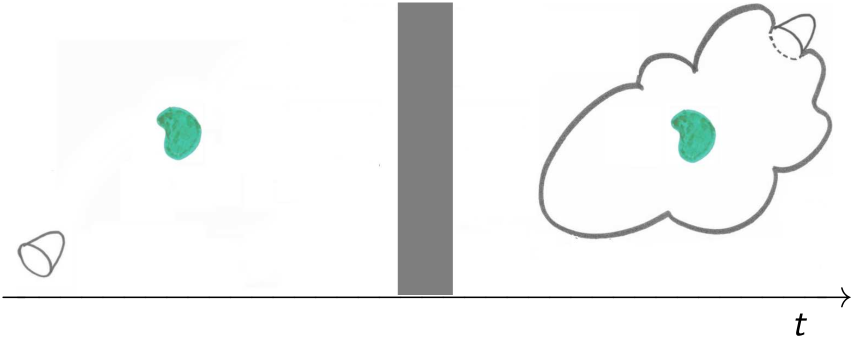
An algebraic approach to light-matter interactions

Ivan Fernandez-Corbaton

Institute of Nanotechnology, Karlsruhe Institute of Technology

Karlsruhe Institute of Technology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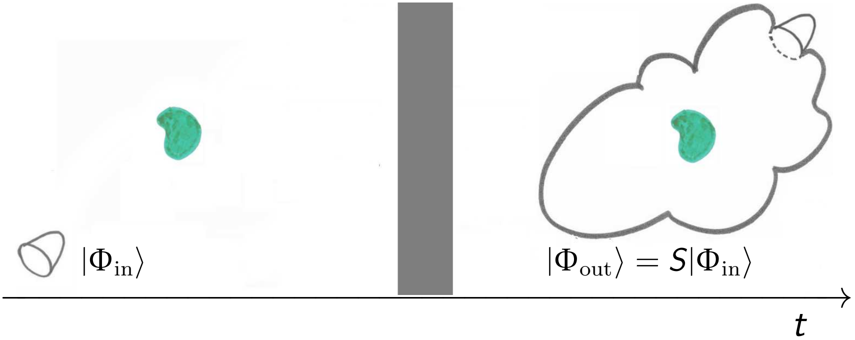


Algebraic approach to light-matter interactions:

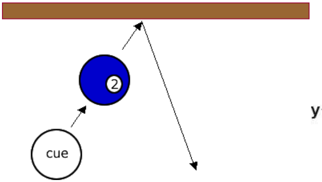
- Incoming field $|\Phi_{\text{in}}\rangle \in \mathbb{M}^{in}$
  - Vector space of **incoming** solutions of Maxwell's equations
- Outgoing field $|\Phi_{\text{out}}\rangle \in \mathbb{M}^{out}$
  - Vector space of **outgoing** solutions of Maxwell's equations
- Effect of the object represented by scattering operator S
  - Linear and bounded map between incoming and outgoing fields

- Easy to use symmetries and light-matter conservation laws
  - Symmetry results are general
  - "(Breaking of) Symmetry X is needed for effect Y"
  - Provide design guidelines
- Algebraic tools ease the solution of complicated problems
  - Given an object, find the best beam to exert force on it
  - Find eigenvalues of matrices
- Algebraic $\Longrightarrow$ very well suited for computer implementation

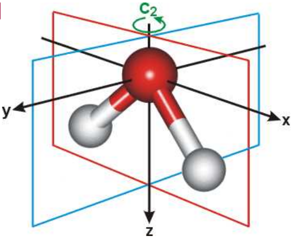
Allows straightforward use of **symmetry arguments**.

Symmetry

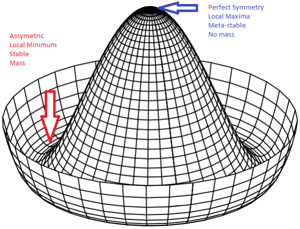
Beautifully general concept, central in theoretical physics ...



Big

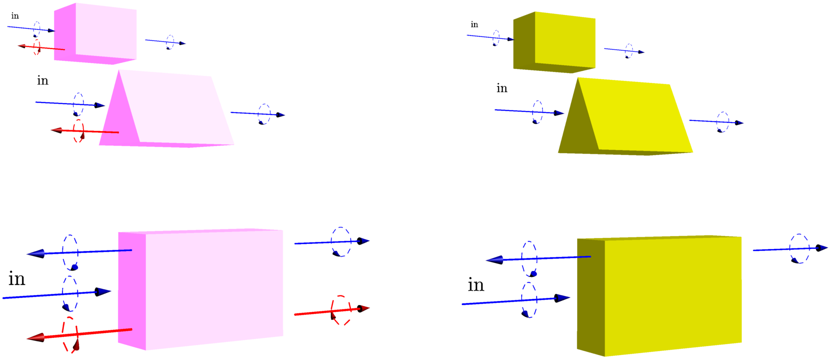


Small

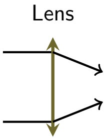


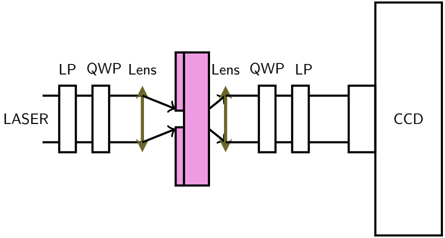
Very Small

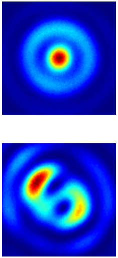
How can symmetry help in the study of light-matter interactions?



Why does zero back scattering happen?







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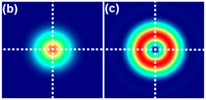
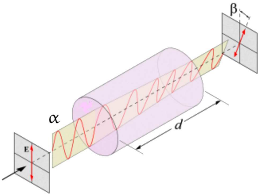
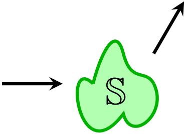
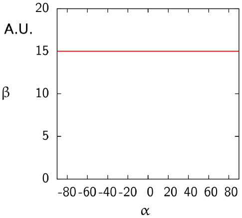


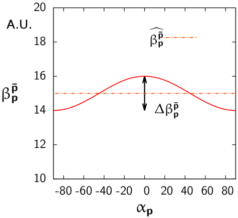
Why do optical vortices appear?

Solution of chiral molecules

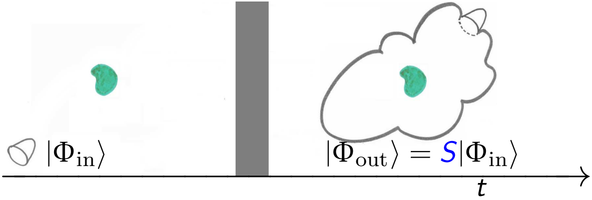








What is the origin of this difference?



- Symmetry is the invariance upon transformations
- Translations, rotations, electromagnetic duality, time translation, parity, time inversion, ...
- Transformations represented by unitary operators in $\mathbb{M}^{in(out)}$
- Symmetry of S under transformation X means $XSX^{-1} = S$

$$XSX^{-1} = S \iff SX - XS = [S, X] = 0$$

$$\text{If } X|\Phi_{\mathrm{in}}\rangle = x|\Phi_{\mathrm{in}}\rangle \implies X|\Phi_{\mathrm{out}}\rangle = x|\Phi_{\mathrm{out}}\rangle$$

For continuous symmetries

$$X_\theta = \exp(-i\theta\Gamma) \text{ with } \Gamma^\dagger = \Gamma$$

$$[S, X_\theta] = 0 \iff [S, \Gamma] = 0$$

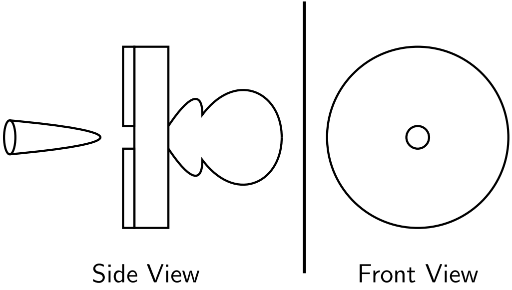
$$\text{If } \Gamma|\Phi_{\rm in}\rangle = \gamma|\Phi_{\rm in}\rangle \implies \Gamma|\Phi_{\rm out}\rangle = \gamma|\Phi_{\rm out}\rangle$$

In light-matter interactions:

- Symmetry: Non-coupling eigenstates with different eigenvalue
- Broken symmetry: Coupling eigenstates with different eigenvalue

Generator Γ	Transformation $\exp(-i\theta\Gamma)$	Expression
Linear momentum $\mathbf{P}$	Spatial translations	$-i\nabla$
Hamiltonian H	Time translations	$i\partial_t$
Angular momentum $\mathbf{J}$	Rotations	$-i\mathbf{r} \times \nabla - i\epsilon_{knm}$
Helicity Λ	Electromagnetic duality	$\frac{\nabla\times}{k}$

Example 1: Angular momentum and Rotational symmetry



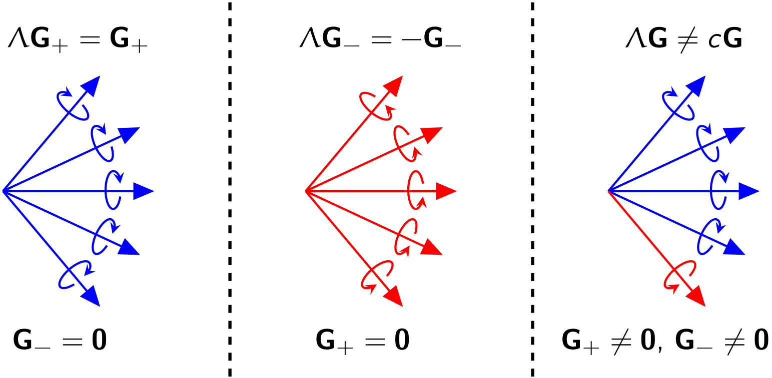
- If the input is an eigenstate $J_z|\Phi_{\text{in}}\rangle = m|\Phi_{\text{in}}\rangle$,
- so is the output: $J_z|\Phi_{\text{out}}\rangle = m|\Phi_{\text{out}}\rangle$
- **with the same eigenvalue**

Example 2: Helicity and Duality symmetry

- Operator:

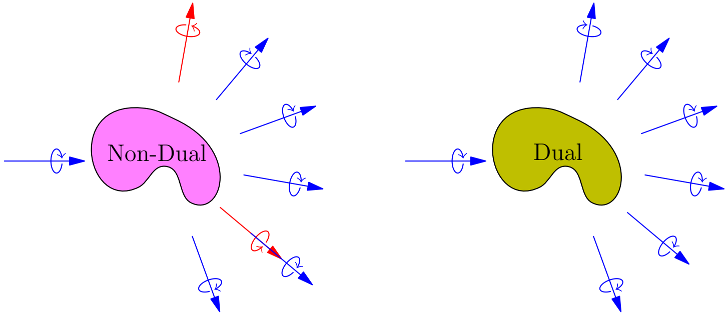
$$\Lambda = \frac{\mathbf{J} \cdot \mathbf{P}}{|\mathbf{P}|} \equiv \frac{\nabla \times}{k}$$

- Eigenstates: $\Lambda \left(\mathbf{E} \pm iZ\mathbf{H} \right) = \pm \left(\mathbf{E} \pm iZ\mathbf{H} \right) = \pm \mathbf{G}_{\pm}$
- Interpretation in the plane wave decomposition



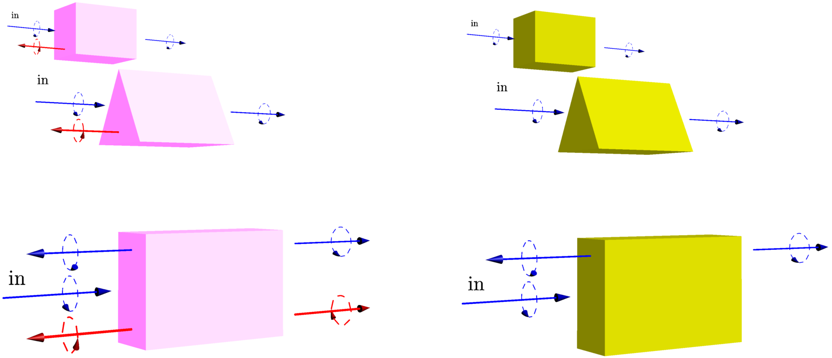
Electromagnetic duality transformation: $D(\theta) = \exp(-i\theta\Lambda)$

$$\mathbf{G}_\pm \to \exp(\mp i\theta)\,\mathbf{G}_\pm$$
$$\mathbf{E} \to \mathbf{E}\cos\theta - Z\mathbf{H}\sin\theta$$
$$Z\mathbf{H} \to \mathbf{E}\sin\theta + Z\mathbf{H}\cos\theta$$



Coupling

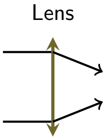
Non-coupling

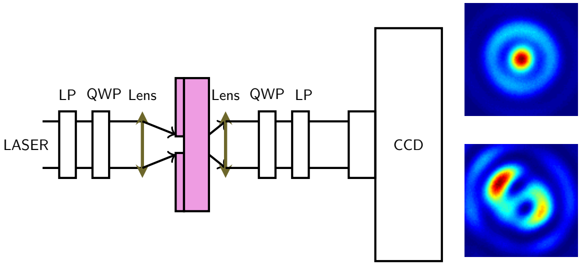


Why does zero back scattering happen?

Duality symmetry and discrete rotational symmetry $n \geqslant 3$

- Prisms on the left are not dual symmetric
- Prisms on the right are dual symmetric
- Common behavior due to discrete rotational symmetry





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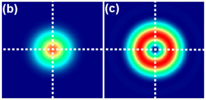
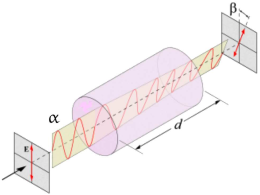
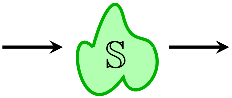


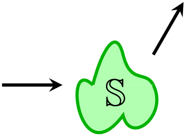
Why do optical vortices appear?

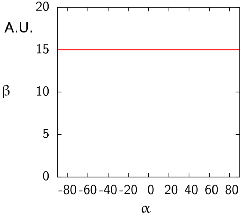
- Focusing: Breaking of transverse translational symmetry
- Scattering: Breaking of duality symmetry

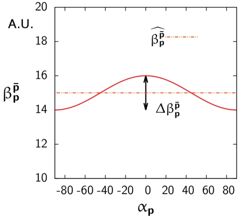
Solution of chiral molecules











What is the origin of this difference?

- Lack of duality symmetry of the sample
- **Chirality is necessary but not sufficient for optical activity**

Algebraic approach: Joint light-matter conservation laws

Properties like energy, momentum, angular momentum, etc ..
are conserved in the **joint system of fields+matter**.

During the light-matter interaction

- Field looses(gains) momentum $\Longrightarrow$ Matter gains(looses) it
- This results in optical force

Algebraic approach: Joint light-matter conservation laws

Properties like energy, momentum, angular momentum, etc ..
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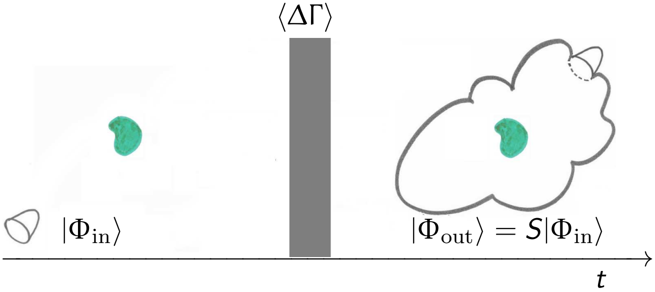
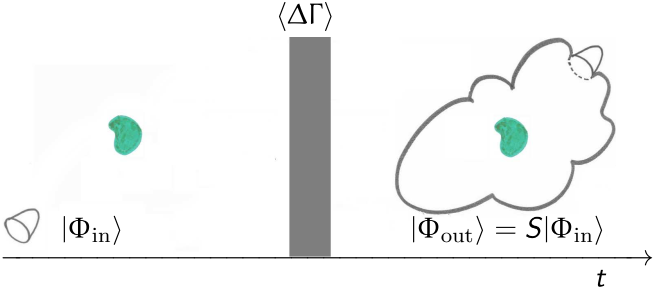
During the light-matter interaction

- Field looses(gains) angular momentum $\Longrightarrow$ Matter gains(looses) it
- This results in optical torque

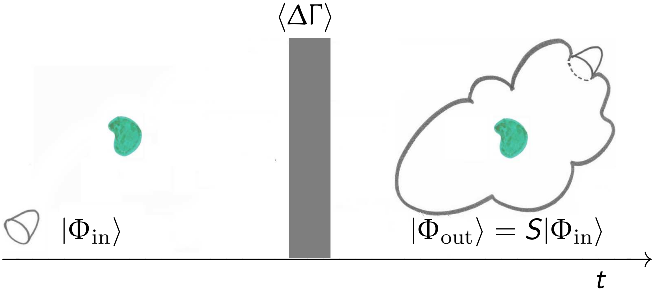


Γ is a given property (momentum, helicity, energy ...)

1. There is some light-matter interaction
2. Γ changes for the field
3. There is a corresponding change of Γ in the matter
4. Objective: **Quantify such change**



1. Measure property Γ in incoming field
2. Measure property Γ in outgoing field
3. Difference is the amount of Γ gained(lost) by the matter



1. **Measure** property Γ in incoming field
2. **Measure** property Γ in outgoing field
3. The difference must be the amount of Γ gained by the matter

Γ represented by a Hermitian operator
Measure of Γ in $|\Psi\rangle$:
scalar product between $|\Psi\rangle$ and $\Gamma|\Psi\rangle$: $\langle \Psi|\Gamma|\Psi\rangle$

$$\boxed{\langle \Delta\Gamma \rangle = \langle \Phi_{\text{in}}|\Gamma|\Phi_{\text{in}}\rangle - \langle \Phi_{\text{out}}|\Gamma|\Phi_{\text{out}}\rangle = \langle \Phi_{\text{in}}|\Gamma - S^{\dagger}\Gamma S|\Phi_{\text{in}}\rangle}$$

Introduce basis (e.g. multipoles, plane waves, ...).
Convenient choice: $\Gamma|\eta\ \gamma\rangle^{\mathrm{in(out)}} = \gamma|\eta\ \gamma\rangle^{\mathrm{in(out)}}$

$$|\Phi_{\mathrm{in}}\rangle = \sum_{\eta,\gamma} \alpha_\gamma^\eta |\eta\ \gamma\rangle^{\mathrm{in}},\ |\Phi_{\mathrm{out}}\rangle = \sum_{\eta,\gamma} \beta_\gamma^\eta |\eta\ \gamma\rangle^{\mathrm{out}}$$

Incoming and outgoing fields represented by colum vectors

$$\underline{\alpha} \equiv \begin{bmatrix} \vdots \\ \alpha_\gamma^\eta \\ \vdots \end{bmatrix},\ \underline{\beta} \equiv \begin{bmatrix} \vdots \\ \beta_\gamma^\eta \\ \vdots \end{bmatrix}$$

Operators S and Γ are represented by matrices

$$\underline{\underline{S}}\,(\eta\gamma,\bar{\eta}\bar{\gamma}) = {}^{\mathrm{out}}\langle\bar{\gamma}\,\bar{\eta}|S|\eta\,\gamma\rangle^{\mathrm{in}},\ \underline{\underline{\Gamma}}\,(\eta\gamma,\bar{\eta}\bar{\gamma}) = {}^{\mathrm{in(out)}}\langle\bar{\gamma}\,\bar{\eta}|\Gamma|\eta\,\gamma\rangle^{\mathrm{in(out)}} = \gamma\delta_{\gamma\bar{\gamma}}$$

In practice:

- Need to compute $\underline{\underline{S}}$
- Compute T-matrix: $\underline{\underline{S}} = \underline{\underline{I}} + 2\underline{\underline{T}}$
- There are publicly available T-matrix computer codes

Abstract operations become vector/matrix algebra in a computer

$$\langle \Delta\Gamma \rangle = \langle \Phi_{\rm in} | \Gamma - S^\dagger \Gamma S | \Phi_{\rm in} \rangle \implies \langle \Delta\Gamma \rangle = \underline{\alpha}^\dagger \left(\underline{\underline{\Gamma}} - \underline{\underline{S}}^\dagger \underline{\underline{\Gamma}}\, \underline{\underline{S}} \right) \underline{\alpha},$$

and complicated things become simple.
Measure the incoming energy

$$E_{\rm in} = \underline{\alpha}^\dagger \underline{\underline{H}}\, \underline{\alpha}$$

The expression

$$\frac{\underline{\alpha}^\dagger \left(\underline{\underline{\Gamma}} - \underline{\underline{S}}^\dagger \underline{\underline{\Gamma}}\, \underline{\underline{S}} \right) \underline{\alpha}}{\underline{\alpha}^\dagger \underline{\underline{H}}\, \underline{\alpha}}$$

is the transfer of $\langle \Delta\Gamma \rangle$ per Joule of energy of the incoming field

$$\frac{\underline{\alpha}^\dagger \left(\underline{\underline{\Gamma}} - \underline{\underline{S}}^\dagger \underline{\underline{\Gamma}}\,\underline{\underline{S}}\right) \underline{\alpha}}{\underline{\alpha}^\dagger \underline{\underline{H}}\,\underline{\alpha}}$$

Q: **Most efficient beam to transfer Γ to a given object ?**

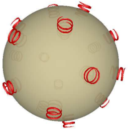
$$\max_{\underline{\alpha}} \frac{\underline{\alpha}^\dagger \left(\underline{\underline{\Gamma}} - \underline{\underline{S}}^\dagger \underline{\underline{\Gamma}}\,\underline{\underline{S}}\right) \underline{\alpha}}{\underline{\alpha}^\dagger \underline{\underline{H}}\,\underline{\alpha}}$$

Known solution using generalized eigenproblem ($\underline{\underline{A}}\,\underline{v} = \lambda \underline{\underline{B}}\,\underline{v}$):

- Optimal beam $\underline{\alpha}_{\max}$:
  Eigenvector corresponding to maximum generalized eigenvalue of $\left(\underline{\underline{\Gamma}} - \underline{\underline{S}}^\dagger \underline{\underline{\Gamma}}\,\underline{\underline{S}}\right)$ and $\underline{\underline{H}}$
- Optimal transfer efficiency: Maximum generalized eigenvalue

- Multipolar decomposition / angular spectrum of $\underline{\alpha}_{\max}$
- Absolute bound for Γ transfer

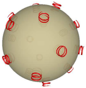
Example with a challenging object:



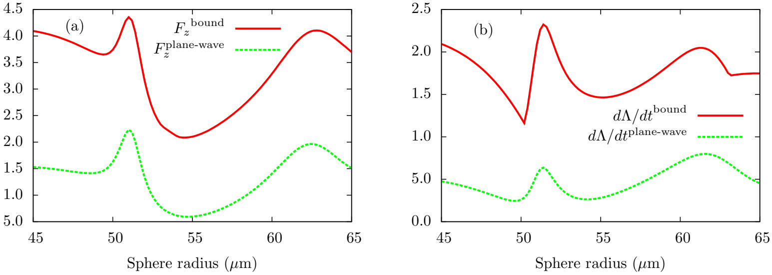
- CdSe sphere radius $\approx$ 55 μm
- 20 helical objects in an icosahedral arrangement

Incoming beams:

- LCP plane wave at 222.8 μm
  - Object not electromagnetically small
- Optimal monochromatic beam at 222.8 μm



Force per incoming Watt (10^{-9} N/W) | $d\Lambda/dt$ per incoming Watt (10^{-13} N·m/W)



Also useful for theoretical considerations

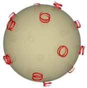
$$\boxed{\langle \Delta\Gamma \rangle = \langle \Phi_{\rm in} | \Gamma - S^\dagger \Gamma S | \Phi_{\rm in} \rangle}$$

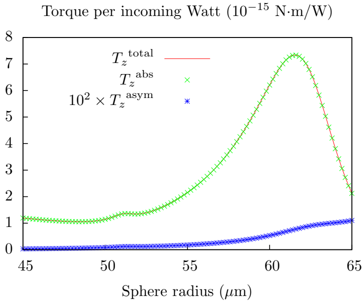
$$\begin{aligned} \langle \Delta\Gamma \rangle &= \langle \Phi_{\rm in} | \frac{A\Gamma + \Gamma A}{2} | \Phi_{\rm in} \rangle + \langle \Phi_{\rm in} | \frac{S^\dagger [S, \Gamma] + [S, \Gamma]^\dagger S}{2} | \Phi_{\rm in} \rangle \\ &= \langle \Delta\Gamma \rangle_{\rm absorption} + \langle \Delta\Gamma \rangle_{\rm asymmetry}. \end{aligned}$$

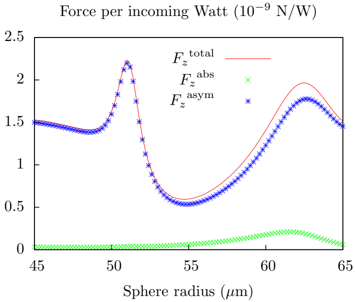
Symmetry $\iff$ $[S, \Gamma] = 0$: Non-coupling of Γ eigenstates
$\langle \Delta\Gamma \rangle_{\rm absorption}$: $A = I - S^\dagger S$ is the non-unitary part of S

$$\langle \Phi_{\rm in} | \Phi_{\rm in} \rangle - \langle \Phi_{\rm out} | \Phi_{\rm out} \rangle = \langle \Phi_{\rm in} | \Phi_{\rm in} \rangle - \langle \Phi_{\rm in} | S^\dagger S | \Phi_{\rm in} \rangle = \langle \Phi_{\rm in} | I - S^\dagger S | \Phi_{\rm in} \rangle$$

Absorption can be responsible for $\langle \Delta\Gamma \rangle \neq 0$ even if $[S, \Gamma] = 0$




Torque per incoming Watt (10^{-15} N·m/W)
T_z^{total}
T_z^{abs}
$10^2 \times T_z^{\text{asym}}$
0
1
2
3
4
5
6
7
8
45
50
55
60
65
Sphere radius (μm)


Force per incoming Watt (10^{-9} N/W)
F_z^{total}
F_z^{abs}
F_z^{asym}
0
0.5
1
1.5
2
2.5
45
50
55
60
65
Sphere radius (μm)

Algebraic approach to light-matter interactions

- Easy to use symmetries and light-matter conservation laws
- Algebraic tools ease the solution of complicated problems
- Algebraic $\implies$ very well suited for computer implementation
- Single Hilbert space $\mathbb{M}$: Classical fields or single photon
- Should be extensible to multiphoton states of light:
  $\mathbb{M} \otimes \mathbb{M} \otimes \dots$

Algebraic approach

[1] I.F-C. Helicity and duality symmetry in light matter interactions: Theory and applications. *Doctoral Thesis, Macquarie University*, 2014.

[2] I.F-C, and Carsten Rockstuhl. A unified theory to describe and engineer conservation laws in light-matter interactions. *Phys. Rev. A*, Accepted (2017).

Optical Activity

[1] I.F-C, X. Vidal, N. Tischler, and G. Molina-Terriza. Necessary symmetry conditions for the rotation of light. *J. Chem. Phys*, 138(21), 2013.

[2] X. Vidal, I.F-C, A. Barbara, and G. Molina-Terriza. Polarization properties of light scattered off solutions of chiral molecules in non-forward direction. *App. Phys. Lett. 107, 211107 (2015).*

[3] I.F-C, M. Fruhnert, and C. Rockstuhl. Dual and Chiral Objects for Optical Activity in General Scattering Directions. *ACS Photonics*, 2(3), 2015.

Zero backscattering

[1] I.F-C. Forward and backward helicity scattering coefficients for systems with discrete rotational symmetry. *Optics Express*, 21, Dec. 2013.

Symmetries in Optical Vortices

[1] I.F-C, X. Zambrana-Puyalto, and G. Molina-Terriza. Helicity and angular momentum: A symmetry-based framework for the study of light-matter interactions. *Phys. Rev. A*, 86, Oct. 2012.

[2] I.F-C et al. Electromagnetic duality symmetry and helicity conservation for the macroscopic maxwell's equations. *Phys. Rev. Lett.*, 111, Aug. 2013.

I am looking for a tenured(-track) position**Thank you for your time !**