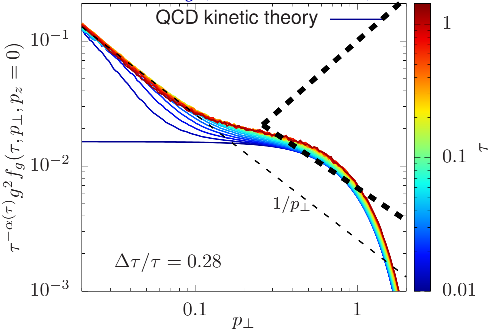
AM and Berges (2018) [8], cf. Micha and Tkachev (2004) [12]

*Scaling exponents $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$ can be time dependent!*

scaling exponents



$\tau^{-\alpha(\tau)} f_g(p_\perp, p_z = 0, \tau)$



pre-scaling

scaling

*Much earlier collapse to scaling solution $f_S$ — pre-scaling regime.*

# Extracting scaling exponents from integral moments

- Pre-scaling evolution $f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$ imposes relations between integral moments

$$n_{m,n}(\tau) \equiv \int_{\mathbf{p}} p_\perp^m |p_z|^n f_g(p_\perp, p_z, \tau) \sim \tau^{\alpha(\tau)-(m+2)\beta(\tau)-(n+1)\gamma(\tau)}$$

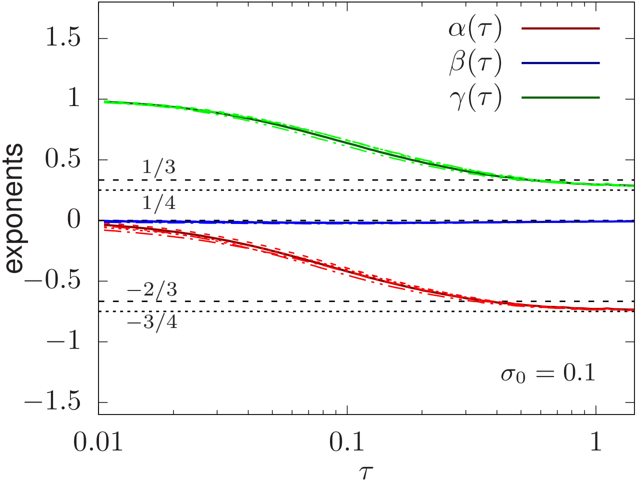
- *Momentum range* of scaling ⇔ number of moments obeying scaling.
- Consider 5 triples of moments: $\{1, p_\perp, |p_z|\}$, $\{1, p_\perp^2, p_z^2\}$, $\{p_\perp, p_\perp^2, p_\perp |p_z|\}$, $\{p_\perp^2, p_\perp^3, p_\perp |p_z|\}$, $\{1, p_\perp^3, |p_z|^3\}$
- Integrals of Boltzmann equation ⇒ equations of motion for moments

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad / \qquad \tau \log\tau\, \dot{\alpha} + \alpha = \alpha_\infty \tau^{\mu(\tau)-\alpha(\tau)+1}.$$

Relaxation to hydrodynamic solution / relaxation to scaling solution

# Time dependent exponents

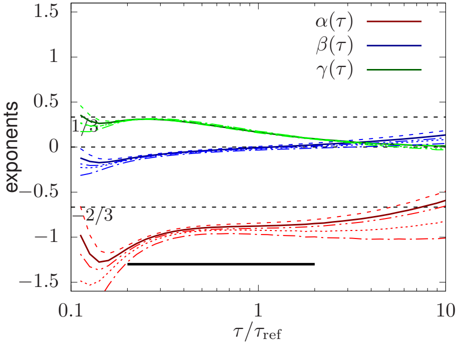
$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$



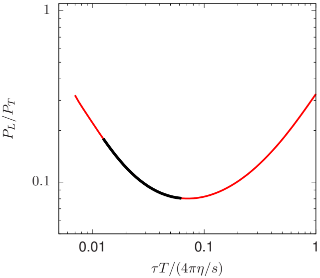
*Time evolution encoded into a few hydrodynamic degrees of freedom.*

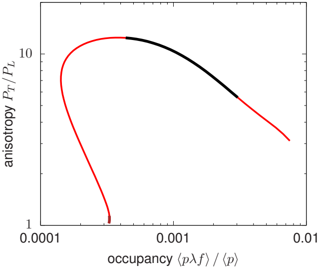
# The onset of thermalization

Consider larger couplings $g = 0.1$ and evolve until equilibration.



- Pre-scaling before isotropization
- Thermal scaling seen at late times.





*Early time pre-scaling disconnected from late time hydrodynamics.*

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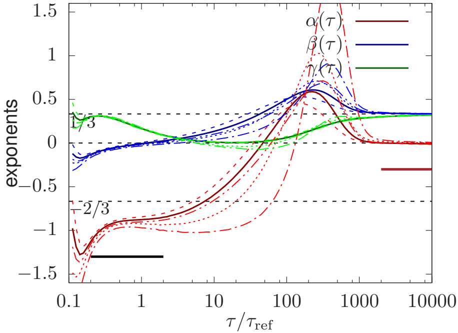
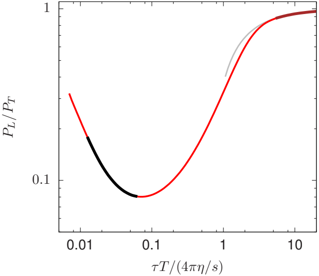
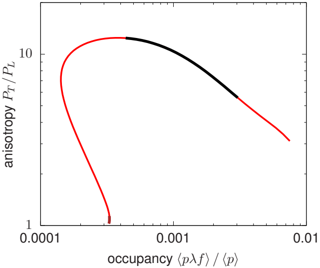






- Pre-scaling before isotropization
- Thermal scaling seen at late times.

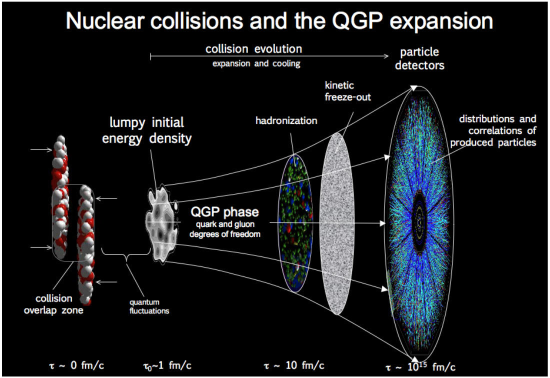
*Early time pre-scaling disconnected from late time hydrodynamics.*

# Part II: Entropy production and hydrodynamic attractors

# Far-from-equilibrium QCD in nucleus-nucleus collisions

## Experiments indicate formation and equilibration of Quark-Gluon Plasma

$$\boxed{\left\langle \frac{dE_\perp}{d\eta} \right\rangle_0 \Rightarrow}$$



Sorensen, Quark-gluon plasma 4, 2010

$$\boxed{\Rightarrow \left\langle \frac{dN_{\mathrm{ch}}}{d\eta} \right\rangle}$$

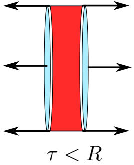
- Non-equilibrium initial-state: tractable in weak coupling QCD (CGC)
- Final-state observables: produced particle spectra and correlations

*Most basic question: how many particles will be produced in a collision?*

# Boost-invariant equations of motion of 1D expansion at early times

Energy-momentum conservation $T^{\mu}{}_{\nu} = \text{diag}\,(e, P_T, P_T, P_L)$

$$\partial_\tau e = -\frac{e + P_L}{\tau},$$



*Need microscopic input: constitutive relation $P_L = P_L(e, \tau)$.*

- Equilibrium: equation of state

$$\frac{P_L}{e} \approx \frac{1}{3} \quad \Longrightarrow \quad e \propto \tau^{-\frac{4}{3}}.$$

- Near-equilibrium: viscous constitutive equations

$$\frac{P_L}{e} = \frac{1}{3} - \frac{16}{9}\frac{\eta/s}{\tau T} + \ldots .$$

$\eta/s$ —specific shear-viscosity.

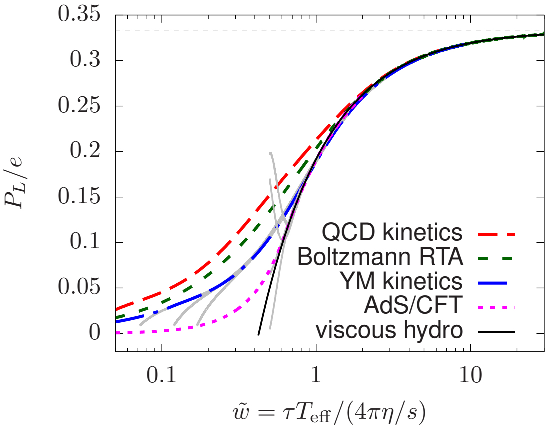
*Macroscopic evolution far from equilibrium?*

# Macroscopic theory of equilibration: hydrodynamic attractors

## Apparent emergence of constitutive relations far-from-equilibrium

Heller and Spalinski (2015)

$$\frac{P_L}{e} = f\left[\tilde{w} = \frac{\tau T_{\text{eff}}}{4\pi\eta/s}\right], \quad \text{where} \qquad T_{\text{eff}} \propto e^{1/4}.$$



see reviews by Florkowski, Heller and Spalinski (2017), Romatschke and Romatschke (2017)

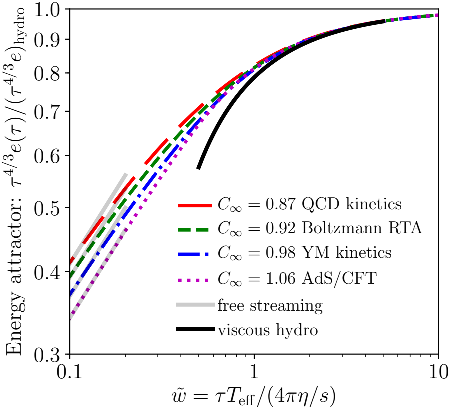
# Similarities of energy evolution in different theories

**Integrating equations of motion:**

Giacalone, AM, Schlichting (2019)

$$e(\tau_{\text{therm}}) = e_0 \exp\left(-\int_{\tilde{w}_0}^{\tilde{w}_{\text{therm}}} \frac{d\tilde{w}}{\tilde{w}} \frac{1+f(\tilde{w})}{\frac{3}{4}-\frac{1}{4}f(\tilde{w})}\right) .$$

**Final-state entropy:** $\frac{dS}{dy} = A_\perp (s\tau)_{\tau_{\text{therm}}} \propto \left(e\tau^{\frac{4}{3}}\right)^{\frac{3}{4}}_{\tau_{\text{therm}}} \equiv \left(e\tau^{\frac{4}{3}}/\mathcal{E}(\tilde{w})\right)^{\frac{3}{4}}$



**Universal early/late asymptotics**

Viscous hydro:

$\mathcal{E}(\tilde{w} \gg 1) = 1 - \frac{2}{3\pi\tilde{w}}$

Free-streaming ($e \sim \tau^{-1}$):

$\mathcal{E}(\tilde{w} \ll 1) = C_\infty^{-1}\tilde{w}^{4/9}$

# Entropy-production from hydrodynamic attractor

Substitute the early time asymptotics

$$(s\tau)_{\tau_{\text{therm}}} = \frac{4}{3}\left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/4}\left(\frac{e\tau^{4/3}}{C_\infty^{-1}\left(\frac{T\tau}{4\pi\eta/s}\right)^{4/9}}\right)^{3/4}.$$

Final state entropy density:

$$(s\tau)_{\tau_{\text{therm}}} = \frac{4}{3}C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3}(e\tau)_0^{2/3}.$$

Consider nucleus transverse overlap area $A_\perp$

$$\underbrace{\left\langle\frac{dN_{\text{ch}}}{d\eta}\right\rangle}_{\text{final-state}} \approx A_\perp\ \frac{N_{\text{ch}}}{S}\underbrace{\frac{4}{3}C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3}}_{\text{medium properties}}\underbrace{\left(\frac{1}{A_\perp}\left\langle\frac{dE_\perp}{d\eta}\right\rangle_0\right)^{2/3}}_{\text{initial-state}}$$

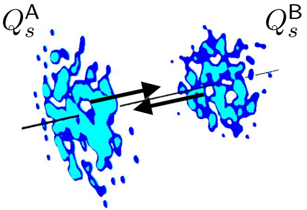
*All relevant-prefactors and powers included!*

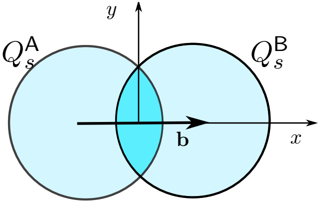
Important to model initial-state energy density $(e\tau)_0$

$\Longrightarrow$ tractable with first principle theory of high-energy QCD.

# Energy deposition in high energy nucleus-nucleus collisions

## Collisions of glasma sheets in color-glass condensate effective theory





Local saturation scale is proportional to nuclear thickness

$$Q_s^2(\mathbf{x}_\perp) \propto T(\mathbf{x}_\perp).$$

Gluon liberation (up to $\log$–corrections)

$$\text{gluon number} \quad (n\tau)_0(\mathbf{x}_\perp) \propto \; T^<(\mathbf{x}_\perp),$$

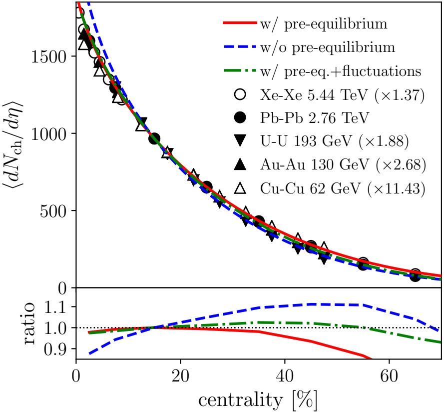
$$\text{gluon energy} \quad (e\tau)_0(\mathbf{x}_\perp) \propto \; T^<(\mathbf{x}_\perp)\sqrt{T^>(\mathbf{x}_\perp)}.$$

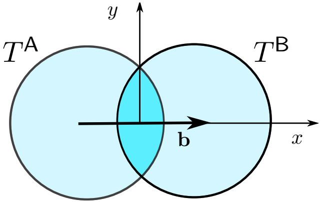
*Can now determine centrality dependence of $dN_{ch}/d\eta$*

# Universal centrality dependence of particle multiplicity

Collapse of rescaled multiplicity $\Longrightarrow$ compare with theory models

$$\left\langle \frac{dN_{\mathsf{ch}}}{d\eta} \right\rangle \propto \underbrace{\frac{dS_{\mathsf{therm}}}{d\eta}}_{\text{equilibration}}, \quad \underbrace{\frac{dN_{\mathsf{gluons}}}{d\eta}}_{\text{no equilibration}}, \quad \underbrace{\left\langle \frac{dS_{\mathsf{therm}}}{d\eta} \right\rangle}_{\text{e-by-e fluctuations}}.$$





$$centrality = \pi b^2/\sigma_{\mathsf{AA}}$$

*Entropy production and e-by-e fluctuations improve agreement with data.*

# Initial state energy density

Bjorken formula for initial state energy density

$$e_0^{\mathsf{Bjorken}} \approx \frac{1}{\tau_0 A_\perp} \frac{dE_\perp^{\mathsf{final}}}{dy}.$$

*Does not include work done during expansion!*

$$\frac{dE_\perp^{\mathsf{initial}}}{dy} = A_\perp (\tau e)_0 > \frac{dE_\perp^{\mathsf{final}}}{dy}.$$

Including the longitudinal work during expansion in central Pb-Pb get

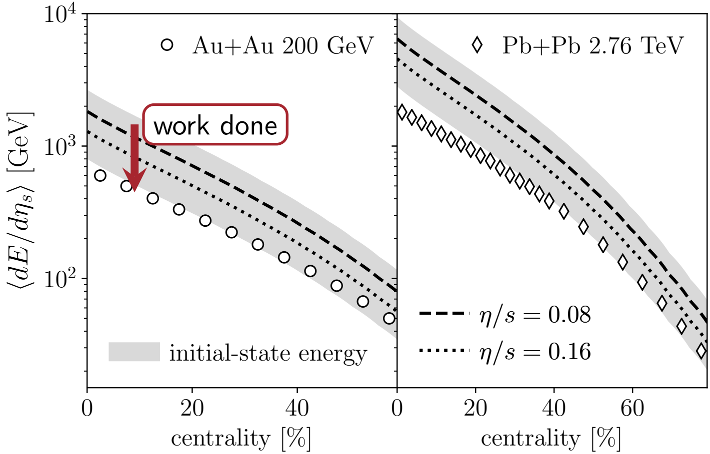
$$e_0 \approx 270 \text{ GeV/fm}^3 \left(\frac{\tau_0}{0.1\text{fm}/c}\right)^{-1} \left(\frac{C_\infty}{0.87}\right)^{-9/8} \left(\frac{\eta/s}{2/4\pi}\right)^{-1/2} \left(\frac{A_\perp}{138\text{fm}^2}\right)^{-3/2} \left(\frac{dN_{\rm ch}/d\eta}{1600}\right)^{3/2} \left(\frac{\nu_{\rm eff}}{40}\right)^{-1/2} \left(\frac{S/N_{\rm ch}}{7.5}\right)^{3/2},$$

c.f. $e \approx 0.3 \text{ GeV}/\mathsf{fm}^3$ near QCD cross-over.

# Centrality dependence of initial state energy

Matching multiplicity allows to infer the initial-state energy per rapidity

Bands are variations of $C_\infty = [0.8\text{-}1.15]$, $\eta/s = [0.08\text{-}0.24]$



*Initial state energy ⇔ non-equilibrium properties of QGP.*

# Summary and Outlook

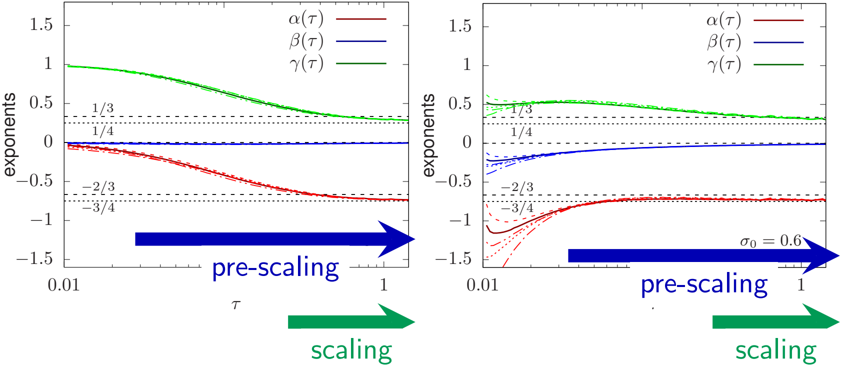
- Scaling and pre-scaling present in full QCD kinetic theory evolution.
- $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$—new hydrodynamic-like degrees of freedom for evolution *not around equilibrium.*
- Hydrodynamic attractors as a direct link between initial and final states: *simple formula for final state entropy.*
- Universal centrality dependence of particle multiplicity and quantitative estimation of initial state energy.

Outlook:

- Can “bottom-up” thermalization be understood as (pre)-scaling $+$ hydrodynamic attractor?
- Equilibration of event-by-event spectra of fluctuation, e.g. with KøMPøST

# Dependence on initial conditions

Vary initial gluon occupation $\sigma_0 = 0.1, 0.6$: $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$



pre-scaling

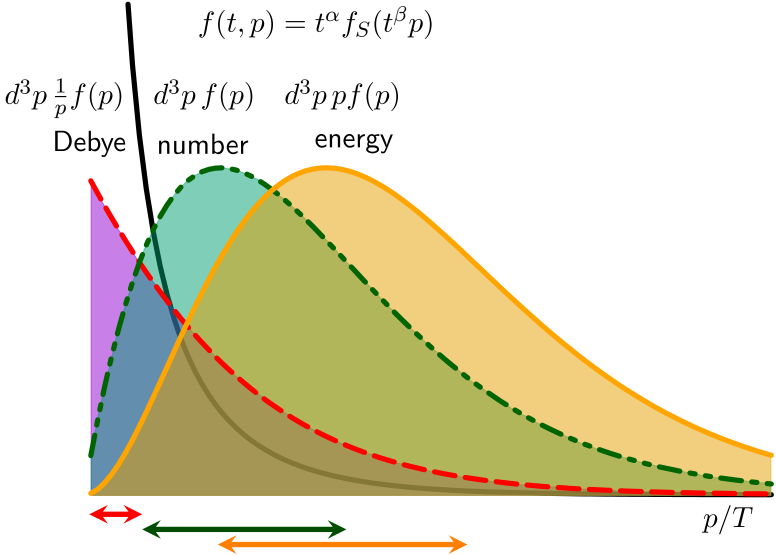
scaling

pre-scaling

scaling

*Non-universal pre-scaling evolution of $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$*

# Weighted momentum distribution



*Moments of distribution function probe different momentum scales.*

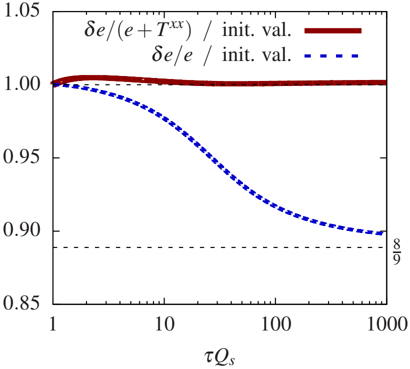
# Equilibration of perturbations

Non-linearities change the perturbation spectra

$$s_{\text{therm}} \propto e_0^{\frac{2}{3}} \Longrightarrow \frac{\delta s_{\text{therm}}}{s_{\text{therm}}} = \frac{2}{3}\frac{\delta e}{e_0}.$$

$k = 0$ perturbation evolution in kinetic theory: $\delta e/(e + T^{xx}) = \text{const.}$

Keegan, Kurkela, AM and Teaney (2016) [3]



$$\frac{\delta e_{\text{therm}}}{e_{\text{therm}}} = \frac{8}{9}\frac{\delta e_0}{e_0}$$

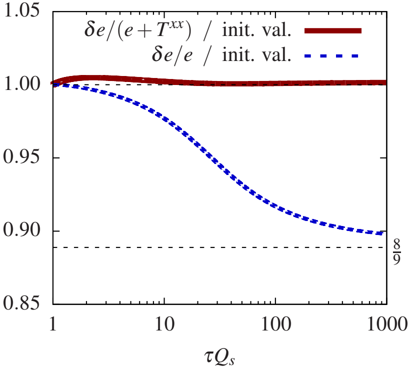
# Equilibration of perturbations

Non-linearities change the perturbation spectra

$$s_{\text{therm}} \propto e_0^{\frac{2}{3}} \Longrightarrow \frac{\delta s_{\text{therm}}}{s_{\text{therm}}} = \frac{2}{3}\frac{\delta e}{e_0} = \frac{3}{4} \times \frac{8}{9}\frac{\delta e_0}{e_0}.$$

$k = 0$ perturbation evolution in kinetic theory: $\delta e/(e + T^{xx}) = \text{const.}$

Keegan, Kurkela, AM and Teaney (2016) [3]



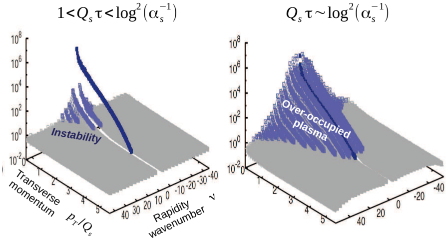
$$\frac{\delta e_{\text{therm}}}{e_{\text{therm}}} = \frac{8}{9}\frac{\delta e_0}{e_0}$$

# Non-thermal fixed point (NTFP) for gauge theories

For $f \sim A^2 \gg 1$ classical-statistical Yang-Mills describes gluon evolution

Aarts, Berges (2002), Mueller, Son (2004), Jeon (2005)

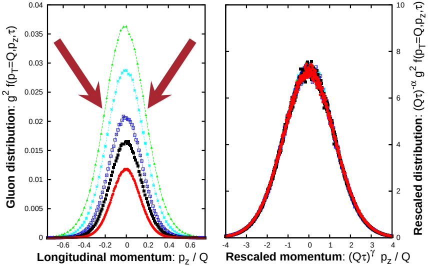
initial plasma instabilities



later evolution

scaling

rescaled



Berges, Schenke, Schlichting, Venugopalan (2014) [13] Berges, Boguslavski, Schlichting, Venugopalan (2014) [9]

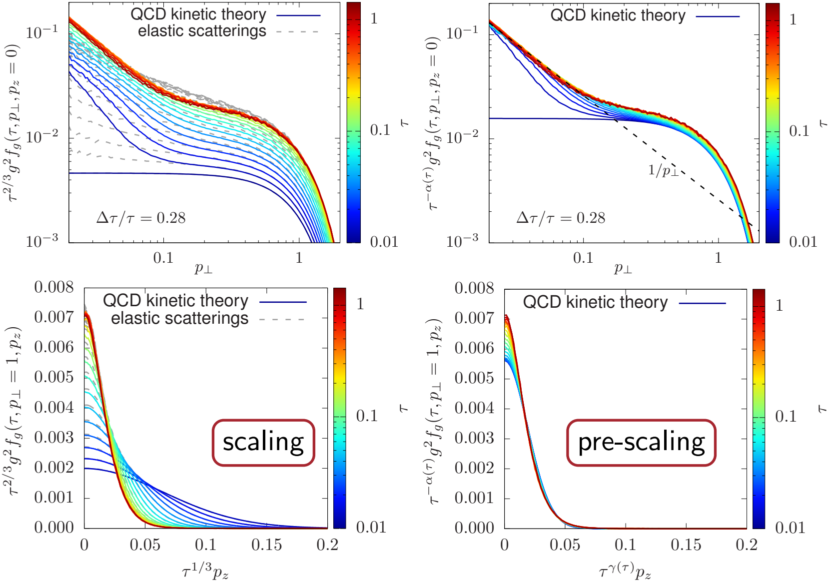
*Self-similar scaling $\Longrightarrow$ simplification of non-equilibrium physics*

$$f_g(p_\perp, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_\perp, \tau^\gamma p_z), \quad \tau = \sqrt{t^2 - z^2}$$

Universal exponents: $\alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$

scaling in other systems: Orioli et al. (2015) [14], Mikheev et al. (2018) [15], Prüfer et al. (2018) [16], Erne et al. (2018) [17]

# Comparison between constant and time dependent exponents



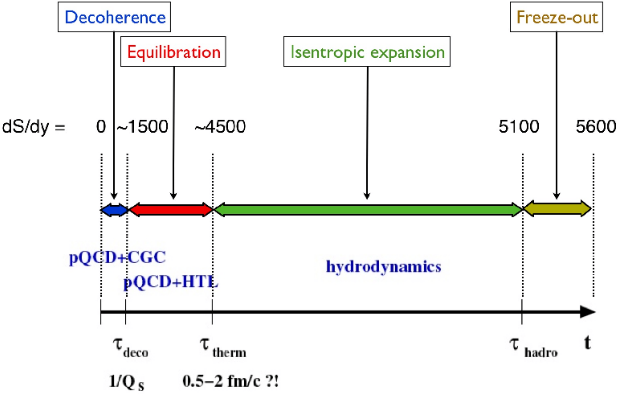
scaling

pre-scaling

# Estimates of entropy production in central Au-Au collisions at RHIC

Particle multiplicity is directly proportional to entropy at thermalization

$$\left\langle \frac{dS}{dy} \right\rangle_{\tau_{\text{therm}}} = \langle s\tau A_\perp \rangle_{\tau_{\text{therm}}} \approx \frac{S}{N_{\text{ch}}} \left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle .$$



Muller and Schafer (2011)

*Most of entropy production occurs at early times during equilibration.*

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