MODEL THEORY OF OPERATOR ALGEBRAS:
QUESTION SESSION LIST

Ilijas Farah Let $\mathcal{Q}$ be the universal UHF algebra and let $\mathcal{R}$ be the hyperfinite II_1 factor. Are $\mathcal{Q}$ and $\mathcal{R}$ elementarily equivalent *in the language of* C^**-algebras*? In other words, do $\mathcal{Q}$ and $\mathcal{R}$ have isomorphic ultrapowers (again, as C^*-algebras)? Note that a positive answer implies that $\mathcal{R}$ is an MF-algebra.

Ben Hayes Call a tracial C^*-algebra (A, τ) *Hayesian* if there is a trace-preserving embedding $A \hookrightarrow \prod_{\mathcal{U}} M_n(\mathbb{C})$, where the latter ultraproduct is the C^*-algebra ultraproduct equipped with the trace obtained by taking the $\mathcal{U}$-ultralimit of the normalized traces on the $M_n(\mathbb{C})$. Call a discrete group Γ *Hayesian* if the tracial C^*-algebra $(C^*_r(\Gamma), \tau_\Gamma)$ is Hayesian, where τ_Γ is the canonical trace. Which groups are Hayesian? Here are some facts about Hayesian groups:

- Amenable groups are Hayesian.
- $\mathbb{F}_2$ is Hayesian. (Haagerup-Thjorbornsen)
- Free products of Hayesian groups are Hayesian (Reference?)
- Direct products of exact Hayesian groups.

Are there any non-Hayesian groups? Is the amalgamated free product of Hayesian groups over an amenable amalgam once again Hayesian?

Chris Phillips Fix $p \in (1, \infty)$. A *unital* L^p*-operator algebra* is a Banach algebra $\mathcal{A}$ such that there is an L^p-space $L^p(X, \mu)$ and an isometric unital Banach algebra homomorphism $\mathcal{A} \hookrightarrow \mathcal{B}(L^p(X, \mu))$. They appear to be closed under ultraproducts and are clearly closed under ultraroots (in fact substructures), so form an axiomatizable class in the language of unital Banach algebras. What are natural axioms?

Alessandro Vignati A result of K.P. Hart implies that if X and Y are two nontrivial continua, then $C(X)$ embeds into an ultrapower of $C(Y)$. It is also known that there is no metrizable continuum X such that $C(Y)$ embeds into $C(X)$ for all other metrizable continua Y. In particular, this implies that for every continuum X, there is a continuum Y such that $C(X) \equiv C(Y)$ but $X \not\cong Y$. For specific

X, find examples of such Y. For example, find Y such that $C([0,1]) \equiv C(Y)$ but $[0,1] \not\equiv Y$.

In another direction, suppose that X and Y are locally compact spaces such that $C(\beta X \setminus X) \equiv C(\beta Y \setminus Y)$. What can we say about $C_0(X)$ vs. $C_0(Y)$. Also, under the same assumption, if one assumes CH, do we know that in fact $C(\beta X \setminus X) \cong C(\beta Y \setminus Y)$?

Isaac Goldbring Call a McDuff II_1 factor *strongly McDuff* if it is isomorphic to one of the form $M \otimes \mathcal{R}$ for M a non-Gamma II_1 factor. Can an existentially closed (e.c.) II_1 factor ever be strongly McDuff? As partial progress, if the non-Gamma factor M is *bc-good* (to be defined shortly), then $M \otimes \mathcal{R}$ is not e.c. Here, M is bc-good if it has a w-spectral gap subfactor N (meaning that $N' \cap M^{\mathcal{U}} = (N' \cap M)^{\mathcal{U}}$ for which $(N' \cap M)' \cap M \neq N$. This leads to the question: is every non-Gamma factor bc-good?

Wilhelm Winter A major open question is whether or not all ssa algebras satisfy UCT. Towards that goal, here are some intermediate questions. View the set of ssa algebras as a category whose morphisms are unital $*$-homomorphisms up to approximate unitarily equivalence. This category has an initial object, namely the Jiang-Su $\mathcal{Z}$, which has a Cartan masa. Can you prove that the initial object has a Cartan masa without actually knowing that it is $\mathcal{Z}$? Also, one can ask the same question for the category of ssa algebras of the form $A \otimes M_{2^\infty}$ where A is ssa. This also has an initial object, M_{2^∞}, which also has a Cartan masa.

Ilan Hirshberg First question: is there a C^*-algebra A such that $A \not\equiv A^{op}$?

Secondly, is there any natural model-theoretic meaning to looking at structures that resemble ultrapowers except one uses βX for X an arbitrary locally compact space (e.g. $\mathbb{R}_+$, which shows up i practice) rather than just βI for I a discrete set? Is there a corresponding logic for which this is well-behaved? Are there parallels to usual model-theoretic facts about ordinary ultrapowers? What uses does this construction have?

Chris Phillips Suppose that A is a unital, simple, purely infinite C^*-algebra. Is there a state on A which can be distinguished up to unitary equivalence?