Flexibility of Semiparametric Choice Models in Traffic Equilibrium

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Agenda

- Discrete Choice Models
- Route/Link Choice in Traffic: (Stochastic) User Equilibrium
- New SUE models arising from DRO framework

Discrete Choice Models



Which route is being used?



Which message is effective?



Which policy is sustainable?



Which product is popular?

Discrete Choice Models

- Random Utility Model (RUM)
- Representative Agent Model (RAM)
- Semiparametric Choice Model (SCM)
- Relations: $RUM \subsetneq RAM = SCM$

Random Utility Model (RUM)

Let $\mathcal{N} = \{1, 2, \dots, n\}$ be the set of alternatives.

The random utility of alternative k is defined as:

$$\tilde{U}_k = \mu_k + \tilde{\epsilon}_k, \quad \forall k \in \mathcal{N}.$$

The deterministic/systemic component of the utility captures many observable attributes affecting the choice.

Often a linear-in-parameters model is used to model the deterministic component in terms of observed attributes.

 $\tilde{\epsilon}_k$ accounts for the unobserved/random component.

Random utilities form a random vector $\tilde{\epsilon}$ that follow a known joint distribution θ .

Random Utility Model (RUM)

When θ is absolutely continuous,

$$p_k = \mathbb{P}_{\tilde{\epsilon} \sim \theta} \left(k = \arg \max_{l \in \mathcal{N}} \mu_l + \tilde{\epsilon}_l \right), \quad \forall k \in \mathcal{N},$$

is the probability of alternative k to be the best choice.

We refer p_k as the choice probability of alternative k.

Choice probabilities depend on the choice of the distribution θ .

For given θ , we can calculate the expected utility as

$$Z^{ heta}(\mu) = \mathbb{E}_{ ilde{\epsilon} \sim heta} \left(\max_{k \in \mathcal{N}} \mu_k + ilde{\epsilon}_k
ight).$$

Representative Agent Model (RAM)

A representative agent chooses between products in ${\cal N}$ to maximize the expected utility while keeping some level of diversity.

He solves the following optimization problem:

 $\max_{x\in\Delta_{n-1}}\mu^{\mathsf{T}}x-V(x).$

V(x) is a (strictly) convex regularization term promoting diversification.

Optimal solution, if it is unique, gives the choice probabilities.

Semiparametric Choice Model (SCM)

RUM is a special case of SCM, where the distribution θ of $\tilde{\epsilon}$ is not given but it is known to lie in a set of distributions, say Θ .

Under this model, maximum expected utility is defined as

$$Z^{\Theta}(\mu) = \sup_{ heta \in \Theta} \mathbb{E}_{ ilde{\epsilon} \sim heta} \left(\max_{k \in \mathcal{N}} \mu_k + ilde{\epsilon}_k
ight).$$

The corresponding choice probabilities are calculated using the extremal distribution θ^* .

$$p_k = \mathbb{P}_{\tilde{\epsilon} \sim \theta^*}\left(k = \arg \max_{l \in \mathcal{N}} \mu_l + \tilde{\epsilon}_l\right), \quad \forall k \in \mathcal{N}.$$

Relationships between different choice models

- MNL \subset RAM (Anderson et al., 1988)
- RUM ⊊ RAM (Hofbauer and Sandholm, 2002)
- RAM = SCM (Feng et al., 2017)

Two special cases where Θ and V(x) are given explicitly:

- MMM, MDM ⊂ RAM (Natarajan et al., 2009)
- CMM ⊂ RAM (Ahipasaoglu et al., 2018)

Choice Models (that are of interest to us)

Model	θ	V(x)	Θ	p_k
MNL (logit)	i.i.d Gum(θ)	$\frac{1}{4}\sum x_i \log x_i$	Marginals: $E_{xp}(0, \frac{1}{2})$	$\frac{e^{-\theta\mu_k}}{\sum_{l\in\mathcal{N}}e^{-\theta\mu_l}}$
MNP (probit)	Ν(0,Σ)	exists	Ν(0,Σ)	$\int \int \int \dots$ (simulation)
MMM (marginal moment)	none	$-\sum_i \sigma_i \sqrt{x_i(1-x_i)}$	Marginals: mean 0, std σ_i	$\frac{\frac{1}{2} + \frac{\mu_k - \lambda}{2\sqrt{(\mu_k - \lambda)^2 + \sigma_k^2}})}{\text{(bisection over }\mathbb{R})}$
MDM (marginal distr.)	none	$-\sum_i\int_{1-x_i}^1F_i^{-1}(t)dt$	Marginals: <i>F_i</i> (.)	$1 - F_k(\lambda - \mu_k)$ (bisection over \mathbb{R})
CMM (cross moment)	none	$-\mathrm{tr}\left(\Sigma^{1/2} \mathcal{S}(x) \Sigma^{1/2} ight)^1$	^{/2} Mean 0, cov Σ	Gradient descent (locally linear)

Route Choice and Traffic Equilibrium

Assumptions:

- Multiple origin-destination pairs with fixed demand
- Multiple available routes (possibly overlapping) for each OD pair
- Arc costs as a function of arc flows
- Additive model for path/route costs



Setup and Notation

$$\mathcal{G} = (N, \mathcal{A})$$

$$\mathcal{W}$$

$$(r_w, s_w)$$

$$\mathcal{K}_w$$

$$\mathcal{K}$$

$$d_w$$

$$\mathbf{x} = (x_{kw})_{k \in \mathcal{K}_w, w \in \mathcal{W}}$$

$$\mathbf{f} = (f_a)_{a \in \mathcal{A}}$$

$$c_a(f_a)$$

$$\mathbf{c}(\mathbf{f}) = (c_{kw}(\mathbf{f}))_{k \in \mathcal{K}_w, w \in \mathcal{W}}$$

$$\mathbf{x}_w(\mathbf{f}) = (x_{kw}(\mathbf{f}))_{k \in \mathcal{K}_w}$$

$$\mathbf{c}_w(\mathbf{f}) = (c_{kw}(\mathbf{f}))_{k \in \mathcal{K}_w}$$

Directed graph with nodes N and arcs/links \mathcal{A} Set of origin-destination (OD) pairs in \mathcal{G} The wth OD pair Directed simple paths between r_w and s_w Set of all simple paths in \mathcal{G} , i.e., $\bigcup_{w \in \mathcal{W}} \mathcal{K}_w$ Demand associated with the wth OD pair The path flow vector The arc flow vector The det. cost of arc $a \in \mathcal{A}$ with f_a units¹ The path cost vector The wth path flow vector The wth path cost vector

¹We slightly abuse the notation and use the same symbol for arc costs and path costs. We assume that $c_a(f_a)$'s are non-decreasing and continuous.

Traffic Equilibrium

Introduced by Wardrop (1952):

- Deterministic Wardropian User Equilibrium: The travel costs on all routes that are actually used are equal to or less than those which would be experienced by a user on any unused route.
- System Optimum: Traffic is distributed (by a central planner) to minimize the average journey time.

Wardropian User Equilibrium

Convex formulation by Beckman, McGuire, and Winsten (1956):

$$\min_{x,f} \qquad \sum_{a \in \mathcal{A}} \int_0^{f_a} c_a(t) dt$$

s.t.
$$\sum_{k\in\mathcal{K}_w} x_{kw} = d_w, \quad \forall w\in\mathcal{W},$$

$$x_{kw} \geq 0, \qquad \forall k \in \mathcal{K}_w, w \in \mathcal{W},$$

$$f_a = \sum_{w \in \mathcal{W}} \sum_{k \in \mathcal{K}_w: k \ni a} x_{kw}, \quad \forall a \in \mathcal{A}.$$

Equilibrium flows can be calculated by a linearisation algorithm based on Frank-Wolfe.

Stochastic User Equilibrium

Introduced by Daganzo and Sheffi (1977):

• Stochastic User Equilibrium: No user can improve his *perceived* travel time by unilaterally changing routes.

Route choice by passengers can be modelled as a discrete problem.

Stochastic Route Choice with RUM

Traditionally, stochastic choice is modeled using a random utility model:

$$\tilde{U}_{kw} = -c_{kw}(f) + \tilde{\epsilon}_{kw}, \forall k \in \mathcal{K}_w, \forall w \in \mathcal{W},$$

where $\boldsymbol{\widetilde{\epsilon}_{w}} \sim \theta_{w}.$

Additive cost model is used to calculate the path costs:

$$c_{kw}(\boldsymbol{f}) = \sum_{a \in k} c_a(f_a),$$

 c_a is estimated from characteristics of link a.

Stochastic Route Choice with RUM

We define

$$p_{kw}(\boldsymbol{f}) = \mathbb{P}_{ heta_w} \Big(-c_{kw}(\boldsymbol{f}) + \tilde{\epsilon}_{kw} \ge -c_{lw}(\boldsymbol{f}) + \tilde{\epsilon}_{lw}, \quad \forall l \neq k, l \in \mathcal{K}_w \Big),$$

to be the route choice probabilities. I.e., the probability of route k to be the best choice among all possible routes for OD pair w.

Choice probabilities depend on the choice of the distribution θ_w .

The corresponding flow on a path is given by:

$$x_{kw} = d_w p_{kw}(f), \quad \forall k \in \mathcal{K}_w, w \in \mathcal{W}.$$

Stochastic User Equilibrium with RUM

In a SUE model, the equilibrium arc flow vector f is the solution to the fixed point equation:

$$f_a = \sum_{w \in \mathcal{W}} d_w \sum_{k \in \mathcal{K}_w: k \ni a} p_{kw}(f), \quad \forall a \in \mathcal{A}, \quad \forall w \in \mathcal{W}.$$



Stochastic User Equilibrium with RUM

SUE can be calculated using one of the following formulations:

In terms of arc flow variables (Sheffi and Powell, 1982):

$$\min_{\boldsymbol{f}} \quad \sum_{w \in \mathcal{W}} d_w \mathbb{E}_{\theta_w} \left(\max_{k \in \mathcal{K}_w} (-c_{kw}(\boldsymbol{f}) + \tilde{\epsilon}_{kw}) \right) + \sum_{\boldsymbol{a} \in \mathcal{A}} f_{\boldsymbol{a}} c_{\boldsymbol{a}}(f_{\boldsymbol{a}}) - \sum_{\boldsymbol{a} \in \mathcal{A}} \int_0^{f_{\boldsymbol{a}}} c_{\boldsymbol{a}}(t) dt.$$

In terms of arc cost variables (Daganzo, 1982):

$$\min_{c} \sum_{w \in \mathcal{W}} d_{w} \mathbb{E}_{\theta_{w}} \left(\max_{k \in \mathcal{K}_{w}} \left(-\sum_{a \in \mathcal{A}: k \ni a} c_{a} + \tilde{\epsilon}_{kw} \right) \right) + \sum_{a \in \mathcal{A}} \int_{\underline{c}_{a}}^{c_{a}} f_{a}(c) dc.$$

Stochastic User Equilibrium with RUM

Two well-known SUE models based on RUM are:

- MNL-SUE: Multinomial Logit Model Suffers from IIA property, needs to be extended to beyond i.i.d. for successful applications
- MNP-SUE: Multinomial Probit Model Captures correlations but not practical

Semiparametric Route Choice Models

Choice probabilities are evaluated under an extremal distribution:

$$p_{kw}(\boldsymbol{f}) = \mathbb{P}_{ heta_{w}^{*}}\left(-c_{kw}(\boldsymbol{f}) + ilde{\epsilon}_{kw} \geq -c_{lw}(\boldsymbol{f}) + ilde{\epsilon}_{lw}, \quad \forall l \neq k, l \in \mathcal{K}_{w}
ight),$$

where

$$\theta_w^* = \arg \max_{\theta \in \Theta_w} E_{\theta} \left(\max_{k \in \mathcal{K}} \{ \tilde{U}_{kw} \} \right).$$

Choice of the uncertainty set Θ_w leads to different choice models and, therefore, different SUE models.

RAM/SCM - SUE

The distributionally robust counterpart of Daganzo's arc cost formulation:

$$\min_{\boldsymbol{C}} \sum_{w \in W} d_w \max_{\theta_w \in \Theta_w} \boldsymbol{E}_{\theta_w} \left[\max_{k \in K_w} \left(-\sum_{a \in \mathcal{A}: k \ni a} c_a + \tilde{\epsilon}_{kw} \right) \right) + \sum_{a \in \mathcal{A}} \int_{\underline{c}_a}^{c_a} f_a(c) dc.$$

Under this approach, the system planner assumes only limited distributional information and uses a 'worst-case' potential function in computing the equilibrium.

RAM/SCM - SUE

$$\begin{split} \min_{\mathbf{x},\mathbf{f}} & \sum_{a \in \mathcal{A}} \int_{0}^{f_{a}} c_{a}(t) dt + \sum_{w \in \mathcal{W}} d_{w} V(x_{kw}) \\ \text{s.t.} & \sum_{k \in \mathcal{K}_{w}} x_{kw} = d_{w}, & \forall w \in \mathcal{W}, \\ & x_{kw} \geq 0, & \forall k \in \mathcal{K}_{w}, w \in \mathcal{W}, \\ & f_{a} = \sum_{w \in \mathcal{W}} \sum_{k \in \mathcal{K}_{w}: k \ni a} x_{kw}, & \forall a \in \mathcal{A}. \end{split}$$

This is a generalization of the convex formulation of Wardropian UE (Beckman et al., 1956) and MNL - SUE (Fisk, 1980).

RAM/SCM - SUE

Classical RUM - SUE models are:

- MNL-SUE: Multinomial Logit Model
- MNP-SUE: Multinomial Probit Model
- MNW-SUE: Multinomial Weibit Model (relatively new)

Two new RAM/SCM - SUE models are introduced recently:

- CMM-SUE: Cross Moment Model (Ahipasaoglu et al., 2015)
- MDM-SUE: Marginal Distributions Model (Ahipasaoglu et al., 2016)

Summary: CMM-SUE

- Captures correlations between routes (due to overlapping arcs).
- CMM choice probabilities can be calculated efficiently as an SDP or using first-order methods.
- Using the representative agent version of the CMM model, a convex-concave min-max reformulation gives the CMM-SUE flows.
- CMM-SUE flows exist and are unique.
- CMM-SUE flows can be calculated by a gradient-descent type algorithm, similar to the MSA for MNP-SUE.
- CMM-SUE provides a practical alternative to the MNP-SUE model.



Figure: Sioux Falls Network: 24 nodes, 76 links, 552 OD pairs, $c_a = \bar{c}_a \left(1 + B \left(\frac{f_a}{s_a}\right)^t\right).$

Table: Computational times and relative difference of total costs.

Г	OD	No. of paths	3 paths	4 paths	5 paths	6 paths	7 paths	8 paths	9 paths	Γ
Г	10	time CMM	0.07	0.10	0.12	0.15	0.17	0.23	0.27	Г
		time MNP	0.16	0.20	0.26	0.32	0.36	0.40	0.46	
		time ratio	2.11	2.08	2.21	2.16	2.13	1.76	1.75	
		cost difference	4.80e-8	6.19e-8	9.25e-8	9.51e-8	1.31e-7	1.78e-7	2.29e-7	
	50	time CMM	0.30	0.59	0.54	0.64	0.78	0.93	1.06	Γ
		time MNP	0.81	1.02	1.29	1.53	1.83	2.07	2.54	
		time ratio	2.68	1.73	2.40	2.39	2.34	2.23	2.41	
		cost difference	8.25e-6	6.86e-6	7.75e-6	5.09e-6	6.51e-6	8.92e-6	1.01e-5	
	100	time CMM	0.75	1.01	1.29	1.65	2.04	2.71	3.44	Γ
		time MNP	2.30	3.05	3.11	5.92	6.59	8.42	10.05	
		time ratio	3.04	3.02	2.42	3.59	3.23	3.11	2.92	
		cost difference	9.13e-6	1.38e-5	1.14e-5	1.75e-5	2.82e-5	3.49e-5	3.64e-5	
	200	time CMM	3.48	6.06	8.50	11.02	16.48	21.07	26.34	
		time MNP	20.61	34.61	44.71	61.80	81.30	94.36	116.90	
		time ratio	5.92	5.71	5.26	5.61	4.93	4.48	4.44	
		cost difference	1.78e-4	3.24e-4	4.44e-4	5.16e-4	6.11e-4	6.75e-4	7.36e-4	
	400	time CMM	18.88	34.25	52.31	73.87	104.61	129.41	172.64	
		time MNP	136.16	262.13	367.83	506.45	610.91	781.89	957.77	
		time ratio	7.21	7.65	7.03	6.86	5.84	6.04	5.55	
		cost difference	2.58e-3	4.88e-3	6.96e-3	8.60e-3	1.03e-2	1.19e-2	1.31e-2	
	552	time CMM	26.58	47.82	84.11	119.44	156.10	210.10	286.46	
		time MNP	186.64	342.60	483.98	685.20	881.98	1250.48	1535.29	
		time ratio	7.02	7.16	5.75	5.74	5.65	5.95	5.36	
		cost difference	5.16e-3	9.41e-3	1.43e-2	1.79e-2	2.16e-2	2.51e-2	2.81e-2	



Convergence of the algorithms. (Left: 3 paths, Right: 5 paths)



The CMM-SUE (left) and MNP-SUE (right) flows.



Figure: CMM-SUE and MNP-SUE flows when there are 552 OD pairs.

Summary: MDM-SUE

- MDM assumes that the marginal distributions are given, but not the general distribution.
- MDM-SUE exists and is unique when
 - $E|\widetilde{U}_{kw}| < \infty$.
 - \widetilde{U}_{kw} has support on $(-\infty,\infty)$ or $[\underline{u}_{kw},\infty)$.
 - The cumulative distribution function $F_{kw}(\cdot)$ is assumed to be strictly increasing and continuous with a pdf $f_{kw}(\cdot) > 0$ on the support.
 - $c_a(f_a)$ is nondecreasing in f_a and continuous.
- Since V(x) is separable, MDM-SUE flows can be easily calculated.

Summary: MDM-SUE

MDM-SUE is very flexible in terms of capturing the user behaviour:

- A generalization of some important logit and weibit-based models.
- Handles the overlapping and equal variance problems simultaneously.
- Generalizes the scaling approach by allowing route-level scaling.
- Allows assigning perception variances independent of the route costs.
- A practical approach to incorporate normal random variables.
- Allows to use skewness to model different route choice behaviors.
- Extended to use bounded and discrete marginal distributions.
- Allows to distinguish between the used and unused routes.

MDM-SUE

MDM-SUE can generate existing logit and weibit based models.



MDM-SUE with exponential marginals

- MDM-SUE provides modelling flexibility beyond the capabilities of existing extensions of MNL.
- It can extend Chen's OD-level scaling approach to route-level scaling by setting $\theta_{kw} = \theta \pi / \sqrt{6 \eta c_{kw}(0)}$.

Route-specific perception variances become:

$$\sigma_{kw}^2 = \eta c_{kw}(0)/\theta^2, \quad \forall k \in K_w, w \in W.$$

• MDM can scale dispersion variances more smoothly, especially when the routes in OD pairs have significantly varying lengths.

MDM-SUE with normal marginals

- Normal traffic random variables have several nice properties: location and scale stable, and reproductive (Castillo et al., 2014).
- MNP model is expensive.
- MDM-SUE provides a practical approach to use normal distribution.
- MDM allows assigning perception variance independent of the cost.



 $\sigma_1 > \sigma_2$

MDM-SUE with gamma marginals

- Shifted gamma random variables are location-scale stable (Castillo et al., 2014), and share nice properties with normal random variables such as being reproductive.
- Moreover, it can have positive skewness as the observed traffic flows.



MDM-SUE with bounded and discrete marginals

Some recent criticism (by Watling et al, 2015) on SUE models are:

- UE models can be considered to be too restrictive as it may assign no flow to a route that is slightly more costly than the used routes.
- SUE models assign a positive flow to each route in the choice set.
- The continuous and unbounded distribution assumption of SUE is unrealistic.
- MDM-SUE formulation that can be extended to handle bounded and discrete distributions by relaxing some of the assumptions.
- Optimal values of the dual variables behave as reference utilities to distinguish between used and unused routes within the SUE framework.
- Equilibrium flows are not unique in this case.

(Link-based) Markovian Traffic Assignment

- Markovian traffic assignment can be considered as a special case of a dynamic discrete choice model for route choice.
- Each route choice is defined as a sequence of link choices, where p_{ij}^d is the probability of choosing link (i, j) at node *i* if destination is *d*.
- Link choice at a particular node is independent of the previous choices (Markovian property).

(Link-based) Markovian Traffic Assignment



 $M^{d} = \left(I - Q^{d'}\right)^{-1}$: fundamental matrix

 \mathbf{Q}_{ij}^{d} , is equal to p_{ij}^{d} if $(i, j) \in A_{d}$ and zero otherwise.

 \mathbf{M}_{ij}^{d} is the expected number of times a user entering the system at node *i* visits node *j* before reaching the destination.

(Link-based) Markovian Traffic Assignment with RUM

- Link cost of arc (i, j) is defined as t_{ij} − *ϵ*^d_{ij}.
- Let w_j^d denote the expected minimum cost from node j to destination d.
- A Markovian choice model for destination *d* solves:

$$w_i^d = E_{\theta_{id}}\left[\min_{j \in N_d^+(i)} \left\{ t_{ij} - \widetilde{\epsilon}_{ij}^d + w_j^d \right\} \right], \ \forall i \in N_d, \text{and} \ w_d^d = 0,$$

where θ_{id} is the joint distribution of the error terms $\{\tilde{\epsilon}_{ij}^d; j \in N_d^+(i)\}$.



The MTE (defined by Baillon and Cominetti, 2008) is the solution to the fixed point problem:

$$\begin{split} t_{ij} &= \tau_{ij} \left(f_{ij} \right), & \forall (i,j) \in A, \\ w_i^d &= E_{\theta_{id}} \left[\min_{j \in N_d^+(i)} \left\{ t_{ij} - \widetilde{\epsilon}_{ij}^d + w_j^d \right\} \right], & \forall i \in N_d, d \in D, \\ w_d^d &= 0, & \forall d \in D, \\ n_i^d &= h_i^d + \sum_{k \in N_d^-(i)} x_{ki}^d, & \forall i \in N_d, d \in D, \\ x_{ij}^d &= n_i^d \cdot \mathsf{P}_{\theta_{id}} \left\{ j = \operatorname*{argmin}_{k \in N_d^+(i)} \left\{ t_{ik} - \widetilde{\epsilon}_{ik}^d + w_k^d \right\} \right\}, & \forall (i,j) \in A_d, d \in D, \\ f_{ij} &= \sum_{d \in D: (i,j) \in A_d} x_{ij}^d, & \forall (i,j) \in A. \end{split}$$



Distributionally Robust Markovian Traffic Equilibrium

Under mild assumptions on θ and τ , the MTE exists and is unique.

Link costs at equilibrium solve an unconstrained convex optimization;

$$Z(\boldsymbol{\theta}) = \max_{\mathbf{t}} \sum_{d \in D} \sum_{i \in N_d} h_i^d \cdot w_i^d(\mathbf{t}) - \sum_{(i,j) \in A} \int_0^{t_{ij}} \tau_{ij}^{-1}(\omega) d\omega,$$

Only used for the MNL since computing the choice probabilities under RUM is not trivial.

We can also calculate MTE from a constrained convex optimization problem:

$$Z(\boldsymbol{\theta}) = \max_{\mathbf{t}, \mathbf{w}} \sum_{d \in D} \sum_{i \in N_d} h_i^d \cdot w_i^d - \sum_{(i,j) \in A} \int_0^{t_{ij}} \tau_{ij}^{-1}(\omega) d\omega$$

s.t. $w_i^d \leq E_{\theta_{id}} \left[\min_{j \in N_d^+(i)} \left\{ t_{ij} - \hat{\epsilon}_{ij}^d + w_j^d \right\} \right], \quad \forall i \in N_d, d \in D,$
 $w_d^d = 0, \qquad \forall d \in D.$

Semiparametric Markovian Traffic Equilibrium

MTE with semiparametric choice model is defined as:

$$\begin{split} \max_{\mathbf{t},\mathbf{w}} & \sum_{d \in D} \sum_{i \in N_d} h_i^d \cdot w_i^d - \sum_{(i,j) \in A} \int_0^{t_{ij}} \tau_{ij}^{-1}(\omega) d\omega \\ \text{s.t.} & w_i^d \leq E_{\theta_{id}} \left[\min_{j \in N_d^+(i)} \left\{ t_{ij} - \tilde{\epsilon}_{ij}^d + w_j^d \right\} \right], \quad \forall \theta_{id} \in \Theta_{id}, \forall i \in N_d, d \in D, \\ & w_d^d = 0, \qquad \qquad \forall d \in D, \end{split}$$

- MTE-MDM uses the MDM choice model for link choice.
- MTE-MDM exists and is unique when MDM probabilities are unique.
- It provides a distributionally robust optimization perspective on the MTE from the system planner's view.
- Using the equivalent RAM, we propose equivalent convex optimization formulations to obtain the equilibrium flows.
- The model is flexible in capturing a wide range of user choice behavior and can be calculated efficiently even for large networks.

Numerical results - Winnipeg Network²



Figure: 1040 nodes (138 destinations), 2836 links, 4344 OD pairs

²from Bar-Gera's Traffic Assignment Test Problem website:See https://github.com/bstabler/TransportationNetworks

		Sio	ux Falls	W	innipeg	Winnipeg (NRL)
α_1	α_2	Iter.	Time(s)	Iter.	Time(m)	Time(m)
2.00	-1.0	255	8.19	117	10.88	
	-0.5	530	20.21	154	14.63	
	0.0	430	19.78	188	16.37	12.07
	0.5	357	20.73	229	23.15	
1.75	-1.0	450	16.85	152	11.27	
	-0.5	480	20.25	175	15.16	
	0.0	363	17.42	194	16.96	11.35
	0.5	327	19.29	218	23.99	
1.50	-1.0	567	22.41	168	11.32	
	-0.5	356	16.27	174	15.23	
	0.0	319	16.99	206	17.03	12.50
	0.5	288	17.26	223	24.10	
1.25	-1.0	280	12.71	185	11.09	
	-0.5	308	15.71	194	14.91	
	0.0	273	14.87	204	16.68	13.71
	0.5	264	16.69	223	23.59	

Table: MDM-SUE with exponential marginals

Summary: Software

An extensive sofware package that can calculate (almost) all equilibrium models in the literature!

MDM - Exponential Wizard		- 0	×	About	seSue	🕼 seSue - Sioud'allsWithPaths.ove — 🗆
Apply To	Settings	Exponential Distribution Parame	ters	What is GUE1 Sectorie Like Optioner SLO is a second of Vertrain epideler propies which is defined as the bells epidelecer arbents	Peebures Current version of outline allows to associate the following discrete inteller matchs and that combinations with the path based DLE mode,	Elle Export Exh Generation Choice Model Wizard Beport Sensitivity Analysis
Path-level scaling using Scaling Factor	✓ Using Path Size		×	ne aletare sam unlaterally scharge nucles in improve Hacher personnel Intervitions. A path insteal ILLE model along instealerers the following inputs	in addition in the Villeship's (1962) user republic Land Lagd Landel Musicia Villeship Villeship Villeship	Graph Nodes Links OD Pairs Paths MSA
Parameter		v	alue	traffe natural topology personalized into functions, urgan clearing (CC pains and consequencing (CC clements).	Muthomar Lagt (MK) NUTHONIal NUCL MARK NUTHONIal Lag((MK) NUTHONIal NUCL (MK)	MSA(Stander8(1,0),10000,0.01) Run
Dispersion (theta) Proportionality Constant (eta)			0.1	Into Suil Inte and path less. On the other hand, Suil is one of the most popular methods that is used as the halfs associated strength within many other associations.	C Lapi Califirm validit models Oli loval societi validiti Outron togt motion	Iteration RMSE Link Row Array
Coefficient of Variation (nu)			0.3	Spicale, decree more more an associated with the attachment more choice tetrainer of the same in a SUE model. The same sholow	Margina Elizabetiva Musici (ACA) Valuenis	0 -1 14169.01,16063.28,14177.28,14203.99,16055.17,24296.42,31816.07,24278.78,36986.10,23163.8.
Commonality factor multiplier (betal)			0.1	model may be analyzed for all stars or offerent choice models may be analyzed to the stars in offerent GO parts (hybrid). But models are distriguized with wapped to be instantial discrete their models as	Marginal Exponential House (NEN)	1 21166.6 7802.54(15001.02,7001.05,5202.63),15003.27,31267.00,24438.09,31154.07,37487.92,104.07,3747
Commonality factor power (gamma) MDM reaction nationater (variantilen)			0.0001	different room choose barberers and to different equilibrium Nove. Another lander that experimently affects the equilibrium fores in a path- tement NUT prover on choose and parameters in a contained traffic	00 and tools lovel scelect resets	2 12923.5 112 16.57,12396.22,11282.4.
				Interiori, these sold is high number of systeme paths, or round, connecting an arige to a destination, and possibly infinite many paths when system is allowed, in general, a mate shoke and contacting	Path generator agorithms lated below and their continuators can be exercised and the second states and their continuators can be	4 1503.02 113 401 Reations have been executed until 33.98,27070.27,11729.8.
$\begin{cases} 0_{kw} = -c_{kw} + \epsilon_{kw} \\ F_{kw}(t) = 1 - \exp(\theta_{kw} (A_{kw})) \end{cases}$	t))		(k e)	White of studes is permuted for each CE pair a prior to finding the BLE forms.	Path Generation Agostome	5 574.658 1127 Convergence. 17.77,26732.64,12011.5.
$p_{\mathbf{k}w} = 1 - F_{\mathbf{k}w}(\lambda_w + c_{\mathbf{k}w})$	$= \exp(\theta_{kw}(A_{kw} - c_{kw} - \lambda_w))$)		The stude choice and and the studen included play an important rule in the stude choice ands and the studen included play an important rule in the equilibrium Nova, Computationaly, the includes in the state of the	Ners Kalented pails agentin Dar's efficient path garantion algorithm	6 177.5% 112 7 00.1677 1114 OK 5.70 10.106.3 11166.3.
where λ_w solves $\sum_{l \in K_w} \exp(\theta_k)$	$_{w}(A_{kw} - c_{kw} - \lambda_{w})) = 1, \forall$	′w ∈ ₩.		parts and solving the SUE node; while a small those air may have in unwalking and provide.	Lik persity apprition Like adminution algorithm	8 64.667111132/92,75/355/97,71759/74,70280/72,75/2983/07,24980/29/29855.04,26057.80,11841.2.
Philippi (confige factor)				What is solice?	Data first of real siles that can be ready used with reflue are provided in the Downloads realizer. These area free are converted from the data	9 45.2923 11109.79,15322.00,11136.17,10287.01,15295.67,23627.41,24973.80,23599.70,25948.32,11829.6.
(muniplaning factor)	1))			adue la a Pindova Forne Applicator developed with 762° Famewain 4.5.	The public of the constraints of the public of the public of the constraint of the c	10 33.4381 11092.24, 15295.13, 11118.35, 10257.60, 15269.01, 23503.00, 24905.54, 23555.53, 25064.21, 11022.0.
$A_{kw} = \theta_{kw}^{-1} \ln \left(\sum_{i} \left(\frac{s_i}{L_k} \right) \left(\frac{\overline{\Sigma}_{ie}}{\Sigma_{ie}} \right) \right)$	$\frac{1}{\kappa_{w} \delta_{sl}}$)).	`	rk ∈ F	It is designed to carry 52 experiments to antices the effects at a different publicational 25.2 minutes associated with different	radiable measurings. Castory retrains our also be impleted and analysis with selfue. A Macc-analysis Exist scretizeds (sher) that	11 25.6013 11070.40, 15273.22, 11104.42, 10251.20, 15347.30, 23546.50, 24995.84, 23579.60, 11076.7.
0 m				description of the second models (as will as fy/or models, and description of prevator approximation of the second seco	servers symptometry or most use the symptom of the Dovided Inchos.	10 00000000000000000000000000000000000
$\theta_{kw} = \frac{1}{\sqrt{6 \eta c_{kw}}},$		`	fw € '	It also allows to samy and sensitivity analysis to presist the effects of periodultate in the CO detaulut and sensetized that have cost	can be used to fird the equilibrium traves of all provided that insides in order to allow fair companion of convergence performances.	14 13.6742 11050.95, 15226.93, 11076.58, 10239.68, 15201.29,23470.92,25020.28,23443.94,25661.72, 11807.9.
				factors.	while allows insultation of congestion in road indexions by con- celling the trike with select to the amount of traffic, taken calls at	15 11.5136 11044.60,15215.77,11070.19,10237.36,15190.17,23452.71,25026.79,23425.77,25629.81,11806.2.
Back		Net			Frage, a Serething Anderer module is exercise, in particular, it attive to certs out sensitivity analysis with respect to percursations in the OD demands and bia root functions.	16 9.82314 11039.18,15206.05,11064.72,10235.40,15180.51,23436.90,25032.66,23410.02,25602.30,11804.9.

http://people.sutd.edu.sg/~ugur_arikan/seSue/

Future Work

- Dynamic equilibrium
- Congestion pricing
- Sensitivity analysis
- Price of anarchy
- Parameter estimation (Software package for MDM estimation)
- Richer models: multiple vehicle types, elastic demand, etc.

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