Directed complexes.

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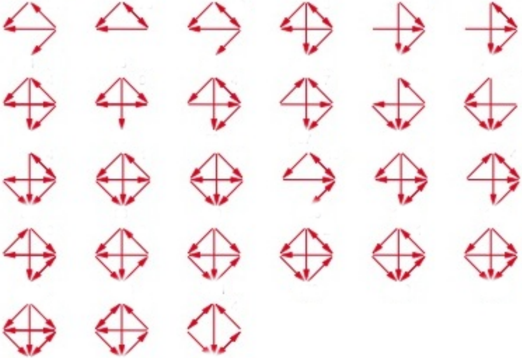
Joint work with Blue Brain Project and EPFL.

The Blue Brain Project

Neuron's connectivity patterns in rat brains

Overexpressed three-neuron connectivity patterns



Overexpressed four-neuron connectivity patterns

Abstract simplicial complex...

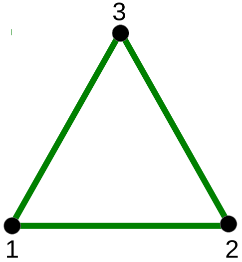
1. ... is a collection of sets $\mathcal{K}$, such that for every $A \in \mathcal{K}$ and for every $B \subset A$, $B \in \mathcal{K}$.

$$\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$$

$$\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,2,3\}\}$$

Abstract simplicial complex...

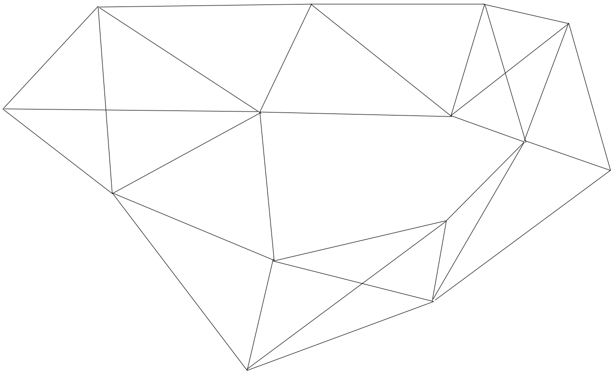
1. ... is a collection of sets $\mathcal{K}$, such that for every $A \in \mathcal{K}$ and for every $B \subset A$, $B \in \mathcal{K}$.
2. Elements $1, 2, 3$ are called vertices.



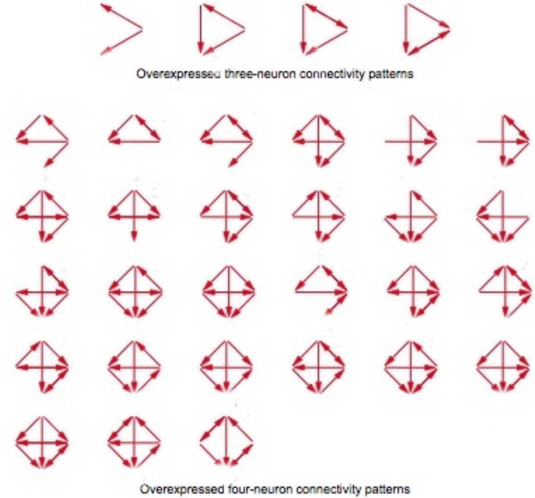
$$\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$$

Flag complexes.

1. G – undirected graph $\to$ abstract simplicial complex.



Connectivity patterns in rat brains



Abstract directed complex.

1. Abstract directed simplicial complex $\mathcal{K}$ is a collection of sequences such that for every ordered set $A \in \mathcal{K}$ every B, subsequence of A belongs to $\mathcal{K}$.

$$\{(1),(2),(3),(1,2),(1,3),(2,3),(1,2,3)\} \ - \ \textit{yes}$$

$$\{(1),(2),(3),(1,2),(3,1),(2,3),(1,2,3)\} \ - \ \textit{no}$$

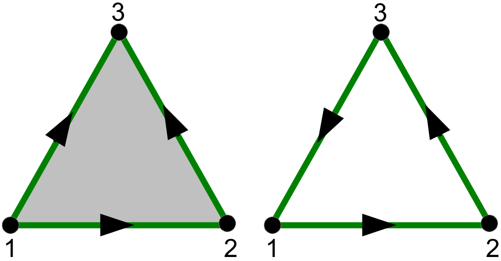
$$\{(1),(2),(3),(1,2),(3,1),(2,3)\} \ - \ \textit{yes}$$

2. Simply speaking, abstract directed complex is a set of ordered sets (sequences) of vertices.

Abstract directed complex.

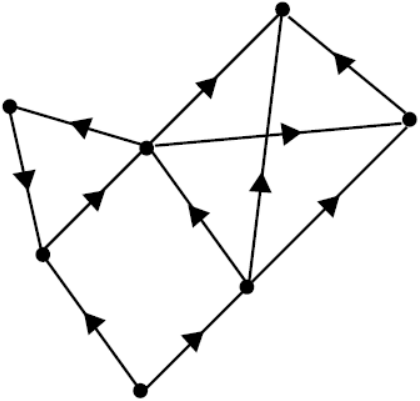
$$\{(1), (2), (3), (1, 2), (1, 3), (2, 3), (1, 2, 3)\}$$

$$\{(1), (2), (3), (1, 2), (3, 1), (2, 3)\}$$

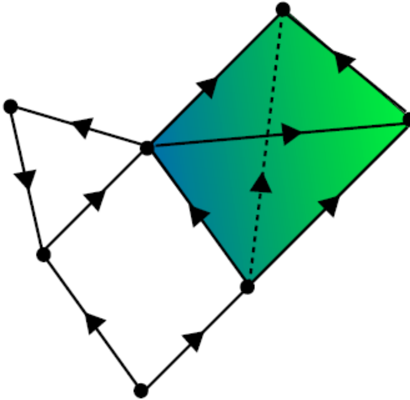


Note that every simplex have a source and a sink.

Directed flag complex.



oriented graph



oriented flag complex

A vague message.

1. Simplices and directed simplices are a kind of ultimate motifs in the network (one cannot have anything more connected than that).
2. Betti numbers and Euler characteristic are meta-motifs. We do not understand what is their precise meaning, but they give a low dimensional characteristic of (organized) network.
3. Directed simplices are the basic information flow units (that can be assembled together).

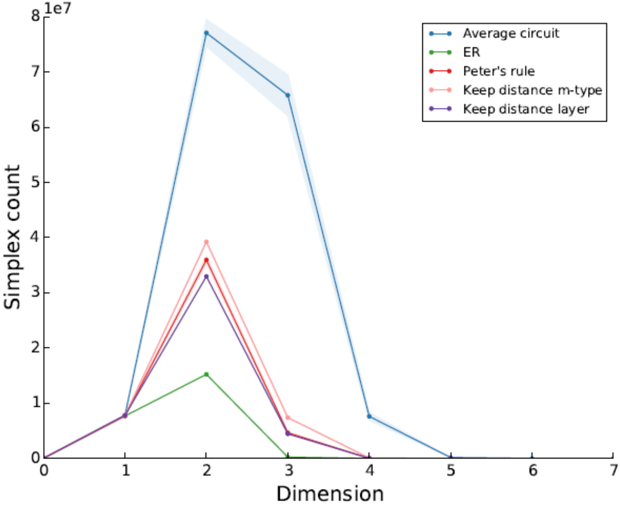
Is there a structure in the BBP microcircuit?

The microcircuit construction algorithm is based on semi stochastic processes. How do we know that its connectivity structure is not random?

Dim	Erdös Renyi	BB
0	31146	31146
1	7764739	7648079
2	15492757	73036616
3	247176	59945205
4	36	6599529
5	0	133115
6	0	529

Number of simplices by dimension in an Erdös Renyi random graph vs. typical Blue Brain graph.

Numbers of simplices.



Structural simplices.

1. Ergo: whatever procedure is used construct the microcircuit, it creates higher dimensional directed simplices.
2. More is true, the Betti numbers are also interesting.
3. $5-$th Betti number is the highest nonzero Betti number for all the microcircuits.
4. By comparison, for a random graphs we analysed, the highest nonzero one is third Betti number.
5. For microcircuit fourth and fifth Betti numbers are considerable.

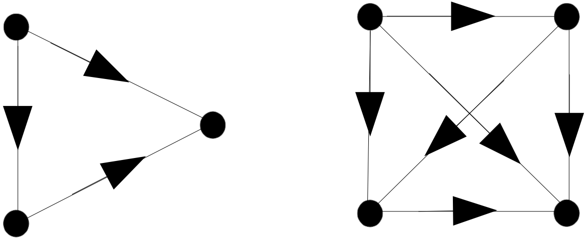
Wired together, fire together.

1. Structure triggers connectivity.
2. When the system is activated, the neurons that are connected in high dimensional directed simplices fire together in a coherent way.

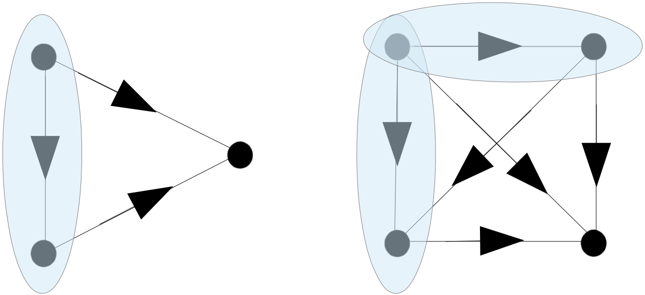
Transmission between neurons.

1. Reliability of transmission between two neurons is very low.
2. ... is below 10%.
3. Yet, despite this, our brains woks in a repeatable, stable way.

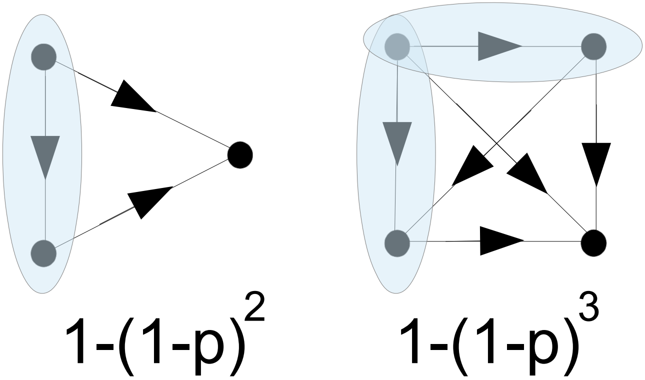
Directed simplices as the basic information flow units.



Directed simplices as the basic information flow units.



Directed simplices as the basic information flow units.



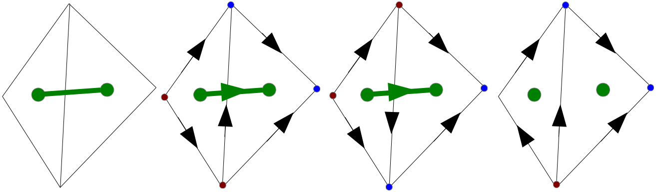
A few numbers.

1. In general $1 - (1 - p)^n$ for n-dimensional highway.
2. Suppose $p = 0.25$.

dimension	probability
1	0.25
2	0.4371
3	0.5781
4	0.6835
5	0.7626
6	0.822

Highway graphs.

1. Vertices: simplices of dimension n.
2. (Directed) edges – if two corresponding simplices intersect in the codim-1 coface (in the right way).



Current status.

1. Highways are there, and in possibly many other places.
2. We know how to find them.
3. They pinpoint regions of reliable information transfer.
4. They can also be use to decompose dynamics into gradient and recurrent one.
5. And generalize concept of segregation and integration of a network.

The code.

http://neurotop.gforge.inria.fr/

Thank you for your time!

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