# Decomposition Methods For Solving Distributionally Robust Programs

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**Part I: Stochastic Binary Programs**

- *Finite support with Wasserstein & Moment Polytopes*

Joint work with Manish Bansal and Kuo-Ling Huang

**Part I: Wasserstein RO – Logistic Regression**

- *Wasserstein Ball*

Joint work with Fengqiao Luo

# Distributionally Robust Two-Stage Stoch. IP

$$\begin{aligned} \min \quad & c^T x + \max_{P \in \mathfrak{P}} \{\mathbb{E}_{\xi_P} [\mathcal{Q}_\omega(x)]\} \\ \text{s.t.} \quad & Ax \geq b \\ & x \in \{0,1\}^p \end{aligned} \tag{1.1}$$

$$\mathcal{Q}_\omega(x) := \min \quad g_\omega^T y_\omega \tag{1.2a}$$

$$\text{s.t.} \quad W_\omega y_\omega \geq r_\omega - T_\omega x \tag{1.2b}$$

$$y_\omega \in \{0,1\}^{q_1} \times \mathbb{R}^{q-q_1}. \tag{1.2c}$$

We assume that

1. $X := \{x : Ax \geq b, x \in \{0,1\}^p\}$ is non-empty,
2. $\mathcal{K}_\omega(x) := \{y_\omega : \text{(1.2b)-(1.2c) hold}\}$ is non-empty for all $x \in X$ and $\omega \in \Omega$,
3. $\mathcal{Q}_\omega(x) < \infty$ for all $x \in X$ and $\omega \in \Omega$ (relatively complete recourse)

# Wasserstein Ball

$$\left\{ v \in \mathbb{R}^{|\Omega|} : \sum_{i=1}^{|\Omega|} \sum_{j=1}^{|\Omega|} \|\omega^i - \omega^j\|_1 k_{i,j} \leq \epsilon, \sum_{j=1}^{|\Omega|} k_{i,j} = v_i, \qquad i = 1, \ldots, |\Omega| \right.$$

$$\sum_{i=1}^{|\Omega|} k_{i,j} = v_j^*, \qquad j = 1, \ldots, |\Omega| \qquad \sum_{i=1}^{|\Omega|} v_i = 1$$

$$v_i \geq 0, \qquad i = 1, \ldots, |\Omega|$$

$$\left. k_{i,j} \geq 0, \qquad i = 1, \ldots, |\Omega|, j = 1, \ldots, |\Omega| \right\}$$

# Moment Set

$$\left\{ \underline{u} \leq \sum_{l=1}^{|\Omega|} v_l f(\omega^l) \leq \overline{u},\ v \geq 0 \right\}$$

*f* (.) is some mapping of a sample vector to another vector. e.g., vector of monomials, etc.

# L-Shaped Formulation

$$\begin{aligned} \min \quad & c^T x + \theta \\ \text{s.t.} \quad & Ax \geq b \\ & \max_{P \in \mathfrak{P}} \left\{ \mathbb{E}_{\xi_P} \left[ \mathcal{Q}_\omega(x) \right] \right\} \leq \theta \end{aligned}$$

$$\Updownarrow$$

$$\begin{aligned} \min \quad & c^T x + \theta \\ \text{s.t.} \quad & Ax \geq b \\ & \mathbb{E}_{\xi_P} \left[ \mathcal{Q}_\omega(x) \right] \leq \theta \ , \ P \in \mathfrak{P} \end{aligned}$$

We want to stay in the space of *x* variables as much as we can.

# A Distributional Cutting Surface Algorithm (I): DR-TS-SLP

$$
\begin{aligned}
\min \quad & c^T x + \theta \\
\text{s.t.} \quad & Ax \geq b \\
& \mathbb{E}_{\xi_P}\left[\mathcal{Q}_\omega(x)\right] \leq \theta \;, \; P \in \mathfrak{P}
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & c^T x + \theta \\
\text{s.t.} \quad & Ax \geq b \\
& \mathbb{E}_{\xi_{P_1}}\left[\mathcal{Q}_\omega(x)\right] \leq \theta \\
& \mathbb{E}_{\xi_{P_2}}\left[\mathcal{Q}_\omega(x)\right] \leq \theta \\
& \quad \ldots \\
& \mathbb{E}_{\xi_{P_k}}\left[\mathcal{Q}_\omega(x)\right] \leq \theta
\end{aligned}
$$

This algorithm will converge in a finite number of iterations if the distributions used to generate “distributional cuts” are “finite”.

Each sub-problem may be solved using “outer linearization” as in the L-shaped method.

# Distribution Separation Problem

we assume that there exists an oracle that provides a probability distribution $P \in \mathfrak{P}$, i.e., $\{p_\omega\}_{\omega\in\Omega}$ where $p_\omega$ is the probability of occurrence of scenario $\omega \in \Omega$, by solving the optimization problem:

$$\max_{P\in\mathfrak{P}} \mathbb{E}_{\xi_P}[\mathcal{Q}_\omega(x)]$$

for a given $x \in X$.

**Moment matching set.**

$$\max_{v\in\mathbb{R}^{|\Omega|}} \left\{ \sum_{l=1}^{|\Omega|} v_l \mathcal{Q}_{\omega^l}(x) \;\middle|\; \underline{u} \le \sum_{l=1}^{|\Omega|} v_l f(\omega^l) \le \overline{u},\ v \ge 0 \right\}$$

**Kantorovich set.**

$$\max \left\{ \sum_{l=1}^{|\Omega|} v_l \mathcal{Q}_{\omega^l}(x) : v \in \mathfrak{P}_K \right\}.$$

The set describing the feasible distributions is a polytope, and an optimum is at its vertex.

# L-Shaped Method for DR-TS-SLP

For a given first stage solution $(x, \theta)$

Let $\pi^*_{\omega,0}(x) \in \mathbb{R}^{m_2}$ be the optimal dual corresponding to

$$\begin{aligned} \mathcal{Q}^s_\omega(x) := \min \ & g_\omega^T y_\omega \\ s.t. \ & W_\omega y_\omega \geq r_\omega - T_\omega x \\ & y_\omega \in \mathbb{R}^q_+. \end{aligned}$$

*optimality cut*

$$\sum_{\omega \in \Omega} p_\omega \left\{ \pi^*_{\omega,0}(x)^T \left(r_\omega - T_\omega x\right) \right\} \leq \theta,$$

where $\{p_\omega\}_{\omega \in \Omega}$ is obtained by solving the distribution separation problem associated to the ambiguity set $\mathfrak{P}$.

Note: In this approach, the distributions generating the cuts are not added explicitly.

We do exactly what we do in the L-shaped Method, but solve one additional linear program to determine the weights (probabilities) corresponding to each scenario.

# L-Shaped Method for DR-TS-Mixed Binary

First, we define subproblem $\mathcal{S}_\omega(x)$,

$$\mathcal{Q}^s_\omega(x) := \min \; g_\omega^T y_\omega$$
$$s.t. \; W_\omega y_\omega \geq r_\omega - T_\omega x$$
$$\alpha^t_\omega y_\omega \geq \beta^t_\omega - \psi^t_\omega x, \quad t = 1, \ldots, \tau_\omega$$
$$y_\omega \in \mathbb{R}^q_+,$$

where $\alpha^t_\omega \in \mathbb{Q}^q$, $\psi^t_\omega \in \mathbb{Q}^p$, and $\beta^t_\omega \in \mathbb{Q}$ are the coefficients of $y_\omega$, coefficients of $x$, right hand side, respectively, of a parametric inequality.

*optimality cut*

For a given

$$\sum_{\omega \in \Omega} p_\omega \left\{ \pi^*_{\omega,0}(x)^T \left(r_\omega - T_\omega x\right) + \sum_{t=1}^{\tau_\omega} \pi^*_{\omega,t}(x) \left(\beta^t_\omega - \psi^t_\omega x\right) \right\} \leq \theta.$$

Observation: The second stage polytope is (as it gets convexified) does not depend on the probability distribution.

# DR-TS-Mixed Binary Programs:
## DR version of Stochastic Server Location; DR-Multiple Knapsacks

DRSLP and DRMKP Instances

| Instance | Stage I | | | Stage II | | | | |
|---|---|---|---|---|---|---|---|---|
| | #Cons | #BinVar | #ContVar | #Cons | #BinVar | #ContVar | $\|\Omega\|$ | RandParam |
| DRSLP.5.25.50 | 1 | 5 | 0 | 30 | 125 | 5 | 50 | RHS |
| DRSLP.5.25.100 | 1 | 5 | 0 | 30 | 125 | 5 | 100 | RHS |
| DRSLP.10.50.50 | 1 | 10 | 0 | 60 | 500 | 10 | 50 | RHS |
| DRSLP.10.50.100 | 1 | 10 | 0 | 60 | 500 | 10 | 100 | RHS |
| DRSLP.10.50.500 | 1 | 10 | 0 | 60 | 500 | 10 | 500 | RHS |
| DRSLP.15.45.5 | 1 | 15 | 0 | 60 | 675 | 15 | 5 | RHS |
| DRSLP.15.45.10 | 1 | 15 | 0 | 60 | 675 | 15 | 10 | RHS |
| DRSLP.15.45.15 | 1 | 15 | 0 | 60 | 675 | 15 | 15 | RHS |
| DRMKP.1 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.2 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.3 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.4 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.5 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.6 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.7 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.8 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.9 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |
| DRMKP.10 | 50 | 240 | 0 | 5 | 120 | 0 | 20 | OBJ |

# DR-TS-Mixed Binary Programs: Full-Distribution Cuts versus L-Shaped on Wasserstein Ball Models

| Instance | L-Shaped $\epsilon = 5.0$ | | | Full-Dist. Cuts $\epsilon = 5.0$ | |
|---|---|---|---|---|---|
| | $z_{opt}$ | #DCs | $T$(s) | #DCs | $T$(s) |
| DRSLP.5.25.50 | 14.0 | 7 | 2.6 | 5 | 4.1 |
| DRSLP.5.25.100 | -40.0 | 10 | 7.3 | 7 | 21.2 |
| DRSLP.10.50.50 | -200.0 | 5 | 240.0 | 3 | 136.2 |
| DRSLP.10.50.100 | -237.0 | 16 | 656.1 | 7 | 712.9 |
| DRSLP.10.50.500 | -159.0 | 7 | 1151.6 | 3 | 611.9 |
| DRSLP.15.45.5 | -252.0 | 5 | 288.5 | 5 | 182.2 |
| DRSLP.15.45.10 | -220.0 | 7 | 518.4 | 5 | 772.5 |
| DRSLP.15.45.15 | -208.0 | 11 | 1203.2 | 4 | 584.0 |
| DRMKP.1 | 9686.0 | 10 | 285.4 | 10 | 243.2 |
| DRMKP.2 | 9388.0 | 9 | 906.0 | 10 | 878.4 |
| DRMKP.3 | 8844.0 | 10 | 1462.2 | 11 | 1345.1 |
| DRMKP.4 | 9237.0 | 23 | 2695.1 | 14 | 10800.0 |
| DRMKP.5 | 10024.0 | 9 | 1656.9 | 11 | 3732.5 |
| DRMKP.6 | 9515.0 | 9 | 257.3 | 10 | 225.0 |
| DRMKP.7 | 10003.0 | 9 | 434.4 | 10 | 386.7 |
| DRMKP.8 | 9427.0 | 28 | 4554.3 | 18 | 2910.4 |
| DRMKP.9 | 10038.0 | 10 | 1090.0 | 18 | 10800.0 |
| DRMKP.10 | 9082.2 | 13 | 4870.0 | 10 | 10800.0 |

Full-Distribution Cut version fails in 3-hours

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# DR-TS-Mixed Binary Programs: Full-Distribution Cuts versus L-Shaped on 3-Moment Models

| Instance | L-Shaped $CI = 80\%$ | | | Full-Dist. Cuts $CI = 80\%$ | | |
|---|---|---|---|---|---|---|
| | $z_{opt}$ | #DCs | $T$(s) | $z_{opt}$ | #DCs | $T$(s) |
| DRSLP.5.25.50 | -93.39 | 99 | 4.2 | -93.39 | 12 | 24.7 |
| DRSLP.5.25.100 | -107.73 | 108 | 10.1 | -107.73 | 11 | 32.8 |
| DRSLP.10.50.50 | -332.79 | 385 | 162.3 | -332.79 | 7 | 248.4 |
| DRSLP.10.50.100 | -325.03 | 472 | 413.6 | -325.03 | 8 | 466.8 |
| DRSLP.10.50.500 | -325.03 | 499 | 7234.2 | -325.03 | 23 | 10191 |
| DRSLP.15.45.5 | -255.03 | 30 | 272.6 | -255.03 | 6 | 63.2 |
| DRSLP.15.45.10 | -242.70 | 119 | 743.7 | -242.70 | 6 | 175.4 |
| DRSLP.15.45.15 | -237.05 | 347 | 650.1 | -237.05 | 8 | 588.7 |
| DRMKP.1 | 9418.71 | 13 | 289.3 | 9418.71 | 5 | 292.5 |
| DRMKP.2 | 9093.42 | 15 | 319.1 | 9093.42 | 6 | 622.5 |
| DRMKP.3 | 8619.42 | 19 | 389.4 | 8619.42 | 6 | 513.6 |
| DRMKP.4 | 8990.40 | 31 | 661.9 | 8990.40 | 6 | 724.0 |
| DRMKP.5 | 9503.67 | 15 | 510.1 | 9503.67 | 8 | 1188.1 |
| DRMKP.6 | 9204.78 | 18 | 486.0 | 9204.78 | 7 | 1129.1 |
| DRMKP.7 | 9709.79 | 13 | 684.0 | 9709.79 | 6 | 1397.0 |
| DRMKP.8 | 9199.72 | 40 | 1555.2 | 9199.72 | 6 | 1610.7 |
| DRMKP.9 | 9830.45 | 50 | 1298.0 | 9830.45 | 6 | 1621.4 |
| DRMKP.10 | 8864.22 | 49 | 4547.7 | 8864.22 | 6 | 3165.9 |

Full-Distribution Cut version fails in 3-hours

WRO + Machine Learning (Logistic Regression):

$$\min_{\theta} \max_{P \in \mathcal{P}} \mathbb{E}_P[h(\theta^{\mathsf{T}} \xi)].$$

# WR0-Logistic Regression

| | $h(\theta,\xi)$ | $\Xi$ | Master | Sep | Method |
|---|---|---|---|---|---|
| E&K (2015) | convex in $\theta$, concave in $\xi$ | convex compact | convex | convex | conjugate |
| S-A (2015) | loss function of log. reg. | $\mathbb{R}^k$ | convex | convex | closed form sol. |
| L&M (2017) | convex in $\theta$ and $\xi$ | convex compact | convex SIP | DC | central cutting-surface |

E&K: Esfahan and Kuhn (2015) S-A: Shafieezadeh-Abadeh et al. (2015)
L&M: Luo and Mehrotra (2017)

# WR0-Primal Conic Reformulation in Joint Probability Space

**Theorem**

*Let $\Theta$ and $\Xi$ be compact sets. The function $h(\cdot,\cdot)$ is bounded on $\Theta \times \Xi$. For every $\theta \in \Theta$, there exists a $C(\theta) > 0$ such that $|h(\theta, s_1) - h(\theta, s_2)| \le C(\theta) d(s_1, s_2)$, $\forall s_1, s_2 \in \Xi$. Then Wass-DRO can be reformulated as a conic linear program as follows:*

$$\min_{\theta \in \Theta} \max_{P \in \mathcal{P}} \mathbb{E}_{\xi \sim P}[h(\theta, \xi)]$$

$$\text{st.} \quad \mathcal{W}(P, P_0) \le r$$

$$\text{(WRO)}$$

**Equivalent to** ⟹

$$\min_{\theta \in \Theta} \max_{\mu} \int_{\Xi} h(\theta, s)\mu(ds \times \Xi)$$

$$\text{st.} \quad \mu(\Xi, \{\widehat{\xi}_i\}) = 1/m, \quad i \in [m]$$

$$\mu(\Xi \times \Xi^{m+1}) \ge 0$$

$$\sum_{i=1}^{m} \int_{\Xi} d(s, s^i)\mu(ds \times \{\widehat{\xi}_i\}) \le r$$

$$\mu \succeq 0 \qquad \text{(ConicLP)}$$

# WR0-Dual Reformulation: Decomposes by Scenario

**Theorem**

*Applying conic duality, (ConicLP) can be reformulated as the following semi-infinite program, and the duality gap is zero.*

$$\min_{\theta\in\Theta} \max_{\mu} \int_{\Xi} h(\theta, s)\mu(ds \times \Xi)$$

$$\text{st.} \quad \mu(\Xi, \{\hat{\xi}_i\}) = 1/m, \quad i \in [m]$$

$$\mu(\Xi \times \Xi^{m+1}) \geq 0$$

$$\sum_{i=1}^{m} \int_{\Xi} d(s, s^i)\mu(ds \times \{\hat{\xi}_i\}) \leq r$$

$$\mu \succeq 0 \qquad \text{(ConicLP)}$$

Dualization ⟹

$$\min_{\theta, v} \frac{1}{m} \sum_{i=1}^{m} v_i + r \cdot v_{m+1}$$

$$\text{st.} \ h(\theta, s) - v_i - v_{m+1} \cdot d(s, \hat{\xi}_i) \leq 0, \quad \forall s \in \Xi, \ i \in [m]$$

$$\theta \in \Theta, v_{m+1} \geq 0$$

$$\text{(WRO-dual)}$$

# WR0-Dual Reformulation as a Semi-infinite Program

Define the following functions:

$$f(x) := \frac{1}{m}\sum_{i=1}^{m} v_i + r_0 \cdot v_{m+1},$$

$$g_i(x, s) := h(\theta, s) - v_i - v_{m+1} \cdot d(s, \xi^i), \quad i \in [m].$$

The problem (WRO-dual) can be rewritten as:

$$\begin{aligned} &\min_x \; f(x) \\ &\text{st. } g_i(x, s) \leq 0, \forall s \in \Xi, i \in [m] \\ &\quad x \in X \end{aligned}$$

(SIP)

$$d(s) \leftarrow v_i + v_{m+1} \cdot d(s, \xi^i),$$

$$g(s) \leftarrow h_\theta(l(\theta, s)) - d(s).$$

Logistic Reg. $\log\left(1 + \exp\left[-y(\theta_0 + \theta^T x)\right]\right)$

# Separation Oracle for a Cutting Surface Algorithm

$$\max_{s\in\Xi} g_i(\widetilde{x}, s) := h(\widetilde{\theta}, s) - \widetilde{v}_i - \widetilde{v}_{m+1} \cdot d(s, \widehat{\xi}_i)$$

- The separation problem is equivalent to the following unconstrained DC optimization:

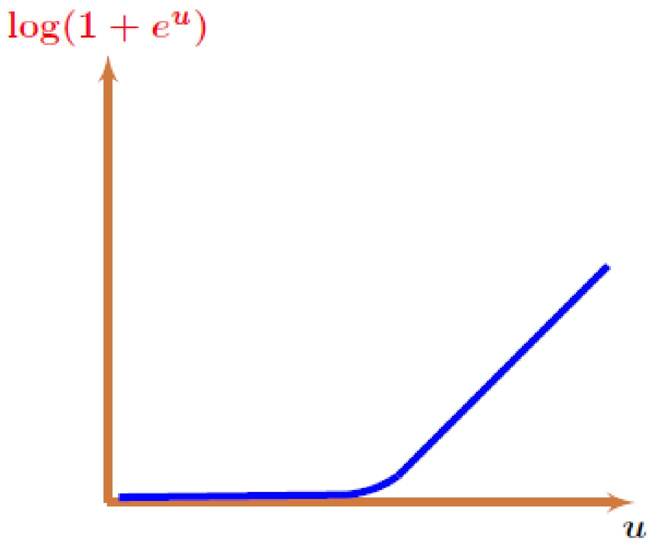
$$\max_{u} \psi(u) := h_\theta(u) - \phi(u),$$

  where

$$\phi(u) = \min_{s\in\Xi} d(s), \qquad \text{s.t. } u = l(\widetilde{\theta}, s).$$

- Assumption: $\Xi$ is a polytope, and the metric $d(s_1, s_1) := \|s_1 - s_2\|_1$ is the 1-norm.
- $\phi(u)$ becomes a univariate piecewise-linear convex function.
- $h_\theta(u)$ is an univariate convex function $\Longrightarrow$ piecewise-linear approximation.
- Subproblem induced by each linear piece is convex optimization.

# WRO-Logistic Regression Separation Problem




$$h_\theta(u) = \log(1 + e^u)$$

$$h_\theta(u) \to u, \quad \text{as} \quad u \to \infty$$

$$h_\theta(u) \to 0, \quad \text{as} \quad u \to -\infty$$

$$h''_\theta(u) = \mathcal{O}(e^{-|u|}).$$

**Theorem**

*For the distributionally robust logistic regression (DRLR) model with (univariate) logistic loss function $h_\theta(u) = log(1 + e^u)$, the separation problem can be solved in at most $O\left(\frac{1}{\sqrt{\varepsilon}} loglog \frac{L}{\varepsilon}\right)$ iterations, where $L := u_{ub} - u_{lb}$.*

# Performance Results: Implementation

- All algorithms are Implemented in C++.
- Master Problem: twice-differentiable convex program $\rightarrow$ Interior Point Method (Ipopt: Wächter and Biegler, 2006).
- Separation Problem: DC optimization $\rightarrow$ Sequence of Parametric Linear Programs (Cplex).

# Data: UCI Repository

## Data sets for numerical study

- Select 11 data sets from UCI machine learning repository.
- Training sample size: 50, 75, 100, 150.
- Candidate Wasserstein radius $r = 0, 0.01, 0.05, 0.1, 0.5, 1$.
- Each experiment is repeated 100 times.

| Data set | Area | No. Attrib. | No. Observ. |
|---|---|---|---|
| BA | Finance | 4 | 1372 |
| VC | Health care | 6 | 310 |
| PID | Health care | 8 | 768 |
| BCW | Health care | 9 | 699 |
| ST-H | Health care | 13 | 270 |
| EES | Health care | 14 | 14980 |
| SPT-H | Health care | 22 | 267 |
| ION | Aerospace | 34 | 351 |
| SPTF-H | Health care | 44 | 267 |
| SPAM | Computer | 57 | 4601 |
| CB | Aerospace | 60 | 208 |

# Performance: Out of Sample Predictability

Compare the mean AUC value between WRLR and LR in 44 cases. With $\alpha = 0.05$, WRLR is better in 55% cases; LR is better in 16% cases; No significant difference in the remaining 29% cases.

| Dataset | $m$ | LR AUC | WRLR AUC | Rel. Diff | p-value |
|---|---|---|---|---|---|
| BCW | 50 | .9716 | .9916 | .7040 | .0000 |
| | 75 | .9773 | .9886 | .4954 | .0000 |
| | 100 | .9790 | .9940 | .7122 | .0000 |
| | 150 | .9889 | .9945 | .5049 | .0000 |
| ST-H | 50 | .8317 | .8808 | .2914 | .0000 |
| | 75 | .8504 | .8903 | .2664 | .0000 |
| | 100 | .8945 | .9064 | .1133 | .0000 |
| | 150 | .8986 | .8990 | .0042 | .4319 |
| ION | 50 | .8429 | .8708 | .1775 | .0000 |
| | 75 | .8582 | .8919 | .2381 | .0000 |
| | 100 | .8606 | .8967 | .2584 | .0000 |
| | 150 | .8715 | .9006 | .2264 | .0000 |

# Performance: Out of Sample Loss Function

| | Non-regularized | | $l_1$-regularized | |
|---|---|---|---|---|
| data set | LR mean loss | WRLR mean loss | LR mean loss | WRLR mean loss |
| BA | 0.0454 | 0.0534 | 0.0689 | 0.0786 |
| VC | 0.3079 | 0.3091 | 0.3150 | 0.3286 |
| PID | 0.5288 | 0.5153 | 0.5184 | 0.5153 |
| BCW | 0.2775 | 0.1110 | 0.1008 | 0.0994 |
| ST-H | 0.4162 | 0.3791 | 0.3970 | 0.3855 |
| EES | 0.6863 | 0.6685 | 0.6747 | 0.6650 |
| SPT-H | 0.8240 | 0.4162 | 0.3831 | 0.3821 |
| ION | 2.7404 | 0.4239 | 0.3656 | 0.3346 |
| SPTF-H | 0.9275 | 0.3787 | 0.4044 | 0.3965 |
| SPAM | 3.5443 | 0.7297 | 0.3679 | 0.3346 |
| CB | 5.4601 | 0.8451 | 0.5058 | 0.4413 |

# Performance: Computational

## Computational performance of solving WR-LogReg

- Number of calls to the master problem: $4 \sim 40$.
- Approximately $2m \sim 20m$ cutting surfaces are added. ($m$: number of training samples)

| Dataset | $m$ | Iters. | Cuts | CPU [sec] | Master (%) | Sep. (%) |
|---|---|---|---|---|---|---|
| BA | 50 | 3.8 | 66.9 | 1.21 | 13.74 | 86.26 |
| | 75 | 4.3 | 90.8 | 0.86 | 17.96 | 82.04 |
| | 100 | 3.9 | 116.7 | 1.83 | 13.89 | 86.11 |
| | 150 | 4.6 | 157.5 | 2.30 | 14.42 | 85.58 |
| BCW | 50 | 8.6 | 251.8 | 6.19 | 31.44 | 68.56 |
| | 75 | 9.4 | 284 | 7.88 | 27.48 | 72.52 |
| | 100 | 8.9 | 501.4 | 12.84 | 34.28 | 65.72 |
| | 150 | 9.6 | 786.1 | 27.31 | 41.79 | 58.21 |
| SPT-H | 50 | 21.5 | 938.8 | 38.87 | 88.91 | 11.09 |
| | 75 | 24.5 | 1031.2 | 53.43 | 83.40 | 16.60 |
| | 100 | 19.4 | 1122.5 | 63.08 | 83.44 | 16.56 |
| | 150 | 13.4 | 1384 | 49.70 | 77.91 | 22.09 |