Measurement schemes for quantum field theory in curved spacetimes

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Motivation and goals

AQFT is founded on the idea of local observables but little has been said about how they would actually be measured.

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Aim: Develop QMT for AQFT, taking measurement theory out of Hilbert space and putting it back in spacetime, where it belongs.

NB We are not attempting to solve the measurement problem, but rather to describe a step in the measurement chain.

Scenario and questions

A QFT (system) is coupled to another QFT (probe) in a compact spacetime region K. The probe is measured elsewhere.

Example: Unruh-deWitt detector – coupling along/near a worldline. Usually a QM system is used, but a QFT will do just as well as a probe, and lives more naturally on spacetime.

An account of traditional UdW is coming soon.

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Questions

- How can the probe measurement be described in terms of local system observables?
- In particular, what can be said about their localisation? Is it:
  - where the probe is measured?
  - where the coupling is located?
  - a combination of the two?
  - something else?

E.g., what is the minimal localisation, as a QFT observable, of a measurement of a UdW probe?

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Questions

- How should the state be updated after a selective/nonselective measurement?
  - Should it change across a surface in spacetime?
  - If so, what surface?
    Backward lightcone of coupling region? Or of probe?
    cf Hellwig and Kraus $\sim$ 1970
- How are multiple measurements handled?
  E.g. ambiguity of order if spacelike separated.

Framework: AQFT in CST

To each glob. hyp. spacetime $\boldsymbol{M}$, there is a unital $*$-algebra $\mathcal{A}(\boldsymbol{M})$. Enough to consider some fixed $\boldsymbol{M}$ and its subregions.

To each open causally convex subregion N of $\boldsymbol{M}$ there is a morphism

$$\alpha_{\boldsymbol{M};\boldsymbol{N}} : \mathcal{A}(\boldsymbol{N}) \to \mathcal{A}(\boldsymbol{M}) \qquad \boldsymbol{N} = \boldsymbol{M}|_N$$

Write $\mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; N) = \operatorname{Im} \alpha_{\boldsymbol{M};\boldsymbol{N}}$ for observables localisable in N.

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Isotony $\alpha_{\boldsymbol{M}_1;\boldsymbol{M}_2} \circ \alpha_{\boldsymbol{M}_2;\boldsymbol{M}_3} = \alpha_{\boldsymbol{M}_1;\boldsymbol{M}_3}$

Timeslice If N contains a Cauchy surface of $\boldsymbol{M}$ then $\alpha_{\boldsymbol{M};\boldsymbol{N}}$ is an iso.

Einstein If N_1 and N_2 are causally disjoint,

$$[\mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; N_1), \mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; N_2)] = 0$$

Haag $$\bigcap_{N \subset K^\perp} \mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; N)^c \subset \bigcap_{L \supset K} \mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; L) \qquad \text{(connected } L\text{)}$$

NB A given observable may be localisable in many distinct regions.

Coupled system

Consider two QFTs, $\mathcal{A}$, α (system) and $\mathcal{B}$, β (probe).

Their uncoupled combination is $\mathcal{A} \otimes \mathcal{B}$, $\alpha \otimes \beta$.

A coupling of $\mathcal{A}$ and $\mathcal{B}$ in K is a theory $\mathcal{C}$, γ
s.t. for each $L \subset K^\perp$ there are interwining isomorphisms

$$\chi_L : \mathcal{A}(\boldsymbol{L}) \otimes \mathcal{B}(\boldsymbol{L}) \to \mathcal{C}(\boldsymbol{L})$$

$$\begin{array}{ccc} \mathcal{A}(\boldsymbol{L}') \otimes \mathcal{B}(\boldsymbol{L}') & \xrightarrow{\alpha_{\boldsymbol{L};\boldsymbol{L}'} \otimes \beta_{\boldsymbol{L};\boldsymbol{L}'}} & \mathcal{A}(\boldsymbol{L}) \otimes \mathcal{B}(\boldsymbol{L}) \\ \downarrow \chi_{\boldsymbol{L}'} & & \downarrow \chi_{\boldsymbol{L}} \\ \mathcal{C}(\boldsymbol{L}') & \xrightarrow{\gamma_{\boldsymbol{L};\boldsymbol{L}'}} & \mathcal{C}(\boldsymbol{L}) \end{array}$$

Diagram commutes for all $\boldsymbol{L}' \subset \boldsymbol{L}$.

Advanced/retarded response, scattering

Define natural in/out regions $M^{-/+} = \boldsymbol{M} \setminus J^{\pm}(K)$, writing morphisms $\alpha^{\pm} = \alpha_{\boldsymbol{M};\boldsymbol{M}^{\pm}}$, $\chi^{\pm} = \chi_{\boldsymbol{M}^{\pm}}$ etc.

Advanced/retarded response $\rho^{-/+} : \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$

$$\rho^{\pm} = \chi^{\pm} \circ (\alpha^{\pm} \otimes \beta^{\pm})^{-1}$$

are isomorphisms providing identifications of the uncoupled and coupled combinations at early/late times.

Scattering morphism Θ, automorphism of $\mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M})$,

$$\Theta = (\rho^{-})^{-1} \circ \rho^{+}$$

(cf relative Cauchy evolution etc).
All constructions are geometrically natural.

Measurement scheme: prepare early, measure late

Describe measurements of $\mathcal{C}(\boldsymbol{M})$ in uncoupled language.

An observable $\widetilde{B} := \rho^+(\mathbf{1} \otimes B)$ tests probe d.o.f. at late times.

Fixing a probe preparation state σ and system state ω, the state

$$\underset{\sim}{\omega}_\sigma = ((\rho^-)^{-1})^*(\omega \otimes \sigma)$$

of $\mathcal{C}(\boldsymbol{M})$ is uncorrelated at early times.

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Measurement of $\widetilde{B}$ in state $\underset{\sim}{\omega}_\sigma$ gives expectation value

$$\underset{\sim}{\omega}_\sigma(\widetilde{B}) = (\omega \otimes \sigma)(\Theta(\mathbf{1} \otimes B)) = \omega(\eta_\sigma(\Theta(\mathbf{1} \otimes B)))$$

where $\eta_\sigma : \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ linearly extends $A \otimes B \mapsto \sigma(B)A$.

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Definition $\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B))$ is the induced system observable corresponding to probe observable B.

Induced system observables

By construction, $\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B))$ obeys

$$\omega(\varepsilon_\sigma(B)) = \underset{\sim}{\omega}_\sigma(\widetilde{B})$$

for probe observable B.

- In QMT language, $(\mathcal{C}, \gamma, \sigma)$ is a measurement scheme for the system observables $\varepsilon_\sigma(B) \in \mathcal{A}(\boldsymbol{M})$ $(B \in \mathcal{B}(\boldsymbol{M}))$
- $\varepsilon_\sigma : \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ is completely positive and linear, but in general is neither injective nor a homomorphism
- In particular, if B is a projection describing a sharp measurement then $\varepsilon_\sigma(B)$ is generally an unsharp observable

$$\varepsilon_\sigma(B)^2 \leq \varepsilon_\sigma(B).$$

Localisation

The general QFT assumptions imply

$$\Theta \circ (\alpha_{\boldsymbol{M};\boldsymbol{L}} \otimes \beta_{\boldsymbol{M};\boldsymbol{L}}) = \alpha_{\boldsymbol{M};\boldsymbol{L}} \otimes \beta_{\boldsymbol{M};\boldsymbol{L}} \qquad \text{for any } L \subset K^{\perp}.$$

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Theorem (a) If $B \in \mathcal{B}^{\mathrm{kin}}(\boldsymbol{M}; L)$ with $L \subset K^{\perp}$ then

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(b) If $A \in \mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then, for any B,

$$[\varepsilon_\sigma(B), A] = [\eta_\sigma(\Theta(\mathbf{1} \otimes B)), A] = \eta_\sigma(\Theta[\mathbf{1} \otimes B, A \otimes \mathbf{1}]) = 0$$

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Corollary By the Haag property,

$$\varepsilon_\sigma(B) \in \mathcal{A}^{\mathrm{kin}}(\boldsymbol{M}; L) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}),$$

where L is any open connected causally convex set containing K.

Localisation (ctd)

The results so far answer the first two general questions

- Any probe observable B corresponds to a system observable $\varepsilon_\sigma(B)$
- $\varepsilon_\sigma(B)$ can be localised in any open connected causally convex region that contains K (and necessarily also contains its causal hull $J^+(K) \cap J^-(K)$).
- The location of the probe observation is irrelevant.

Also: probe observables causally separated from K induce trivial system observables.

Example Measurements of CMB photons correspond to local observables of the quantised gravitational field at the surface of last scattering.

Effect-valued measures

Recall: an effect is an observable s.t. B and $\mathbf{1} - B$ are positive, i.e., $0 \leq B \leq \mathbf{1}$. The effect is a measurement with possible values 0 or 1,

$$\sigma(B) = \mathrm{Prob}(B = 1 \mid \sigma) \qquad (= \mathrm{Prob}(B \mid \sigma))$$

In QMT, observables are effect-valued measures (EVMs)

$$\mathsf{E} : \mathcal{X} \to \mathrm{Effects}(\mathcal{B}(\boldsymbol{M}))$$

for σ-algebra $\mathcal{X}$, with the interpretation

$$\sigma(\mathsf{E}(X)) = \text{probability a result in range } X \text{ is observed in state } \sigma$$

Each such observable of the probe induces an observable $\varepsilon_\sigma \circ \mathsf{E}$ of the system (generally unsharp).

Joint unsharp measurements

In elementary QM one learns that only commuting observables can be measured simultaneously.

Modern QMT: noncommnuting observables can be measured jointly, provided one accepts a joint unsharp measurement.

Our measurement schemes provide a nice illustration and natural construction.

Joint unsharp measurements (ctd)

Consider probe EVMs E_i with causally disjoint localisation.

$$[\mathsf{E}_1(X_1), \mathsf{E}_2(X_2)] = 0, \qquad \forall\ X_i \in \mathcal{X}_i,$$

so (temporarily assuming a C^*-setting) there is a joint EVM

$$\mathsf{E}(X_1 \times X_2) := \mathsf{E}_1(X_1)\mathsf{E}_2(X_2)$$

whose marginals are the E_i

$$\mathsf{E}(X_1 \times \Omega_2) = \mathsf{E}_1(X_1), \qquad \mathsf{E}(\Omega_1 \times X_2) = \mathsf{E}_2(X_2)$$

and which is sharp if the E_i are.

Then $\varepsilon_\sigma \circ \mathsf{E}$ is a joint observable for the $\varepsilon_\sigma \circ \mathsf{E}_i$, and must be a joint unsharp measurement if the $\varepsilon_\sigma \circ \mathsf{E}_i$ are incompatible.

Joint unsharp measurements (ctd)

Significance: resolves the tension between

- the freedom of observers at spacelike separation to make sharp measurements
- the impossibility of jointly and sharply measuring incompatible system observables.

Post-selection and pre-instruments

Suppose a probe-effect B is tested when the system state is ω. We can derive an expression for the post-selected system state, conditioned on the effect being observed.

Post-selection and pre-instruments

Consider a system EVM $\mathsf{E} : \mathcal{X} \to \mathrm{Effects}(\mathcal{A}(\boldsymbol{M}))$

Probability of a joint successful measurement of $\mathsf{E}(X)$ and B is

$$\mathrm{Prob}(\mathsf{E}(X)\&B) = \omega(\eta_\sigma(\Theta(\mathsf{E}(X) \otimes B)))$$

so $$\mathrm{Prob}(\mathsf{E}(X)|B) = \frac{\mathrm{Prob}(\mathsf{E}(X)\&B)}{\mathrm{Prob}(B)} = \frac{(\mathcal{I}_\sigma(B)(\omega))(\mathsf{E}(X))}{(\mathcal{I}_\sigma(B)(\omega))(\mathbf{1})},$$

where $$(\mathcal{I}_\sigma(B)(\omega))(A) := (\omega \otimes \sigma)(\Theta(A \otimes B)).$$

Call $\mathcal{I}_\sigma(B) : \mathcal{A}(\boldsymbol{M})^*_+ \to \mathcal{A}(\boldsymbol{M})^*_+$ a pre-instrument.

If defined, the normalized post-selected state, conditioned on B, is

$$\omega' = \frac{\mathcal{I}_\sigma(B)(\omega)}{(\mathcal{I}_\sigma(B)(\omega))(\mathbf{1})}.$$

Instruments and non-selective measurement

If E is a probe EVM then

$$X \mapsto \mathcal{I}_\sigma(\mathsf{E}(X))$$

is an instrument in the sense of Davies and Lewis, i.e., a measure valued in CP maps on $\mathcal{A}(\boldsymbol{M})^*$.

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A non-selective measurement of the EVM is a post-selection on $\mathsf{E}(\Omega) = \mathbf{1}$. The corresponding updated state

$$\omega'_{\mathrm{ns}}(A) = \mathcal{I}_\sigma(\mathbf{1})(\omega)(A) = (\omega \otimes \sigma)(\Theta(A \otimes \mathbf{1}))$$

is simply the partial trace of $\Theta^*(\omega \otimes \sigma)$

- independent of the specific probe EVM
- depends only on the coupling
- if A is localisable in $K^\perp$ then $\omega'_{\mathrm{ns}}(A) = \omega(A)$.

Locality and post-selection

The pre-instrument may be rewritten

$$\mathcal{I}_\sigma(B)(\omega)(A) = \omega(\eta_\sigma(\Theta(A \otimes B))) = \omega(A\varepsilon_\sigma(B))$$

so also

$$\omega'(A) = \frac{\omega(A\varepsilon_\sigma(B))}{\omega(\varepsilon_\sigma(B))}.$$

Theorem $\omega'(A) = \omega(A)$ iff A is uncorrelated with $\varepsilon_\sigma(B)$ in ω.

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Equality or otherwise of expectation values is not determined by the localisation region of A. E.g., if ω has a Reeh–Schlieder property, and A can be localised in $K^\perp$ then

$$\omega'(A) = \omega(A) \implies \varepsilon_\sigma(B) = \omega(\varepsilon_\sigma(B))\mathbf{1}$$

Post-selection on any nontrivial measurement alters expectation values in $K^\perp$ [and the rest of $\boldsymbol{M}$]

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There seems no good reason to declare that ω changes to ω' across a surface in $\boldsymbol{M}$.

Successive measurement of two probes

For $i = 1, 2$ consider $\mathcal{B}_i$ with coupling regions K_i and scattering morphisms Θ_i.

Also consider $\mathcal{B}_1 \otimes \mathcal{B}_2$ as a combined probe with coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Suppose $K_2 \cap J^-(K_1) = \emptyset$, so K_2 is later than K_1 according to some observers and assume Bogoliubov's formula

$$\hat{\Theta} = \hat{\Theta}_1 \circ \hat{\Theta}_2, \quad \text{where } \hat{\Theta}_1 = \Theta_1 \otimes_3 \mathrm{id} \quad \text{and } \hat{\Theta}_2 = \Theta_2 \otimes_2 \mathrm{id}$$

Theorem Coherence of successive measurement

$$\mathcal{J}_{\sigma_2}(B_2) \circ \mathcal{J}_{\sigma_1}(B_1) = \mathcal{J}_{\sigma_1 \otimes \sigma_2}(B_1 \otimes B_2)$$

Post-selection on B_1 and then B_2 agrees with post-selection on $B_1 \otimes B_2$.

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Corollary If K_1 and K_2 are causally disjoint,

$$\mathcal{I}_{\sigma_2}(B_2) \circ \mathcal{I}_{\sigma_1}(B_1) = \mathcal{I}_{\sigma_1 \otimes \sigma_2}(B_1 \otimes B_2) = \mathcal{I}_{\sigma_1}(B_1) \circ \mathcal{I}_{\sigma_2}(B_2)$$

Summary so far

General questions have been answered:

- induced local observables localised near coupling region
- derivation of post-selected states
- no need to posit state change across surfaces
- successive measurements are coherent even when order is ambiguous

Now turn to a specific model in which induced observables can be computed.

Probe model

Two free scalar fields: Φ (system) and Ψ (probe) coupled via an interaction term

$$\mathcal{L}_{\text{int}} = -\rho\Phi\Psi, \qquad \rho \in C_0^\infty(M), \qquad K = \operatorname{supp}\rho.$$

Uncoupled and coupled field equations:

$$T_0\Xi = \begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix}\begin{pmatrix} \Phi \\ \Psi \end{pmatrix} = 0, \qquad T\Xi = \begin{pmatrix} P & R \\ R & Q \end{pmatrix}\begin{pmatrix} \Phi \\ \Psi \end{pmatrix} = 0$$

where $\qquad P = \Box + m_\Phi^2, \qquad Q = \Box + m_\Psi^2, \qquad R\Phi = \rho\Phi.$

At least for sufficiently weak coupling, T and T_0 are Green-hyperbolic with Green operators $E_{T_0}^\pm$, $E_T^\pm$.

Probe model: quantization

Use T_0 and T and Green operators to quantize as usual

- algebras $\mathcal{C}_0(\boldsymbol{M})$ and $\mathcal{C}(\boldsymbol{M})$
  generators $\Xi_0(F)$, $\Xi(F)$, $F \in C_0^\infty(\boldsymbol{M}; \mathbb{C}^2)$
- $\mathcal{C}_0(\boldsymbol{M}) = \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M})$
  where $\mathcal{A}$ and $\mathcal{B}$ are the KG theories for Φ and Ψ

$$\Xi_0(f \oplus h) = \Phi(f) \otimes \mathbf{1} + \mathbf{1} \otimes \Psi(h)$$

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Scattering morphism ($=$ relative Cauchy evolution) is determined by

$$\Theta(\Xi_0(F)) = \Xi_0(F - (T - T_0)E_T^- F)$$

for all $F \in C_0^\infty(M^+; \mathbb{C}^2)$. Recall: 'out' region is $M^+ = M \setminus J^-(K)$.
Bogoliubov formula holds for composite probes.

Induced system observables $\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B))$

Start with probe observable $\Psi(h)$, with $h \in C_0^\infty(M^+)$. Then

$$\Theta(\mathbf{1} \otimes \Psi(h)) = \Phi(f^-) \otimes \mathbf{1} + \mathbf{1} \otimes \Psi(h^-),$$

$$\begin{pmatrix} f^- \\ h^- \end{pmatrix} = \begin{pmatrix} 0 \\ h \end{pmatrix} - \begin{pmatrix} 0 & R \\ R & 0 \end{pmatrix} E_T^- \begin{pmatrix} 0 \\ h \end{pmatrix}$$

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Θ is a homomorphism, so we obtain formal power series identities

$$\Theta(\mathbf{1} \otimes e^{i\Psi(h)}) = e^{i\Phi(f^-)} \otimes e^{i\Psi(h^-)}$$

$$\varepsilon_\sigma(e^{i\Psi(h)}) = \sigma\left(e^{i\Psi(h^-)}\right) e^{i\Phi(f^-)} = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

if σ is quasifree with two-point function S.

Example $\qquad \varepsilon_\sigma(\Psi(h)^2) = \Phi(f^-)^2 + S(h^-, h^-)\mathbf{1}$

Localisation of induced observables

$$\varepsilon_\sigma(e^{i\Psi(h)}) = \sigma\left(e^{i\Psi(h^-)}\right) e^{i\Phi(f^-)} = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

Localisation of $\varepsilon_\sigma(\Psi(h)^n)$ can be expressed in terms of f^-,

$$f^- = -(E_T^-(0 \oplus h))_2 \rho$$

So the induced observables in this case may be localised in any open causally convex neighbourhood of

$$S = \mathrm{supp}\, \rho \cap J^-(\mathrm{supp}\, h)$$

Q. Can we do better than this? Localisation in S?

Ans: Not in any useful way. E.g., may calculate

$$[\varepsilon_\sigma(\Psi(h)), \varepsilon_\sigma(\Psi(h'))] = [\Phi(f^-), \Phi(f'^-)] = iE_P\left(f^-, f'^-\right)\mathbf{1}$$

The RHS senses the geometry everywhere in the causal hull of $U = \operatorname{supp} f^- \cup \operatorname{supp} f'^-$, where

$$\mathsf{CausalHull}(U) = J^+(U) \cap J^-(U)$$

can be as large as $\mathsf{CausalHull}(\operatorname{supp}\rho)$, for suitable h and h'.

The compatibility of the induced observables cannot be decided in terms of the geometry of (subsets of) $\operatorname{supp}\rho$ alone.

Induced observables have properties that are not local to $\operatorname{supp}\rho$.

Minimal localisation for eternal UdW probe is the full right wedge

Deformed product on the probe system

The map ε_σ is not a homomorphism.
However, we can define a deformed product

$$e^{i\Psi(h)} \star e^{i\Psi(h')} = e^{S_{\text{sym}}(h^-,h'^-)-iE_P(f^-,f'^-)/2} e^{i\Psi(h+h')}$$

in which ε_σ is a homomorphism (though not injective).

This allows the partial representation of the system in the probe algebra.

E.g., $\Psi(h)$'s do not necessarily $\star$-commute at spacelike separation,

$$[\Psi(h), \Psi(h')]_\star = iE_P(f^-, f'^-),$$

showing how long-range correlations can be created.

Weak coupling

If σ has vanishing-one-point function,

$$\epsilon_\sigma(\Psi(h)^*\Psi(h)) = \Phi(f^-)^*\Phi(f^-) + \sigma(\Psi(h^-)^*\Psi(h^-))\mathbf{1}$$

Replacing ρ by $\lambda\rho$ and using a Born expansion,

$$\omega(\epsilon_\sigma(\Psi(h)^*\Psi(h))) = S(\overline{h}, h) + \lambda^2 \left(W(\overline{h_1}, h_1) + 2\mathrm{Re}\, S(\overline{h}, h_2) \right) + O(\lambda^4)$$

where S and W are the two-point functions of σ and ω, and

$$h_1 = \rho E_Q^- h, \qquad h_2 = \rho E_P^- \rho E_Q^- h$$

Lowest order term: noise from spontaneous excitation of the probe.

Weak coupling ctd

If σ is a Fock state and $\Psi(h)$ is (approx.) an annihilation operator,

- $\Psi(h)^*\Psi(h)$ is a number operator, up to normalisation, and
- the terms in S (approx.) vanish.

$$\omega(\epsilon_\sigma(N_h)) = \lambda^2 W(\overline{h_1}, h_1) + O(\lambda^4)$$

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In a limit where ρ is concentrated on timelike curve γ, so that $\rho(F) = (\gamma^* F)(\tilde{\rho})$ for some smooth compactly supported $\tilde{\rho}$,

$$\omega(\epsilon_\sigma(N_h)) = \lambda^2((\gamma \times \gamma)^* W)(\tilde{\rho}\gamma^*\overline{h_1}, \tilde{\rho}\gamma^* h_1) + O(\lambda^4)$$

for $\rho(F) = (\gamma^* F)(\tilde{\rho})$, replicating the first order perturbation result.

Summary

- Operational framework of QMT adapted to AQFT
- Probe observables induce local system observables
- Localisation in the causal hull of coupling region
- Post-selected states, coherence under successive measurements
- No need to invoke state change across a surface
- Computation of induced observables for specific model
- Agreement with first order perturbation theory at weak coupling