BPS state counting in K3 string theories

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Banff, November 1, 2017

Based on

arXiv 1702.05095

in collaboration with:

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Introduction

- String models with 16 supersymmetries arising from type IIA/B on $K3 \times T^2$ and their orbifolds (CHL models)

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- Goal: Calculate space-time index counting 1/4 BPS states for each value of electric-magnetic charges
- Some of the first examples of matching BH black hole entropy vs microscopic degeneracy [Strominger, Vafa 95; Dijkgraaf, Verlinde^2 96 Shih, Strominger, Yin 2005; David, Sen 2006; ...]

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- All these indices organized into a generating function $1/\Phi$
- Meromorphic Siegel modular form of genus 2
- For type IIA/K3$\times T^2$, Φ is Igusa cusp form of weight 10

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  2-dim superconformal field theory
- Elliptic genus of K3 $\phi(\tau, z)$
- If NLSM on K3 has a symmetry g, we can define a ***twining genus*** ϕ_g
- ϕ_g is weak Jacobi form wt 0 ind 1

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- some ϕ_g cannot be computed directly in the NLSM
- Idea: use consistency conditions on $1/\Phi$ from wall-crossing
- Result: for almost all g, ϕ_g is uniquely determined, otherwise only 2 possibilities

Overview

Introduction

NLSM on K3

Strings on $K3 \times T^2$

CHL models

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Basic facts about NLSM on K3

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  [Aspinwall, Morrison '95; Nahm, Wendland '99]
- Elliptic genus of K3:

$$\phi_{K3}(\tau, z) = \mathrm{Tr}_{RR}(q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}} y^{J_0} (-1)^{J_0 + \bar{J}_0})$$

where $(\tau, z) \in \mathbb{H} \times \mathbb{C}$, $q = e^{2\pi i \tau}$, $y = e^{2\pi i z}$

$J_0, \bar{J}_0$ are generators in $su(2)$ in $\mathcal{N} = 4$
[Schellekens, Warner '86; Witten '87;

Eguchi, Ooguri, Taormina, Yang '88]

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- ※ $\phi(\tau, z) = \sum_{n \geq 0, l} c(n, l) q^n y^l$

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- For each $g \in G$, define twining genus

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- Classification: at most 81 independent twining genera ϕ_g

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Can we find all of them?

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$$\mathcal{H}_{r,s} := \{v \text{ in } g^r\text{-twisted sector} \quad \text{s.t. } g(v) = e^{\frac{2\pi i s}{N}} v\}$$

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※ $\phi_{g^n} = \sum_{s\in\mathbb{Z}_N} e^{\frac{2\pi i n s}{N}} \phi_{r,s}$

Strings on $K3 \times T^2$

Strings on K3

$$\text{het}/T^6 \quad \leftrightarrow \quad \text{IIA}/K3 \times T^2 \quad \leftrightarrow \quad \text{IIB}/K3 \times T^2$$

※ 4-dim $\mathcal{N} = 4$ with generic gauge group $U(1)^{28}$

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[Sen '94; Witten '94]

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※ 134-dim moduli space

1/4 BPS index

※ For each (Q, P) consider 1/4 BPS index $D(Q, P)$

$$D(Q,P) = \#\{\text{‘bosonic’ supermultiplets}\} - \#\{\text{‘fermionic’ supermultiplets}\}$$

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- Goal: Find $d(n, m, l)$ for all $n, m, l \in \mathbb{Z}$

1/4 BPS index

- Organize into a generating function

$$\frac{1}{\Phi\left(\begin{smallmatrix}\sigma & z\\ z & \tau\end{smallmatrix}\right)} = \sum_{n,m,l} d(n,m,l)e^{2\pi i(m\sigma+n\tau+lz)}$$

for some complex ‘chemical potentials’ σ, τ, z

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※ Given by exp-lift of elliptic genus

$$\frac{1}{\Phi\left(\begin{smallmatrix}\sigma & z\\ z & \tau\end{smallmatrix}\right)} = \prod_{n,m\geq 0,l} \left(1 - e^{2\pi i(m\sigma + n\tau + lz)}\right)^{-c(mn,l)}$$

($l < 0$ if $m = n = 0$)

where $c(mn, l)$ are Fourier coeffs of ϕ_{K3}

[Dijkgraaf, Verlinde, Verlinde '96; Shih, Strominger, Yin 2005; David, Sen 2006]

Wall crossing

- Some 1/4 BPS states can decay into pair of 1/2 BPS in subregions of the moduli space

  $\Rightarrow D(Q, P)$ 'jumps' across wall of marginal stability [Bergman '97; Bergman, Kol '98; Denef '00]

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$$d(n,m,l) = \oint_{\mathcal{C}} \frac{e^{-2\pi i(m\sigma+n\tau+lz)}}{\Phi\left(\begin{smallmatrix}\sigma & z\\ z & \tau\end{smallmatrix}\right)}$$

where

$$\mathcal{C} := \{0 \leq \sigma_1, \tau_1, z_1 \leq 1,\ \left(\begin{smallmatrix}\sigma_2 & z_2\\ z_2 & \tau_2\end{smallmatrix}\right) \text{ fixed}\}$$

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- $1/\Phi$ is meromorphic $\Rightarrow d(n,m,l)$ ambiguous

Wall crossing

Contour prescription: $\mathcal{C}$ depends on moduli and charges

$$\mathcal{C} := \{0 \leq \sigma_1, \tau_1, z_1 \leq 1, \begin{pmatrix} \sigma_2 & z_2 \\ z_2 & \tau_2 \end{pmatrix} = \epsilon^{-1} \mathcal{Z}(Q, P, \mu)\}$$

where $\epsilon \ll 1$ and $\mathcal{Z}$ is the 'central charge vector'

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- Poles of $1/\Phi$ exactly at walls of marginal stability
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- Independent interpretations

  [Banerjee, Sen, Srinistava 2008; Bossard, Cosnier-Horeau, Pioline 2016]

CHL models

CHL model definition

- Consider type II/$K3 \times S^1 \times \tilde{S}^1$

[Chaudhuri,Hockney,Lykken 95; Sen,Vafa 95]

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- Consider type II/$K3 \times S^1 \times \tilde{S}^1$
- Orbifold by (δ, g), where
  - g is symmetry of NLSM on K3, $ord(g) = N$, preserving $\mathcal{N} = 4$ SUSY
  - δ is a shift by $1/N$ period along S^1

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Obtain 4-d $\mathcal{N} = 4$ theory with

- reduced gauge group $U(1)^d$ ($6 \leq d \leq 28$)
- reduced moduli space
- e-m charges $(Q, P) \in \Lambda_e \oplus \Lambda_m$ ($rk\, \Lambda_{e,m} = d$)
- Duality group $\supset O(\Lambda_e)$

[Chaudhuri,Hockney,Lykken 95; Sen,Vafa 95]

1/4 BPS states

※ 1/4 BPS index depends on $O(\Lambda_e)$-invariants

$$D_g(P,Q) = (-1)^{P\cdot Q+1} d_g(P^2/2, Q^2/2, P\cdot Q)$$

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- Generating function $1/\Phi_g$ of $d_g(n,m,l)$ is exponential lift of twining genera [Jatkar,Sen 05]

$$\frac{1}{\Phi_g\left(\begin{smallmatrix}\sigma & z\\ z & \tau\end{smallmatrix}\right)} = \prod_{(n,m,l)} \left(1-e^{2\pi i(m\sigma+\frac{n\tau}{N}+lz)}\right)^{-c^{(g)}_{n,m}(\frac{nm}{N},l)}$$

$c^{(g)}_{n,m}(\frac{nm}{N},l)$: linear combinations of Fourier coeffs of ϕ_{g^d}, for $d|N$, and their $SL(2,\mathbb{Z})$ transformations

Wall crossing in CHL models

- Similar wall-crossing for ‘decaying’ of 1/4 BPS to pair of 1/2 BPS states [Sen 2007]

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Main assumption: Contour prescription provides the correct $d_g(n,m,l)$ at all points in moduli space

$\Rightarrow 1/\Phi_g$ has only poles at the walls

How to compute the remaining $1/\Phi_g$?

- We can determine walls of marginal stability

[Paquette, Volpato, Zimet 2017]

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- ⁜ By contour prescription $\to$ poles of $1/\Phi_g$
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This info + modularity + q^0 Fourier coeffs

$\downarrow$

ϕ_g and $1/\Phi_g$ determined almost completely (either uniquely or up to 2 possibilities)

[Paquette, Volpato, Zimet 2017]

Open questions

- Can all $1/\Phi_g$ be determined exactly?
- Can $1/\Phi_g$ be fixed without passing through NLSM(K3)?
- Relations to Umbral + Conway moonshine