A micro-structured multi-scale brittle damage model of porous material

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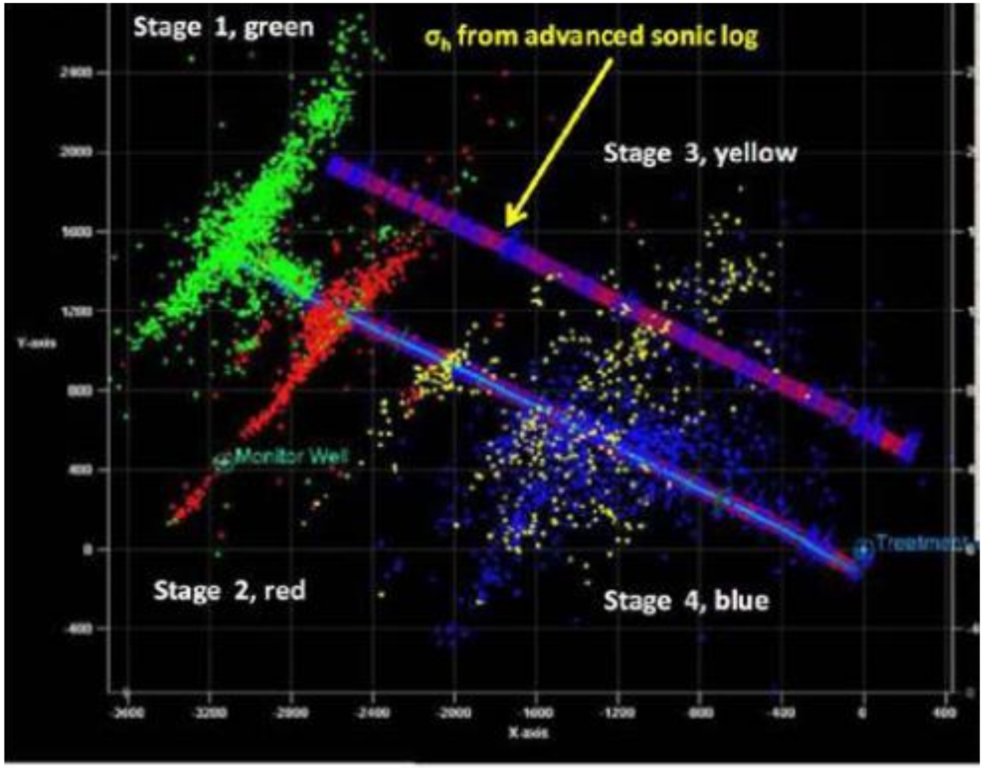
Introduction

- Fractures and discontinuities in natural rocks can evolve due to the action of gravity, superposed localized pressure, and shear tractions

- Actual great interest: damage induced by hydraulic stimulation in oil/gas reservoirs in view of increasing the reservoir production

Complexity of fracking induced faults

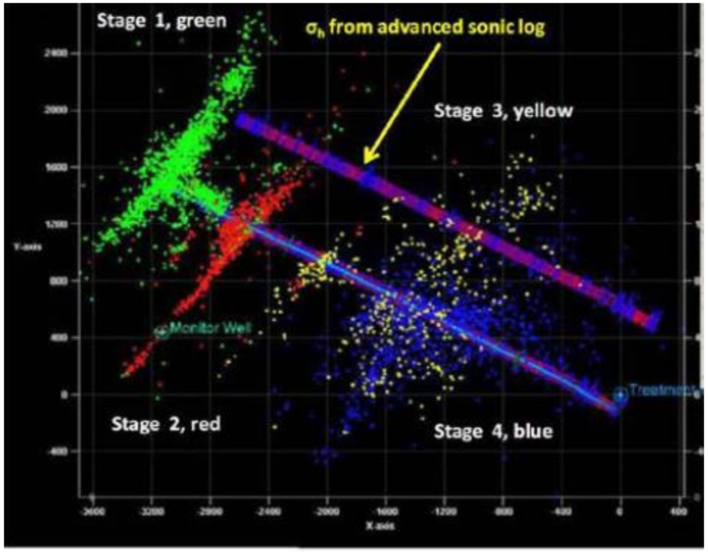
- Present day hydraulic fracture modeling and simulation is, for the most part, based on old technology and unrealistic simplifications and idealizations.
- Fractures are commonly modeled as mathematically sharp and the fluid is accounted for with complex hydrodynamic models.
- Reality is otherwise: acoustic measurements show that HF is a complex phenomenon involving the formation of intricate fracture patterns.

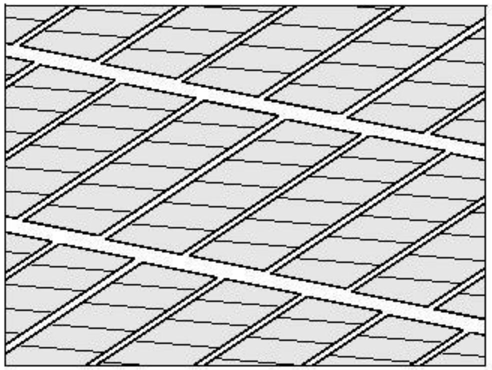


[Chuprakov et al., 2013; Wu et al., 2012]

Which model for HF?

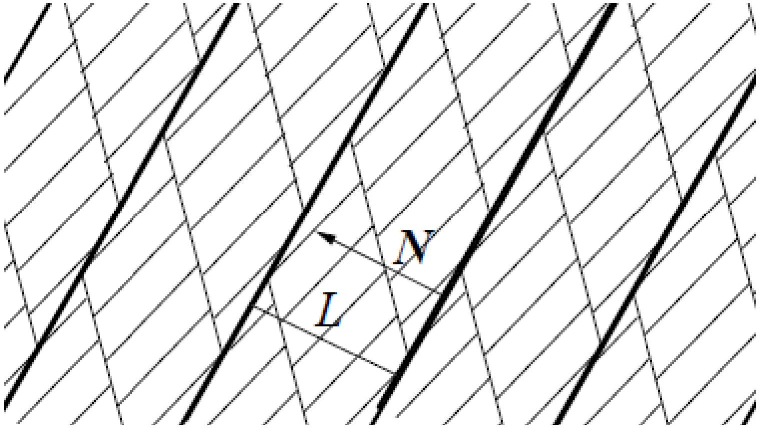
- Rocks in compression often fail through the formation of fine hierarchies of nested shear/frictional cracks.
- In HF operations fine fragmentation patterns contribute significantly to the permeability of rocks.
- Predictive HF simulations need material models that account for mechanical degradation AND permeability changes in the stimulated rocks.
- Resort to a multiscale Brittle Damage Model [Pandolfi et al, JMPS, 2006]





Brittle damage model (linearized version)

- Particular class of microstructures, consisting of nested families of equi-spaced cohesive faults. The faults bound elastic (or any other) matrix material.
- Each family is characterized by an orientation (defined by the normal $\mathbf{N}$ to the faults) and a spacing L.
- L is a microstructural feature of the material that derives from optimality conditions on the system energy.
- The average macroscopic strain tensor admits the additive decomposition:



$$\boldsymbol{\varepsilon} = \mathrm{sym}\nabla\boldsymbol{u} = \boldsymbol{\varepsilon}^{\mathrm{m}} + \boldsymbol{\varepsilon}^{\mathrm{f}}$$

[De Bellis et al, submitted, 2016]

Kinematics of faults

- For a single fault family, the symmetric tensor $\boldsymbol{\varepsilon}^{\mathsf{f}}$, due to the cohesive fault opening, can be evaluated as follows.

  Take a segment $d\boldsymbol{x}$ that spans two material points and define the number of faults:

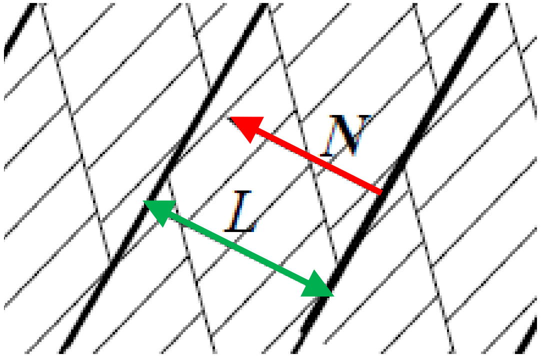
$$n = \frac{1}{L} d\boldsymbol{x} \cdot \boldsymbol{N}$$

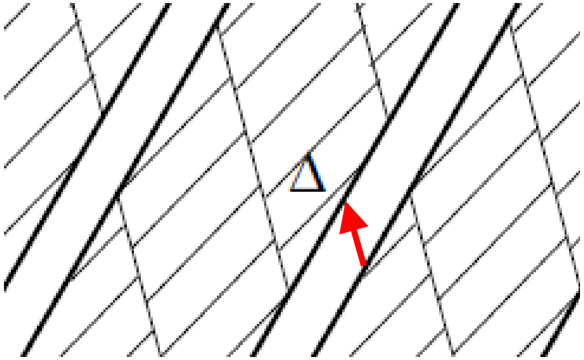
- Superpose the opening displacement $\boldsymbol{\Delta}$ to all the n faults and obtain the displacement:

$$d\boldsymbol{u}^{\mathsf{f}} = n\boldsymbol{\Delta} \equiv \nabla\boldsymbol{u}^{\mathsf{f}} d\boldsymbol{x}$$

- Derive the deformation component due to fault activity:

$$\boldsymbol{\varepsilon}^{\mathsf{f}} = \mathrm{sym}\nabla\boldsymbol{u}^{\mathsf{f}} = \frac{1}{2L}\left(\boldsymbol{\Delta} \otimes \boldsymbol{N} + \boldsymbol{N} \otimes \boldsymbol{\Delta}\right)$$





Elasticity of the matrix (E, ν)

- Here we assume linear elastic isotropic behavior for the underlying matrix

$$W^{\mathrm{m}} = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{m}\,T} \cdot \boldsymbol{D}\boldsymbol{\varepsilon}^{\mathrm{m}}$$

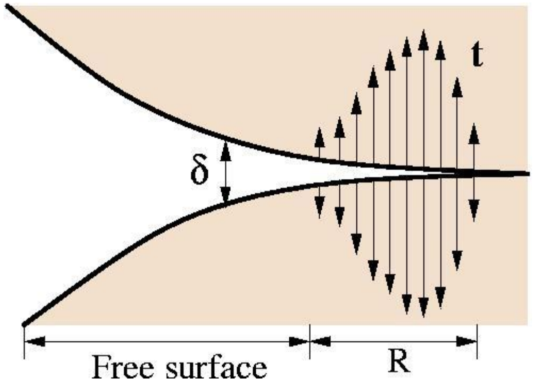
- Cauchy stress tensor and elastic tangents follow as:

$$\boldsymbol{\sigma} = \frac{\partial W^{\mathrm{m}}(\boldsymbol{\varepsilon}^{\mathrm{m}})}{\partial \boldsymbol{\varepsilon}^{\mathrm{m}}} \qquad D\boldsymbol{\sigma} = \frac{\partial^2 W^{\mathrm{m}}(\boldsymbol{\varepsilon}^{\mathrm{m}})}{\partial \boldsymbol{\varepsilon}^{\mathrm{m}} \partial \boldsymbol{\varepsilon}^{\mathrm{m}}}$$

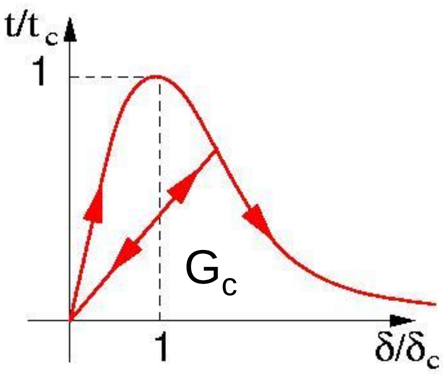
- Any other material model, accounting for plasticity, or viscosity, or other material behavior typical of soils, can be considered for the matrix. In such case, instead of the elastic potential, an incremental work of deformation will be taken into account.

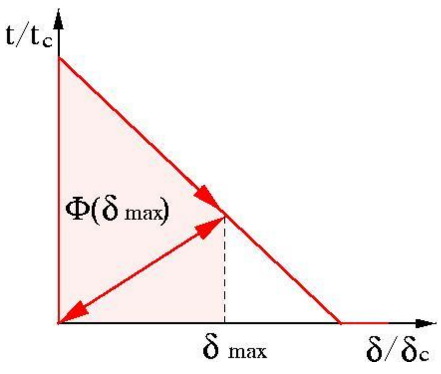
Trivia on cohesive approach to fracture

- Cohesive theories describe fracture evolution as the progressive separation of two surfaces. The displacement jump δ is resisted by tractions $\mathbf{t}$ along the cohesive zone R.



- Cohesive laws express the dependence of the tractions from the opening displacements.
- Simple uniaxial cohesive laws are defined by two parameters, e.g., the cohesive strength t_c of the material and the critical energy release rate G_c.
- Extension to mixed mode fracture, irreversibility, and 3D.





- [Camacho and Ortiz, 1996; Ortiz and Pandolfi, 1999].

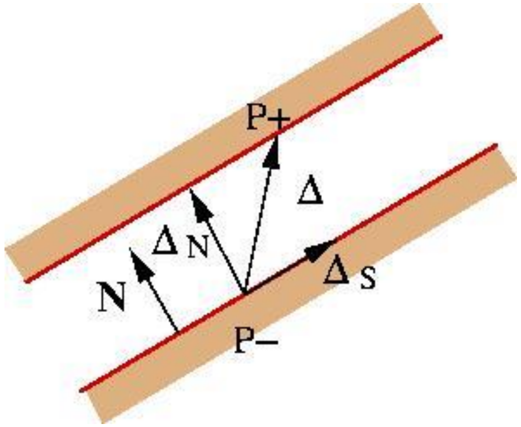
Cohesive faults (T_c, G_c, β)

- Effective opening displacement Δ [Ortiz & Pandolfi, 1999]:

$$\Delta = \sqrt{{\Delta_N}^2 + \beta^2 {\Delta_S}^2} \qquad \Delta_N = \Delta\cdot\mathbf{N} > 0, \qquad \Delta_S = |\Delta - \Delta_N \mathbf{N}|$$

- The constant β governs the shear behavior
- Cohesive energy and effective traction

$$\Phi = \Phi(\Delta, \mathbf{q}), \qquad T = \frac{\partial \Phi(\Delta, \mathbf{q})}{\partial \Delta}$$

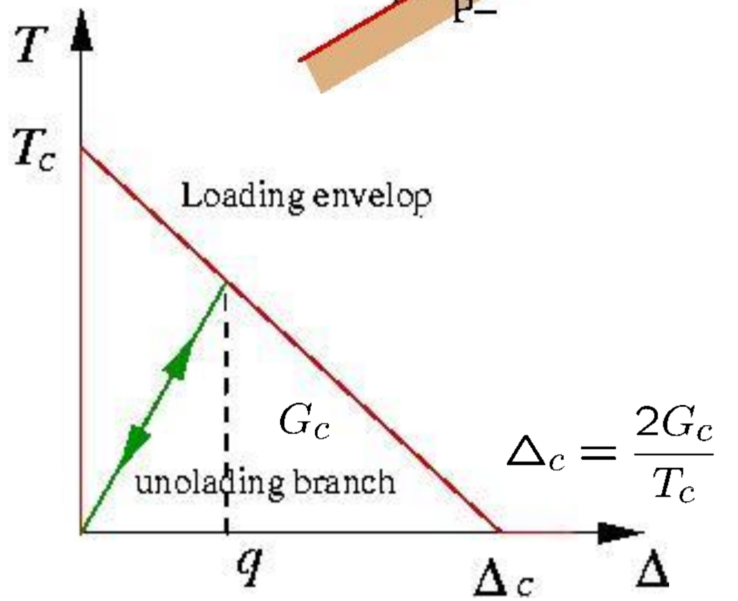


- Cohesive tractions

$$\mathbf{T} = \frac{T}{\Delta}\left[(1-\beta^2)\,\Delta^N\,\mathbf{N} + \beta^2\,\Delta\right]$$

- Irreversibility: unloading to origin, use the maximum Δ as internal variable q with kinetic relations

$$\dot{q} = \begin{cases} \dot{\Delta}, & \text{if } \Delta = q \text{ and } \dot{\Delta} \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



Frictional contact (μ)

- Friction is an essential dissipation mechanism in brittle materials. It is assumed that, upon the attainment of a critical opening displacement, faults loose cohesion and friction remains the only dissipation mechanism.
- Dual kinetic potential ψ^* per unit area [Pandolfi *et al.*, IJNME, 2002]

$$\psi^*(\dot{\boldsymbol{\Delta}}; \boldsymbol{\varepsilon}, \boldsymbol{\Delta}, q)$$

  - If faults undergo opening and are not in contact, then $\psi^* = 0$
  - If faults are closed and the contact tractions are compressive, then ψ^* is convex and minimized at $\dot{\boldsymbol{\Delta}} = 0$.
- Coulomb friction:

$$\psi^*(\dot{\boldsymbol{\Delta}}; \boldsymbol{\varepsilon}, \boldsymbol{\Delta}) = \mu \max\{0, \ -\boldsymbol{N} \cdot \boldsymbol{\sigma} \boldsymbol{N}\} \ |\dot{\boldsymbol{\Delta}}|$$

where $\mu = \tan\phi$ is the friction coefficient.

Variational form within time discretization

- Assume the existence of a single family of faults with known L and $\mathbf{N}$ (case of the presence of an existing fault)
- Variational characterization by time discretization. The state of the material at time t_n is known, and the total $\boldsymbol{\varepsilon}$ at time t_{n+1} is assigned.
- The incremental strain energy density follows from a constraint optimization problem of an incremental work of deformation for the current step [Ortiz & Stainier,1999; Pandolfi *et al.* 2006] :

$$W_n(\boldsymbol{\varepsilon}_{n+1}) = \inf_{\boldsymbol{\Delta}_{n+1},\, q_{n+1}}$$

$$\left\{ W^{\mathrm{m}}({\boldsymbol{\varepsilon}^{\mathrm{m}}}_{n+1}) + \frac{1}{L}\Phi(\boldsymbol{\Delta}_{n+1}, q_{n+1}) + \frac{\Delta t}{L}\psi^{*}\left(\frac{\boldsymbol{\Delta}_{n+1} - \boldsymbol{\Delta}_n}{\Delta t}; \boldsymbol{\varepsilon}_{n+1}, \boldsymbol{\Delta}_{n+1}\right)\right\}$$

subject to the constraints:

$$\boldsymbol{\Delta}_{n+1} \cdot \boldsymbol{N} \geq 0; \quad q_{n+1} \geq q_n$$

- Non standard formulation of friction: symmetry of the tangent moduli.
- If faults are impeded to reclose, the contact constraint is modified to include the minimal opening (case of presence of proppant in the fluid).

Fault orientation – Mohr Coulomb (μ = β)

- If faults are not there, the normal is not defined. It can be computed by way of optimization, including $\boldsymbol{N}$ in the set of optimization variables under the constraint:

$$|\boldsymbol{N}| = 1$$

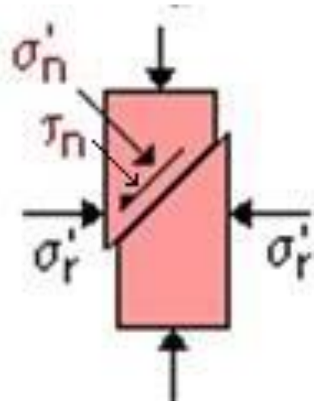
- Obtain an eigenvalue problem, corresponding to two different conditions typical of brittle materials:
- Failure in opening (Rankine criterion)

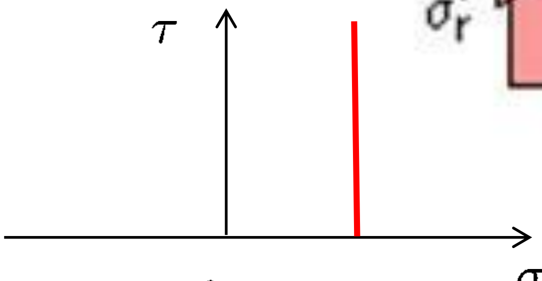
$$\psi^* = 0, \qquad \boldsymbol{\Delta} \cdot \boldsymbol{N} > 0$$

- Failure in sliding (Mohr-Coulomb criterion)

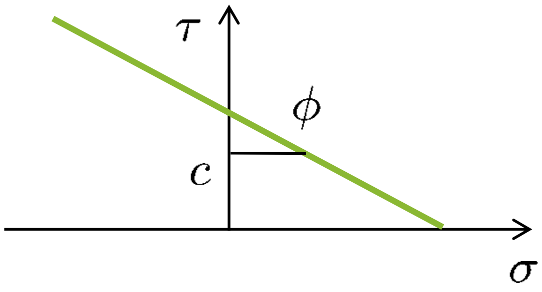
$$\psi^* > 0, \qquad \boldsymbol{\Delta} \cdot \boldsymbol{N} = 0 \qquad \beta = \mu$$

- Faults are inserted if their presence reduces the energy of the system with reference to the unfractured situation.





$\theta = 0$



$\theta = \pi/4 - \phi/2$

Optimal separation (L_1)

- The fault separation L can be intended as a model variable, and be computed by way of energy optimization during the calculation.
- Include an additional (approximated) misfit energy, necessary to accommodate the inner fault family inside the pertinent volume, geometrically defined by the size of the confining container L_n:

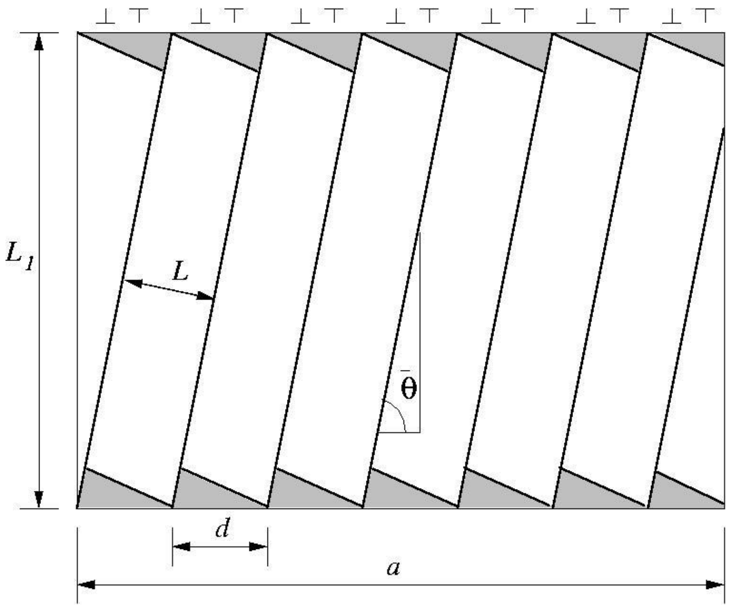
$$W_n(\boldsymbol{\varepsilon}) = \inf_{\boldsymbol{\Delta}, q, L} \left\{ W^{\mathrm{m}}(\boldsymbol{\varepsilon}^{\mathrm{m}}) + \frac{1}{L}\Phi(\boldsymbol{\Delta}, q) + \frac{\Delta t}{L}\Psi^* \left(\frac{\boldsymbol{\Delta} - \boldsymbol{\Delta}_n}{\Delta t}; q, \boldsymbol{\Delta} \right) + E^{\mathrm{mis}}(L) \right\}$$

- Model the boundary layer as an array of dislocations of alternating sign. The Burgers vector of the misfit dislocation is of the order of $|\Delta|$, thus:

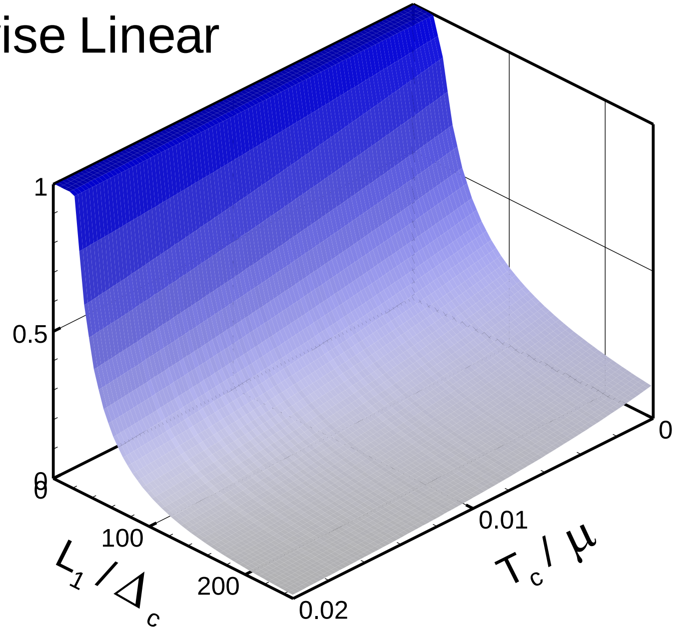
$$E^{\mathrm{mis}}(L_{n+1}) = \frac{C|\boldsymbol{\Delta}|^2}{L_n} \frac{1}{L_{n+1}} \log \frac{L_{n+1}}{L_0}$$

- C is a constant proportional to the shear modulus G, L_n is the size of the confining container, and L_0 (related to $2G_c/T_c$) plays the role of “core cut off”.

Rank-n recursion length scale



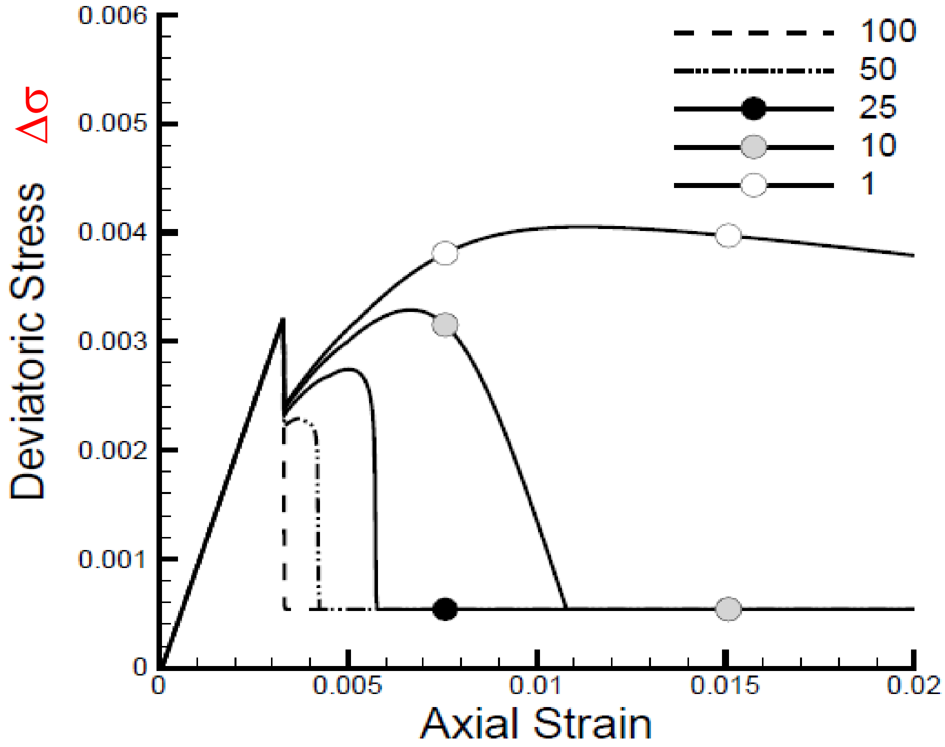
Piecewise Linear

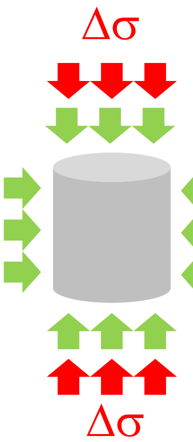


- Minimizing wrt to L_{n+1} , for the linear decreasing cohesive law obtain the explicit expression of the scale at the inner level:

$$L_{n+1} = \frac{L_0 T_c}{2G_c} \exp\left[1 - \frac{L_n}{2G_c}\frac{T_c^2(1+\nu)}{E}\right]$$

Effect of the scale parameter L_0





Recursive Faulting

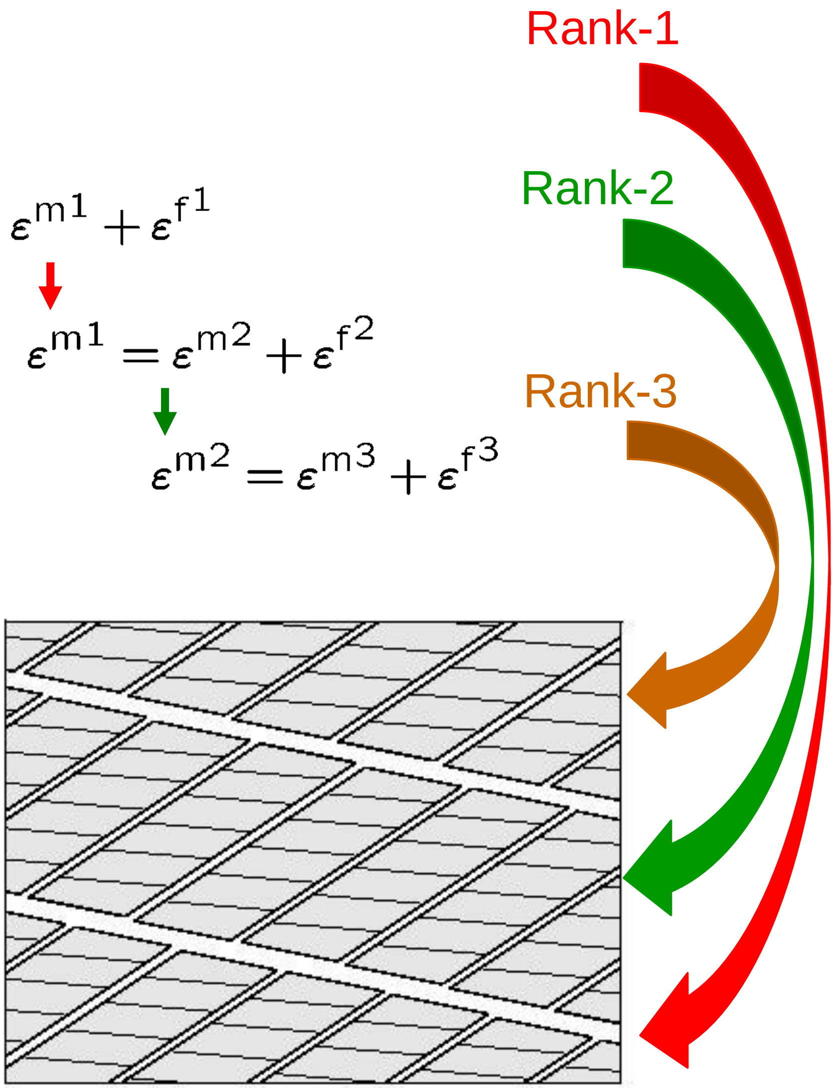
- Once the first fault family has developed, the matrix between faults may experience a tensile/shear state resulting in further faulting on a sublevel.
- The matrix deformation gradient $\boldsymbol{\varepsilon}^{m}$ at the first level can be in turn decomposed into an "elastic" part and a cohesive part.
- This can be repeated several times, by using a recursive procedure, easily supported by C and C++ languages.

$$\varepsilon = \varepsilon^{m3} + \varepsilon^{f1} + \varepsilon^{f2} + \varepsilon^{f3}$$

$$\varepsilon = \varepsilon^{m1} + \varepsilon^{f1}$$

$$\varepsilon^{m1} = \varepsilon^{m2} + \varepsilon^{f2}$$

$$\varepsilon^{m2} = \varepsilon^{m3} + \varepsilon^{f3}$$

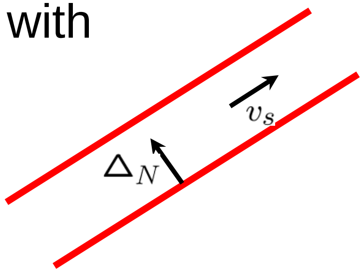


Porosity and permeability of the faults

- Porosity due to a single family and to Q fault families

$$n^{\mathrm{f}} = \frac{\Delta_N}{L} \qquad n^{\mathrm{f}} = \sum_{k=1}^{Q} \frac{\Delta_N^k}{L^k}$$

- Permeability on a plane of normal $\boldsymbol{N}$, derived consistently with Navier-Stokes equation
  - for Q families :



$$\kappa^{\mathrm{f}} = \frac{1}{12} \sum_{k=1}^{Q} \frac{{\Delta_N^k}^3}{L^k} \left(\boldsymbol{I} - \boldsymbol{N}^k \otimes \boldsymbol{N}^k \right)$$

- Accounting for initial porosity and permeability of the matrix, it results:

$$n = n^{\mathrm{m}} + n^{\mathrm{f}} \qquad \kappa = \kappa^{\mathrm{m}} + \kappa^{\mathrm{f}}$$

where :

$$n^{\mathrm{m}} = n_0 + \varepsilon_{\mathrm{v}} \qquad \kappa^{\mathrm{m}} = C_{KC} \frac{(n^{\mathrm{m}})^3}{(1 - n^{\mathrm{m}})^2}$$

Volumetric deformation

Kozeny-Carman type permeability

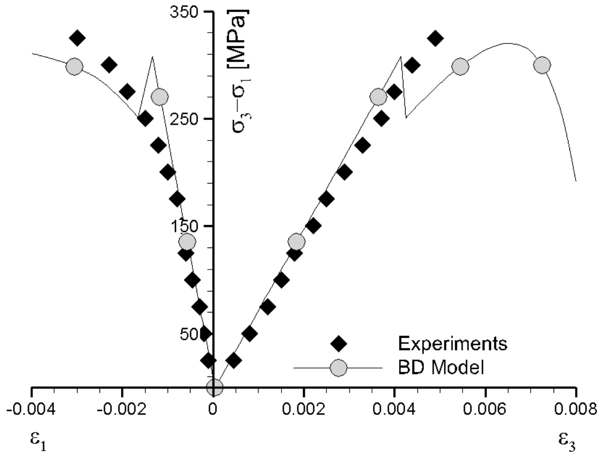
Reduced number of material properties

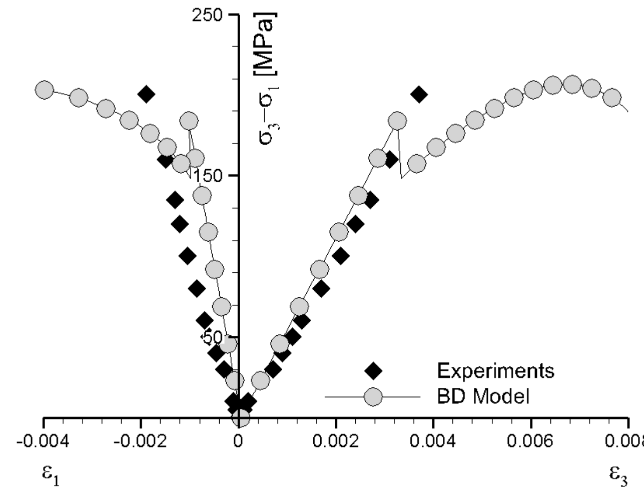
- Two elastic parameters (E, ν)
- Three cohesive parameters (T_c, G_c and $\beta = \mu = \tan\phi$)
- One scale parameter (L_0)
- For coupled problems: initial porosity and matrix permeability coefficient

- In all the following numerical tests, the material parameters have been taken from the experimental papers

Material	E (MPa)	ν	T_c (MPa)	G_c (N/m)	$\mu = \beta$	$\frac{L_0 T_c}{2G_c}$
Lac du Bonnet granite	68,000	0.21	50	10	1.05	0.2
Beishan granite	52,000	0.21	60	10	0.7	0.8
Inada sandstone	68,000	0.2	20	5	1.6	50
Darley dale sandstone	10,000	0.3	15	5	1.07	1
Flechinger sandstone	11,000	0.25	35	0.1	0.7	12.5

Point triaxial test – Permeability validation



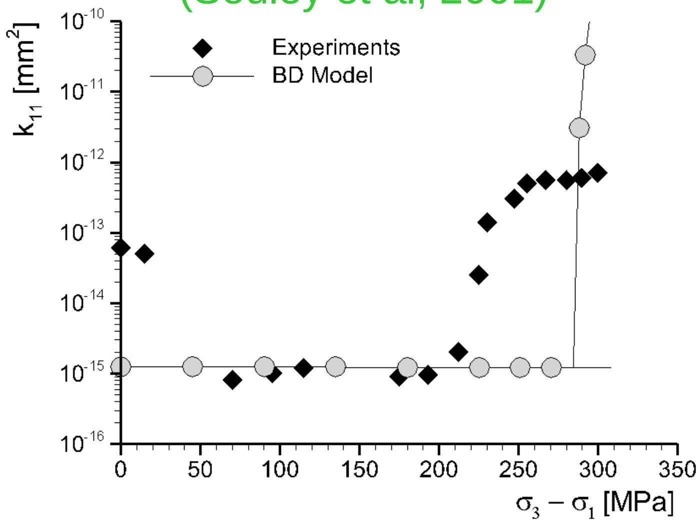


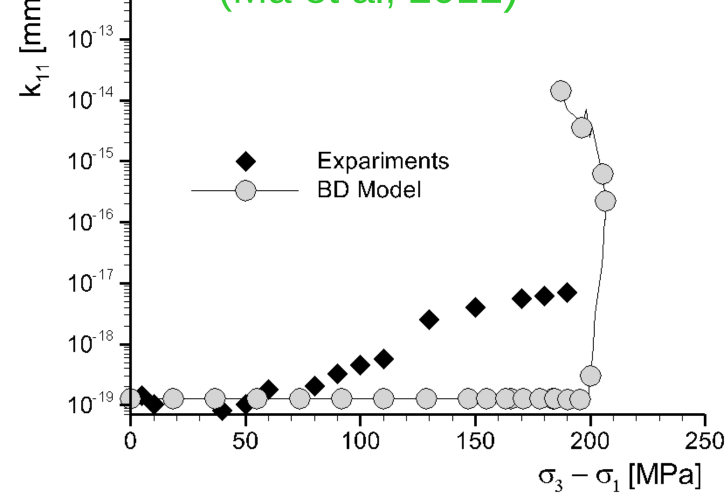
Lac du Bonnet granite

(Souley et al, 2001)

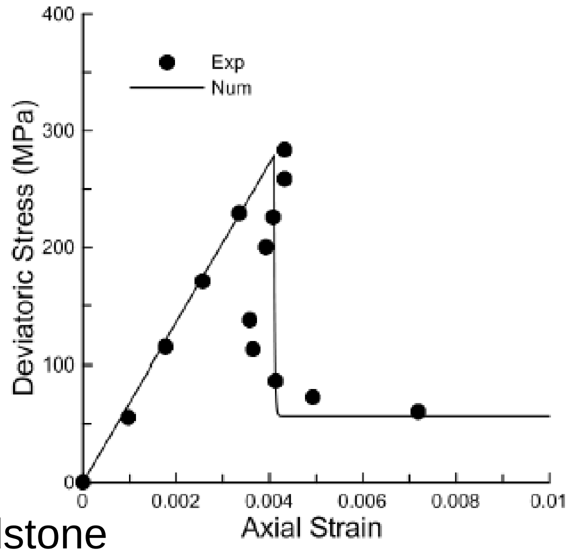
Beishan granite

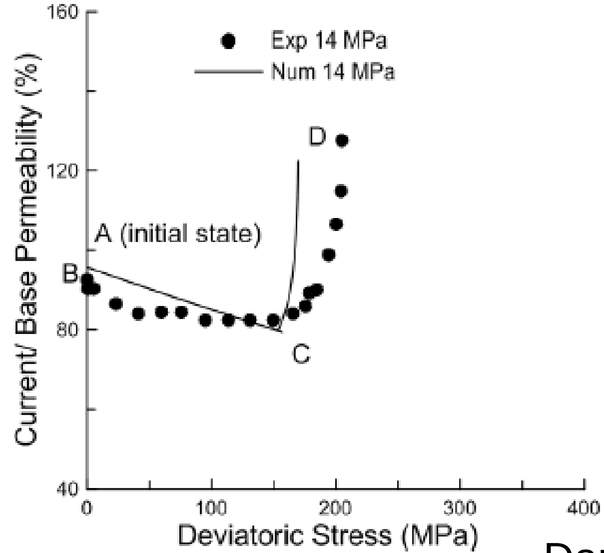
(Ma et al, 2012)





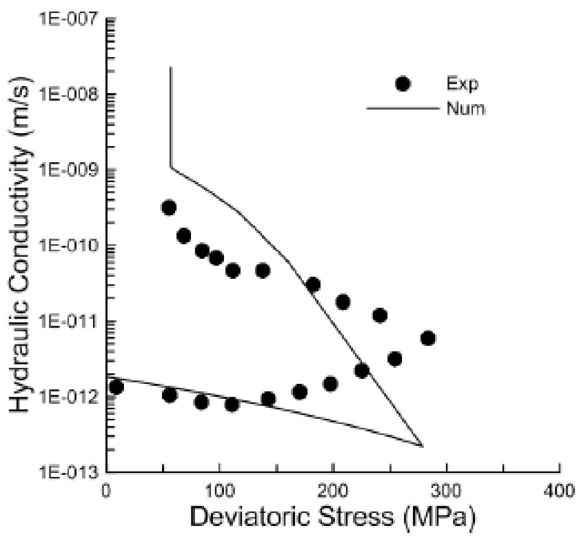
Point triaxial test – Permeability validation

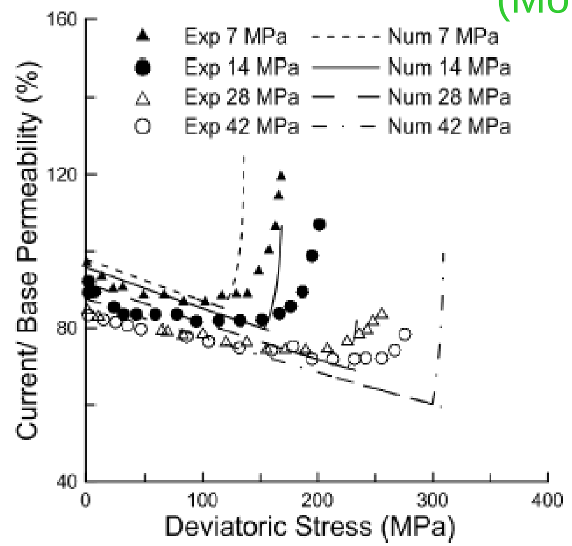




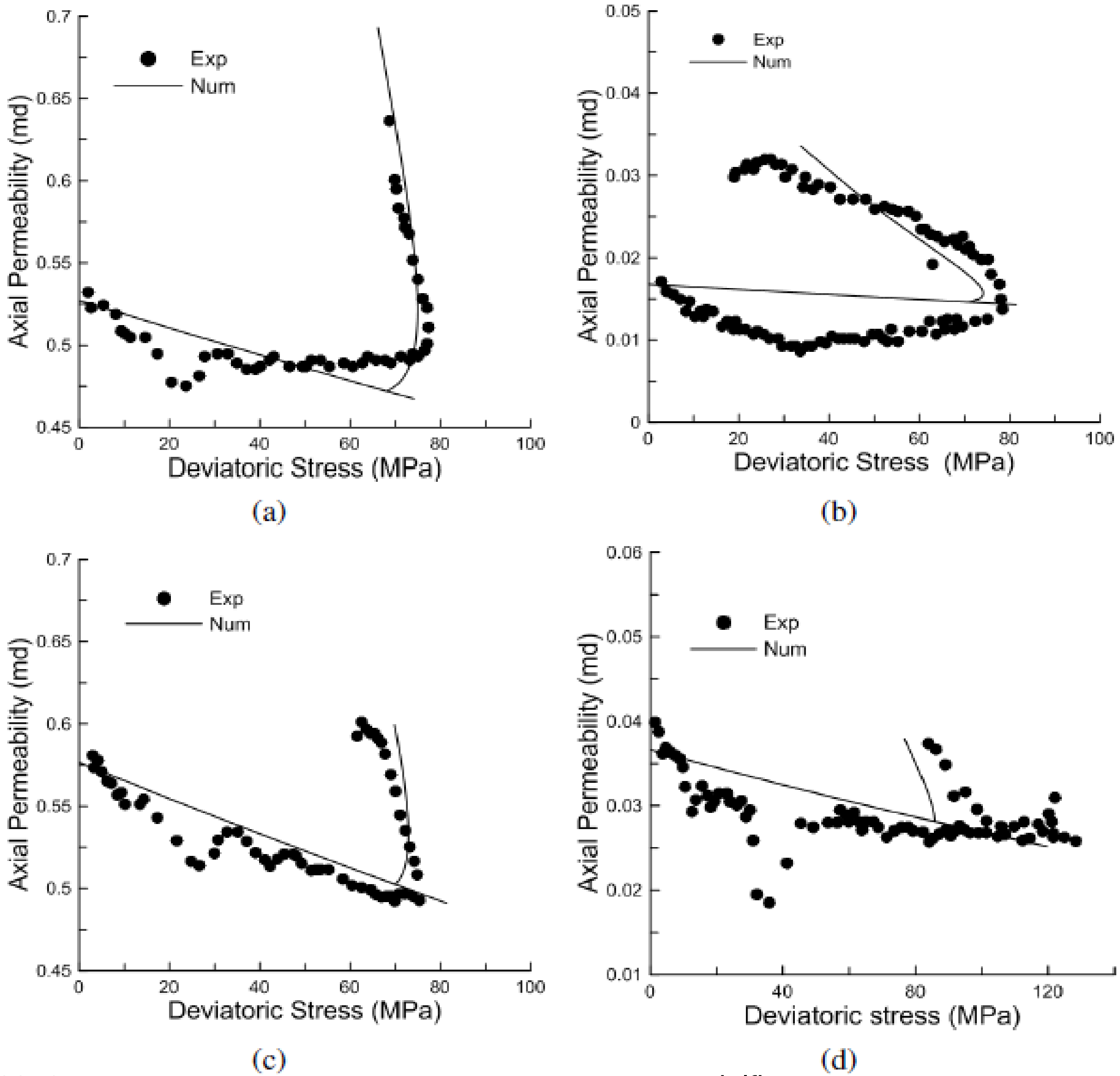
Inada sandstone
(Kiyama et al, 1996)

Darley dale sandstone
(Mordecay, 1970)





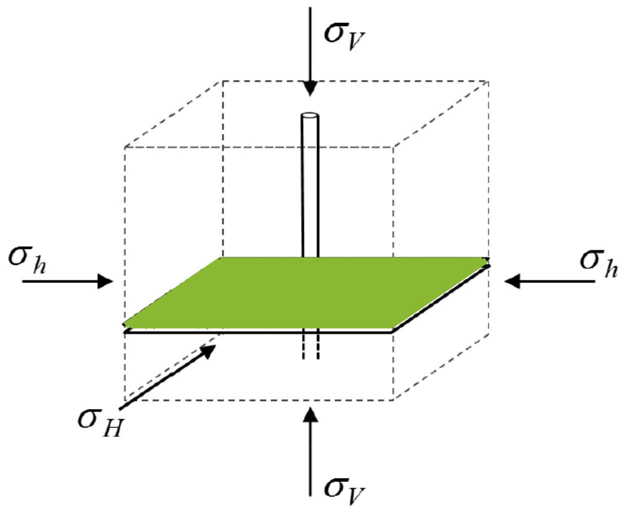
Point triaxial test – Permeability validation

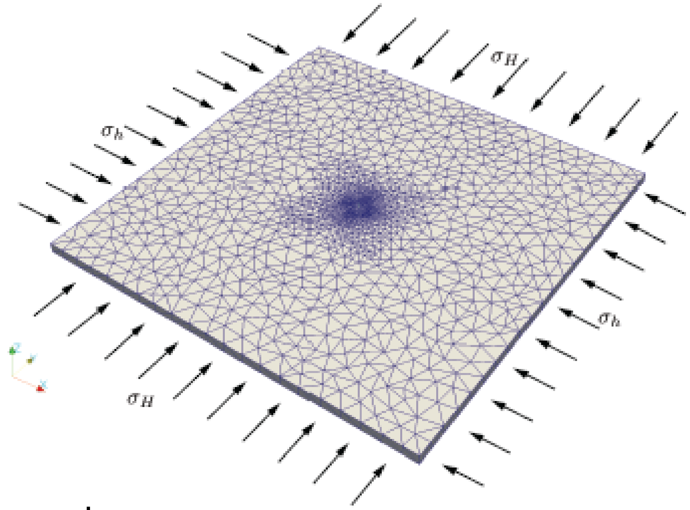


Flechinger sandstone, with increasing confinement (Heiland, 2003)

Borehole excavation (dry model)

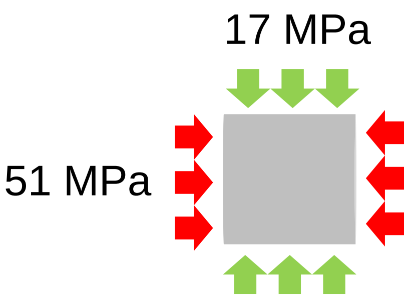
- Case problem of a 2 m diameter vertical borehole excavation in dry material, in a 10 x 10 m field, at a depth of 10 m.
  Compare to a FE study using a cohesive approach with explicit interfaces in a triangular 2D mesh [Lisjack et al, JRMMS 2014].





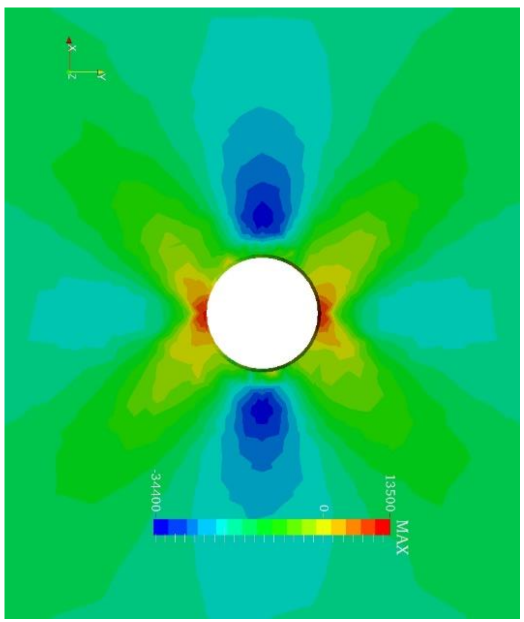
3D mesh
8010 nodes
36086 elements

Moderate difference in horizontal stresses

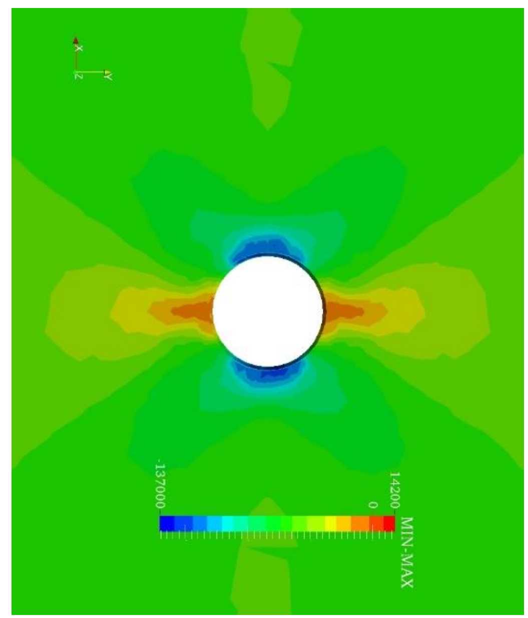


Initial anisotropic stress state in the horizontal plane for the intact mass (K_0 = 3)
Expected typical shear borehole breakouts start to develop along the direction of minimum stress and spread along the hoop direction.

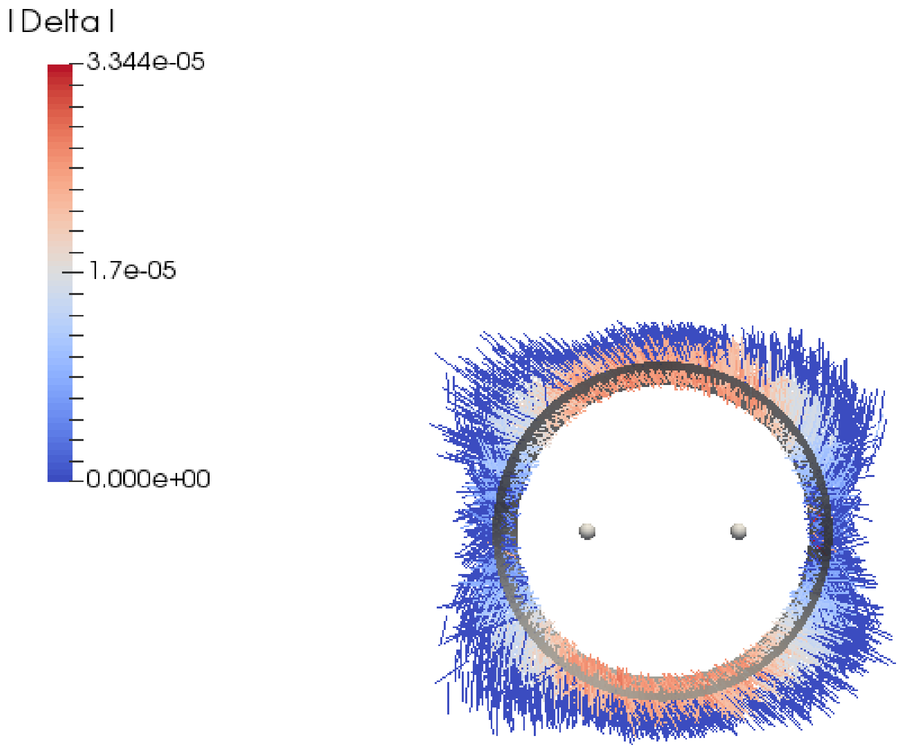
Maximum stress distribution at the first step of excavation



Max-min stress difference at the first step of excavation



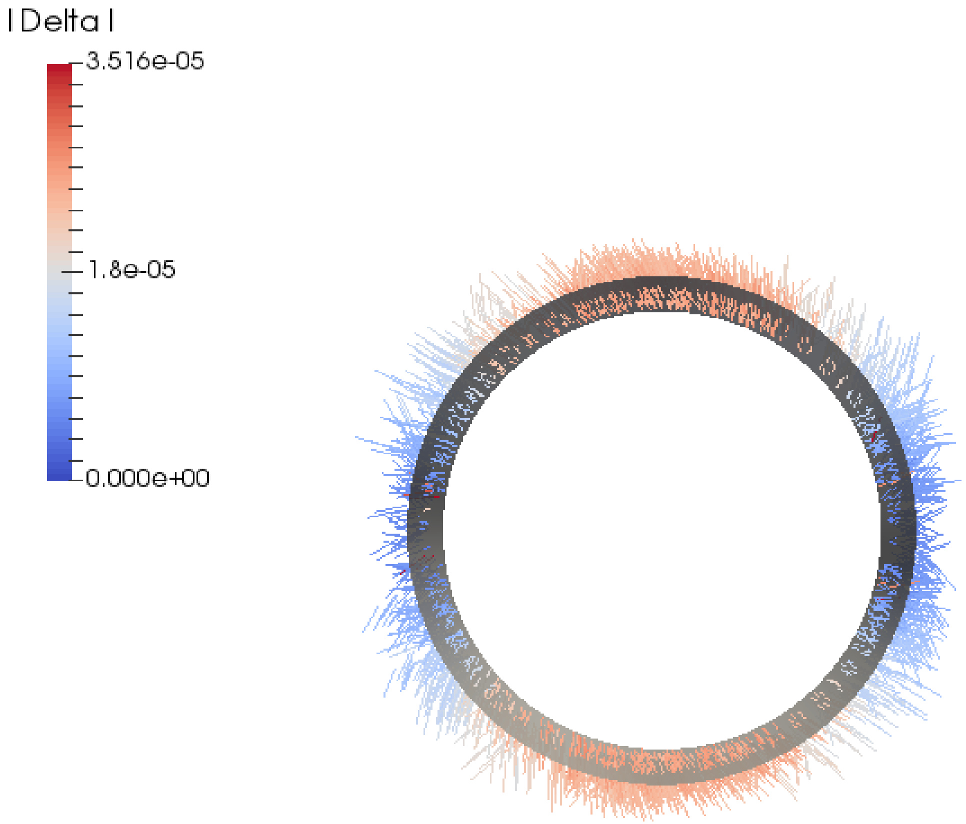
Shear crack distribution



- Arrows oriented as N and colored as Δ

D = 1 m

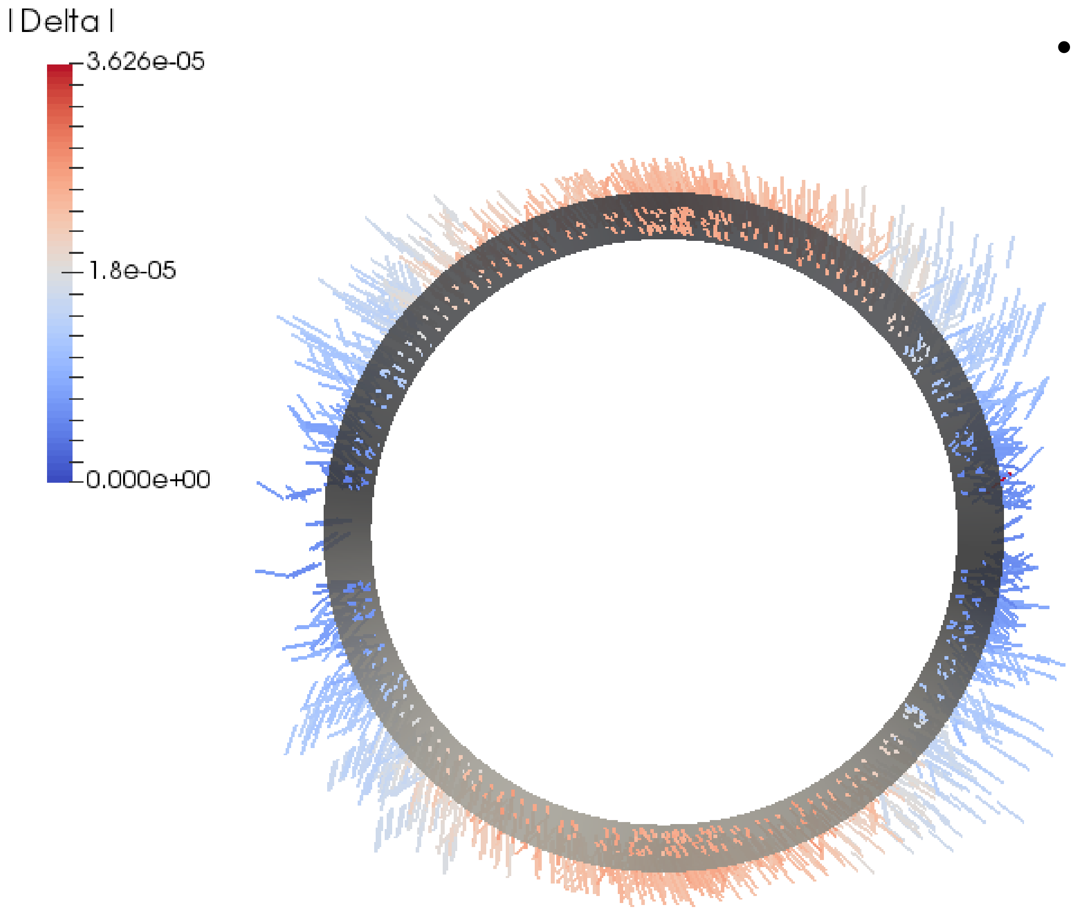
Shear crack distribution



- Arrows oriented as N and colored as Δ

D = 1.5 m

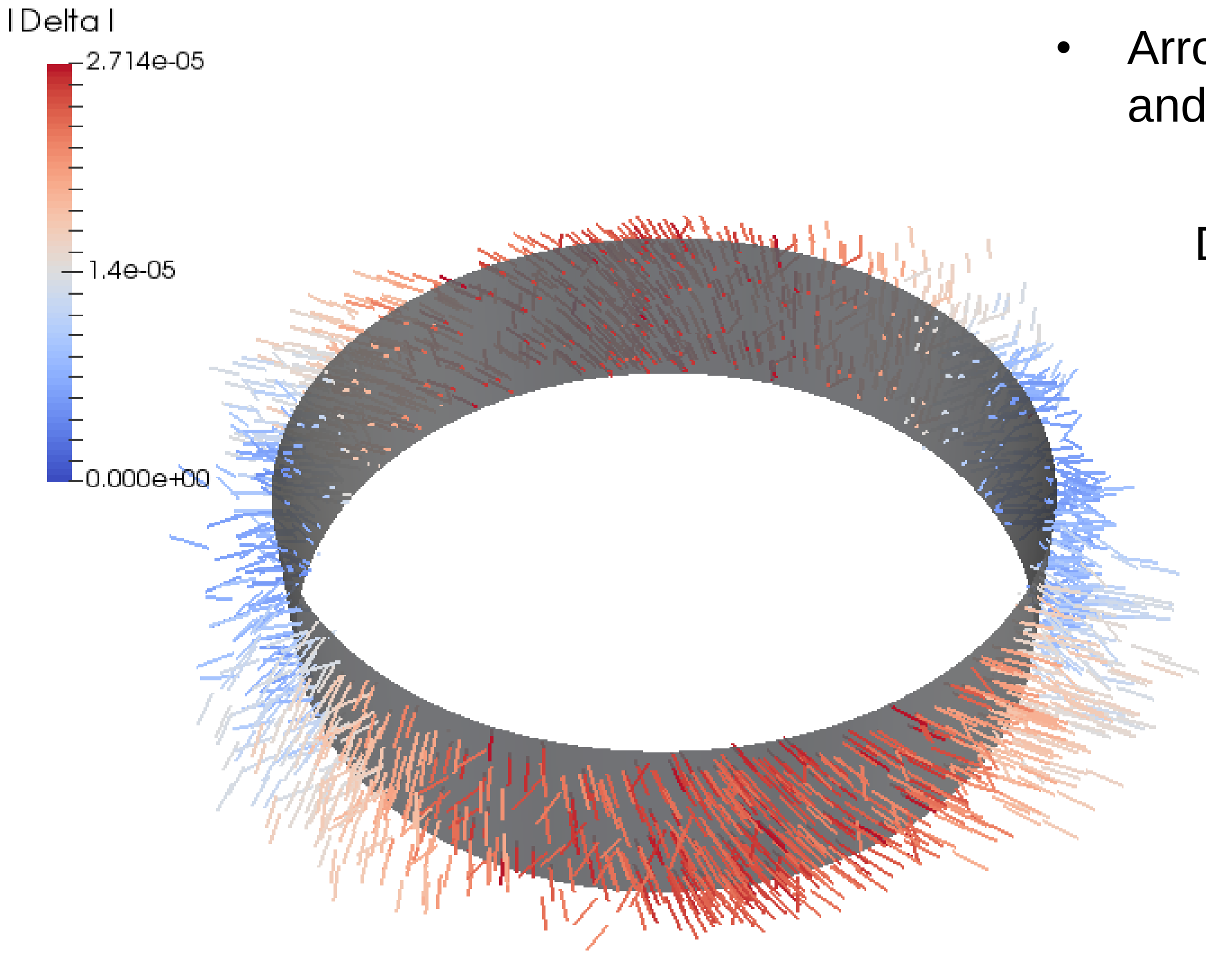
Shear crack distribution



- Arrows oriented as N and colored as Δ

D = 2 m

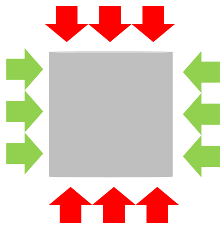
Final shear crack distribution in 3D



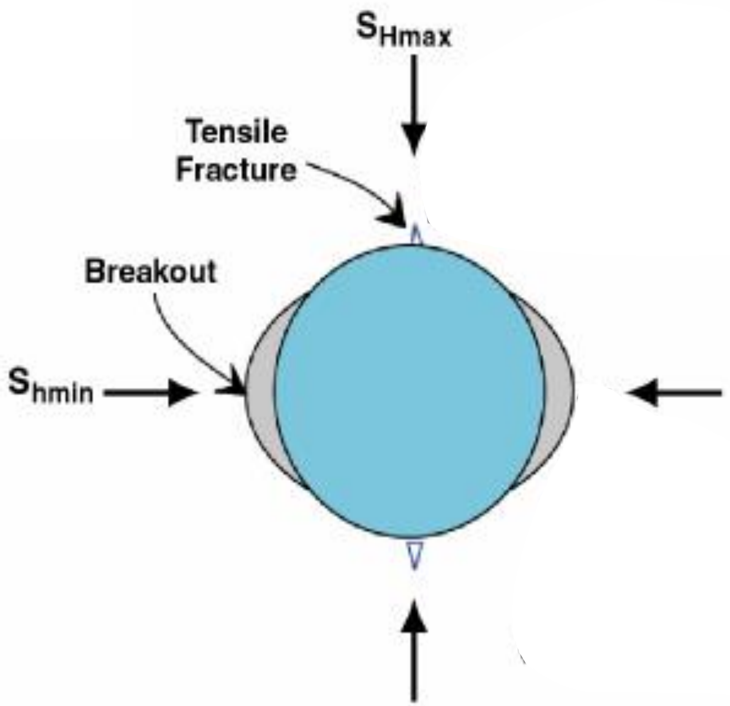
- Arrows oriented as N and colored as Δ

D = 2 m

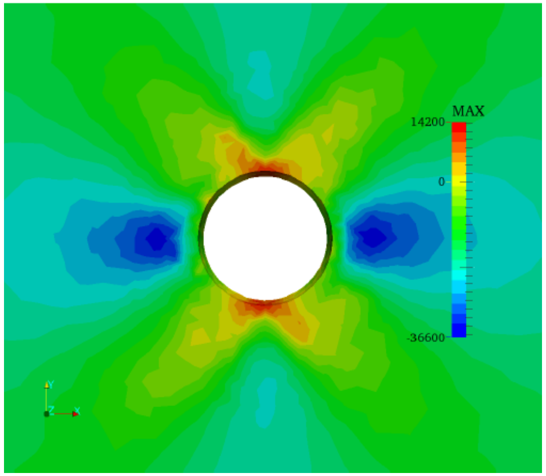
Larger difference in horizontal stresses



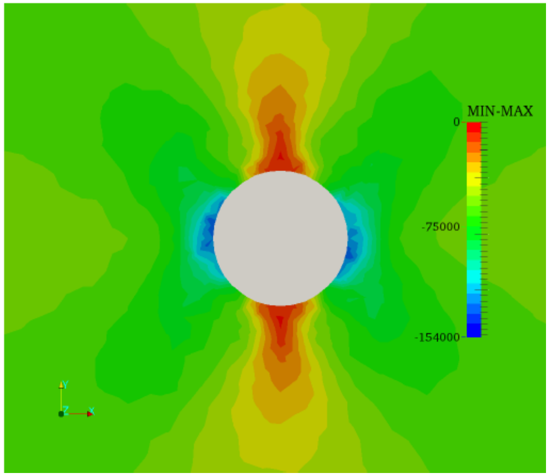
Larger anisotropic stress in the horizontal plane for the intact mass (K_0 = 4)



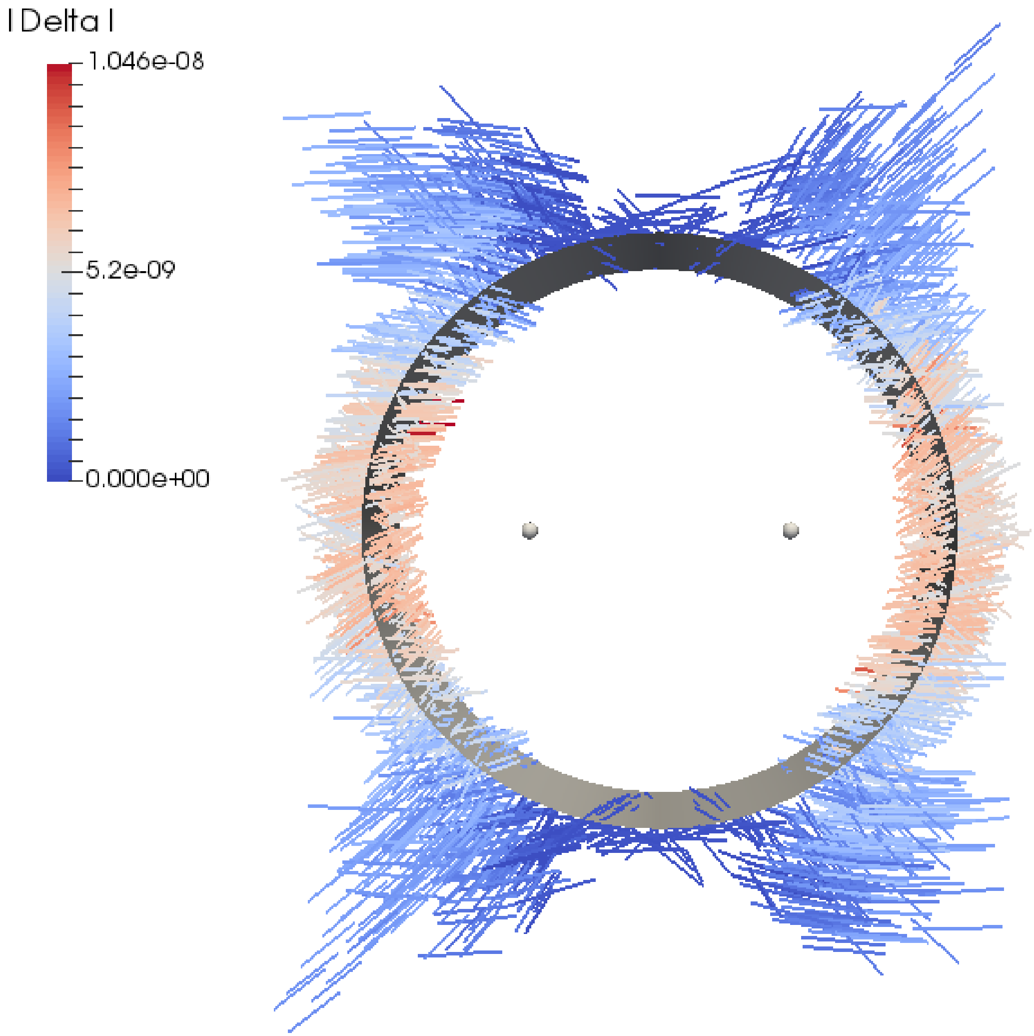
Max stress distribution at the first step of excavation



Max-min stress difference at the first step of excavation



More anisotropy in the stress – Shear fractures

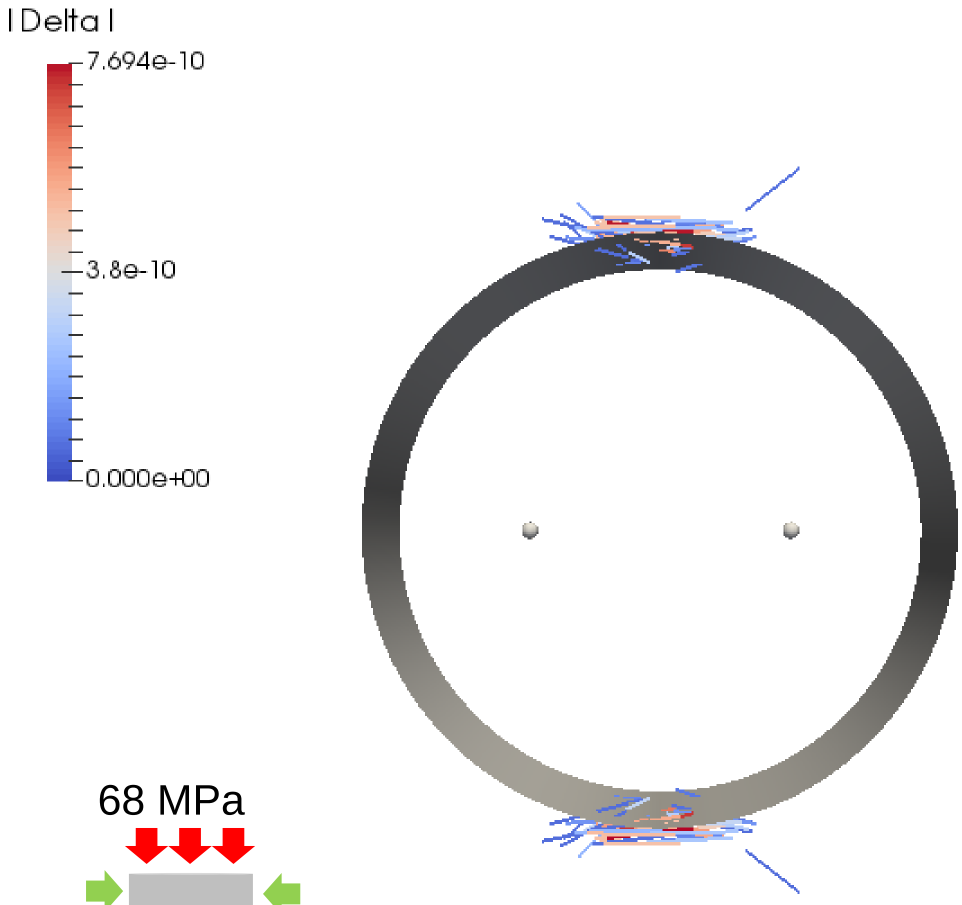


- Arrows oriented as N and colored as Δ

D = 1 m

Expected a typical “eye-shaped” fracture distribution around the hole, with superposition of extended shear fractures and a few tensile fractures.

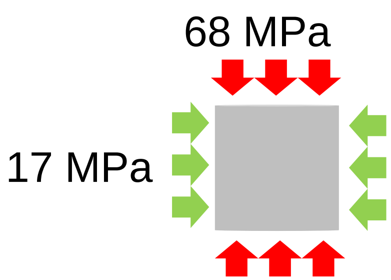
More anisotropy in the stress – Tension fractures



- Arrows oriented as N and colored as Δ

D = 1 m

Presence of tensile fractures in the plane of minimum stress



Porous media equations

- Linear momentum balance

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \mathbf{0}$$

- Continuity equation (fully saturated porous media, incompressible fluid and incompressible soil particles), n porosity, ε_v volumetric strain

$$\frac{\partial n}{\partial t} = -\nabla \cdot \boldsymbol{q} \qquad \frac{\partial n}{\partial t} = \frac{\partial \varepsilon_v}{\partial t}$$

- Terzaghi's effective stress principle, p pore pressure

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + p\boldsymbol{I}$$

- Constitutive relations

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma}'(\boldsymbol{\varepsilon}) \, , \qquad \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{m}} + \boldsymbol{\varepsilon}^{\mathrm{f}}$$

- Constitutive relation for fluid flow in porous media (Darcy law), h hydraulic head, $\boldsymbol{\kappa}$ permeability tensor

$$\mathbf{q} = -\boldsymbol{\kappa}\frac{\rho_f g}{\mu}\nabla h \qquad h = \frac{p}{\rho_f g} + z$$

Coupled field problem solution strategy

- Two field equation: linear momentum balance and continuity equation

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \mathbf{0}$$

$$\frac{\partial \varepsilon_v}{\partial t} = -\nabla \cdot \boldsymbol{q}$$

- Weak form (unknowns $\boldsymbol{u}$ and p, introduce the test functions v and η)

$$\int_V (\sigma'_{ij} + \delta_{ij} p) \frac{\partial v_j}{\partial x_i} dV = \int_{\Gamma_t} \bar{t}_j v_j d\Gamma + \int_V b_j v_j dV \, .$$

$$\int_V \frac{\partial n}{\partial t} \eta dV + \int_V \frac{\partial \eta}{\partial x_j} q_j dV = \int_{\Gamma_q} q_n \, \eta d\Gamma \, .$$

- After spatial discretization obtain the matrix form (similar to the consolidation equations)

$$\boldsymbol{F}^{\text{ext}} - \boldsymbol{F}^{\text{int}}(\boldsymbol{U}) = \boldsymbol{H}^T \boldsymbol{P}$$

$$\boldsymbol{K} \, \boldsymbol{P} = \boldsymbol{Q}^{\text{ext}} + \boldsymbol{H} \dot{\boldsymbol{U}}$$

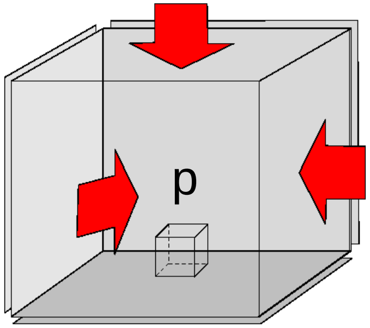
- which is solved with a staggered approach (explicit in $\boldsymbol{u}$, implicit in p).

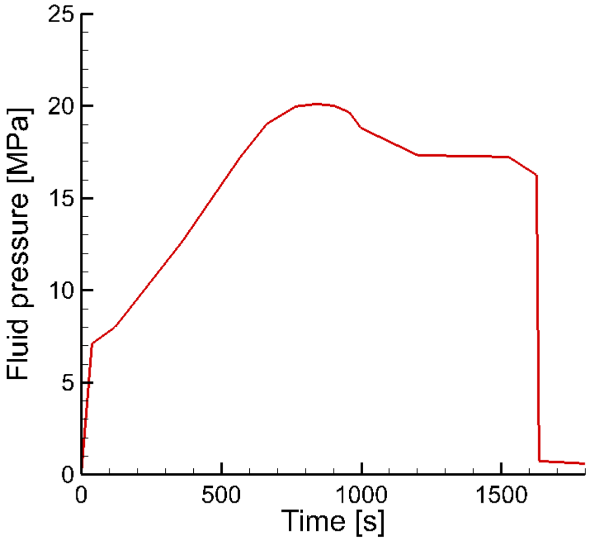
Triaxial loading of a cubic sample of cement

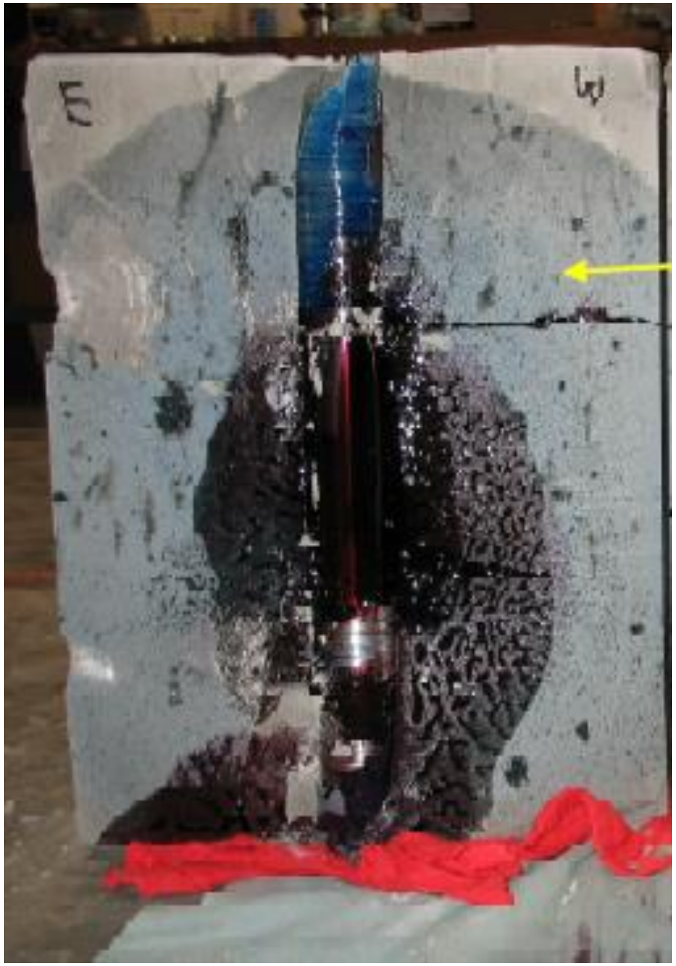
- Experimental data on a compressed block of cement pressurized with a fluid in a small cavity at the low center of the specimen [Athavale & Miskimins 2008]

  σ_v = 24.2 MPa
  σ_H = 17.3 MPa
  σ_h = 10.4 Mpa

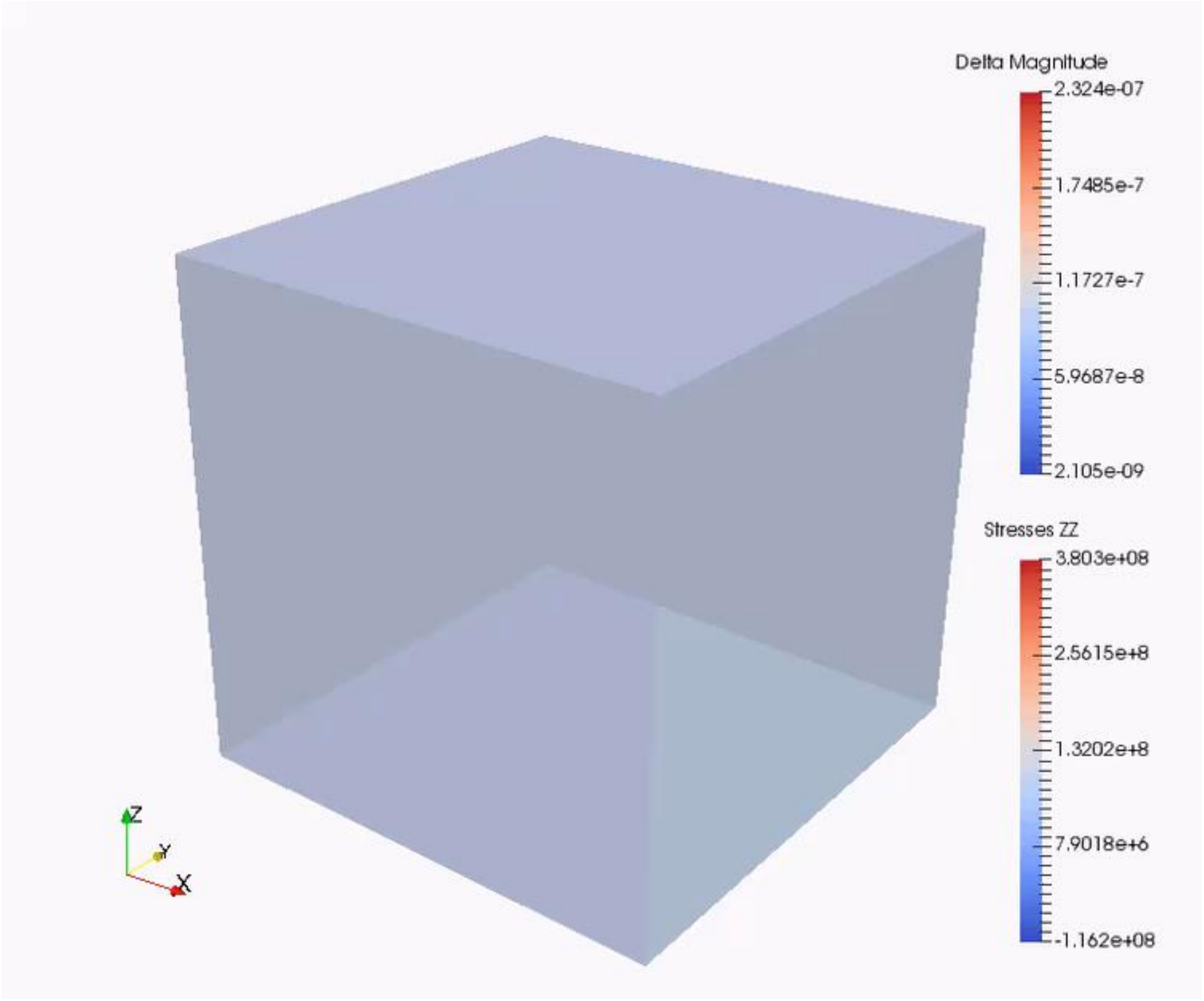
  Max fluid pressure = 20 MPa



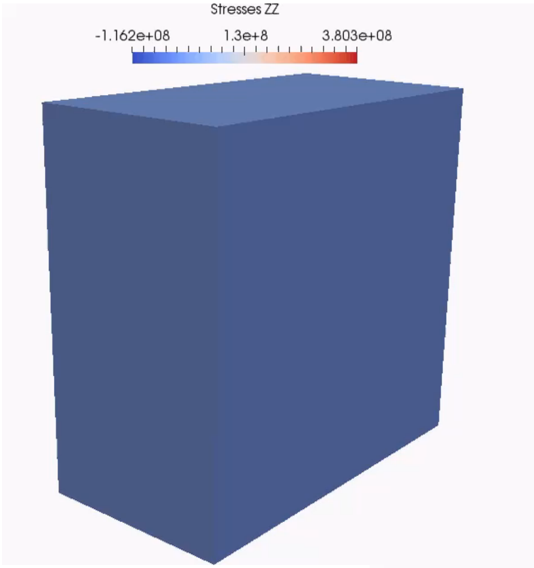


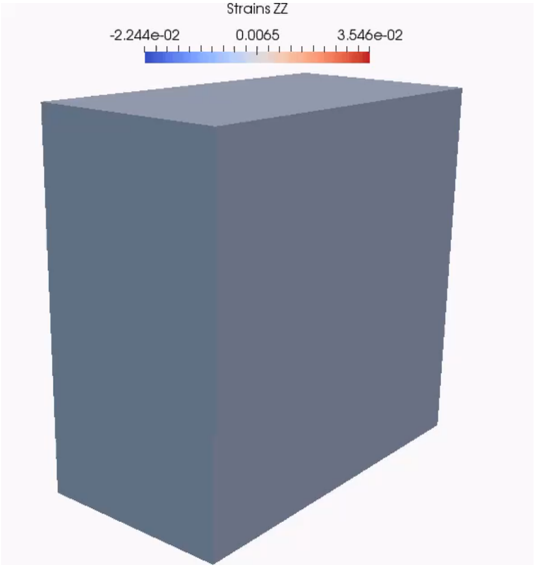


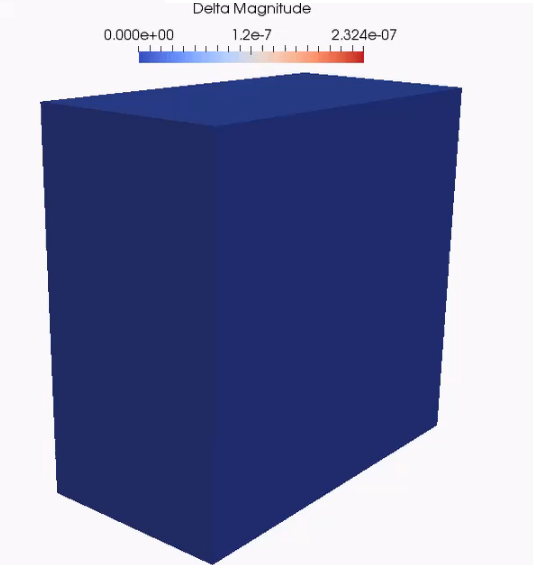
Evolution of the damage (numerical test)

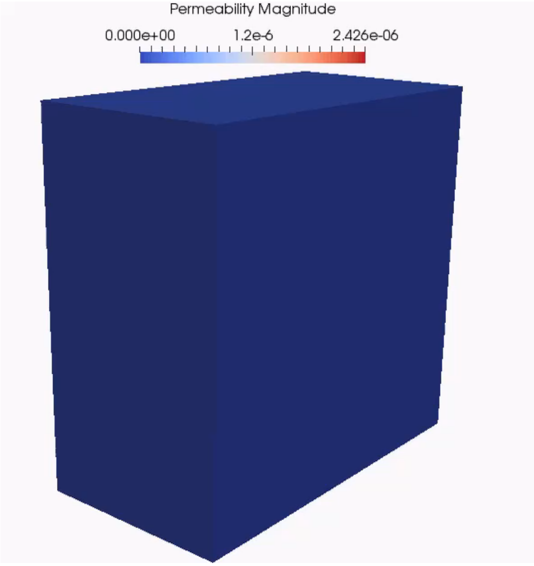


Stress, Strain, Permeability and Delta









Conclusions

- Model of distributed brittle damage, based on additive decomposition of the deformation tensor.
- Features:
  - Recursive faulting
  - Several intrinsic length scales, obtained by energy minimization
  - Accounting for frictional contact
- The microstructures allow for analytical definition of porosity and permeability.
- The effect of proppant is trivial: modify the closure of contact constraint
- Inclusion of pre-existing fractures is trivial: assign the orientation N and a large L to the elements intersected by the crack.
- The theory provides a sound mathematical basis for HF approach.

Porosity and permeability of the faults

- Porosity due to a single family and to Q fault families

$$n^{\mathsf{f}} = \frac{\Delta_N}{L} \qquad n^{\mathsf{f}} = \sum_{k=1}^{Q} \frac{\Delta_N^k}{L^k}$$

- Solution of Navier-Stokes equation, velocity of a laminar flow in direction s within a planar channel of width Δ_N

$$v_s = -\frac{\Delta_N^2}{12}\frac{\rho g}{\mu}\frac{\partial h}{\partial s}$$

- Discharge rate and permeability of a fault in direction s

$$q_s = n^{\mathsf{f}}\, v_s = -\frac{\Delta_N^3}{12}\frac{\rho g}{\mu}\frac{\partial h}{\partial s} \longrightarrow \kappa_s = \frac{\Delta_N^3}{12}$$

- In tensor form, if $\boldsymbol{d}$ denotes the unit vector in in direction s

$$\kappa^{\mathsf{f}} = \frac{\Delta_N^3}{12}\,(\boldsymbol{d} \otimes \boldsymbol{d})$$

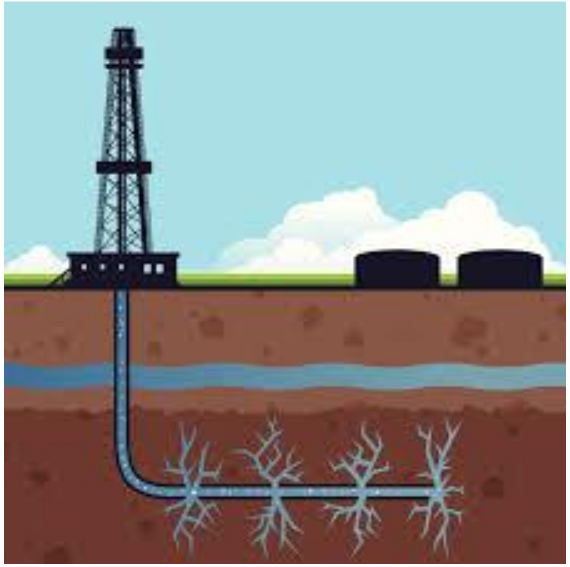
Simulation of a multistage multiaxial test

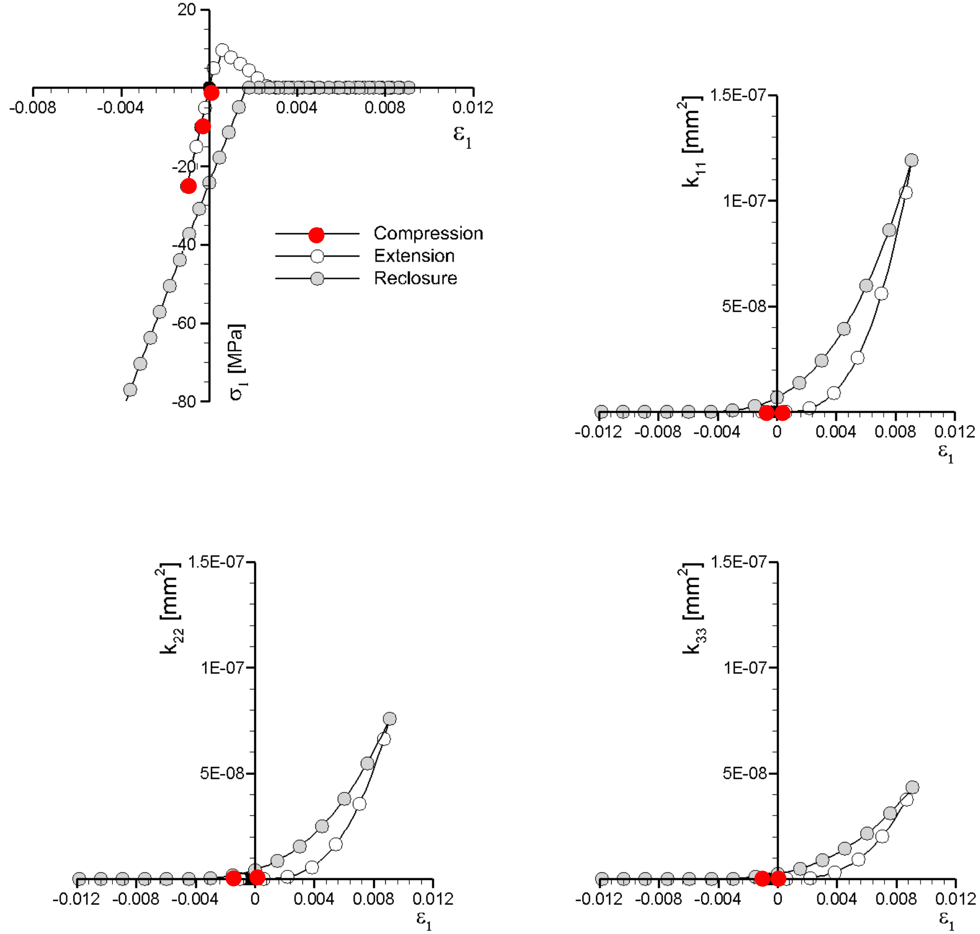
Mimic the variation of stress and permeability in the field due to a fracking job:

Stage 1: Isotropic compression

Stage 2: Isotropic extension

Stage 3: Anisotropic compression





λ (MPa)	G (MPa)	T_c (MPa)	G_c (N/mm)	φ	k_0	n_0
2778	4167	10.0	0.1	45.0	0	0

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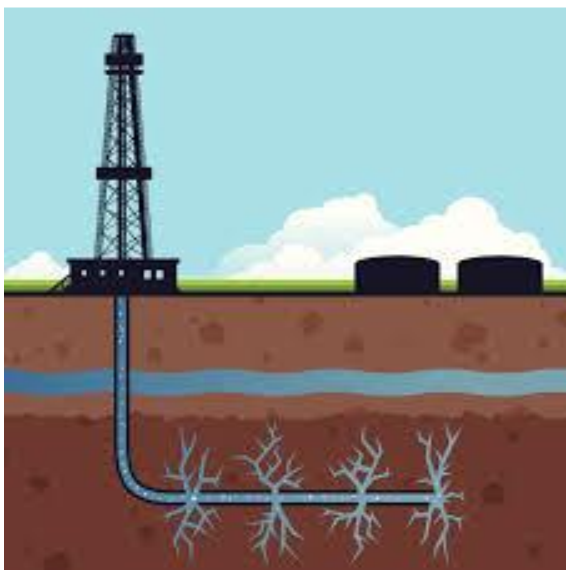

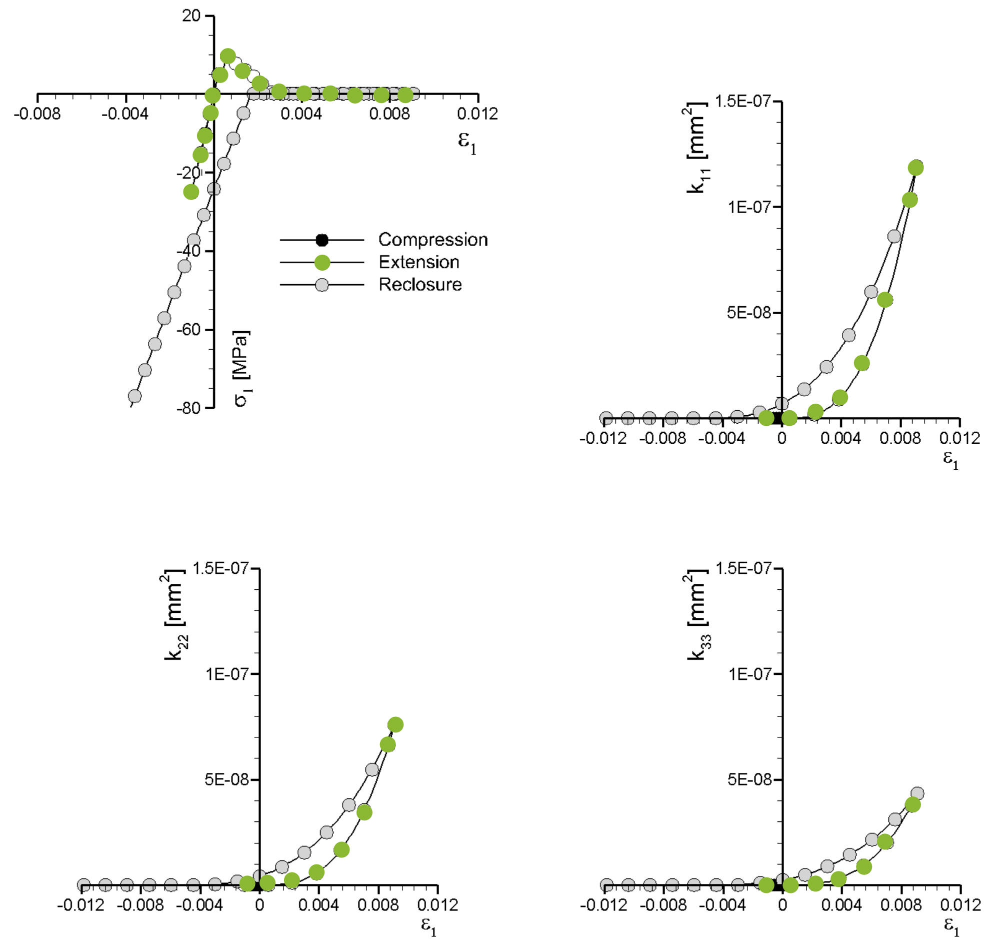

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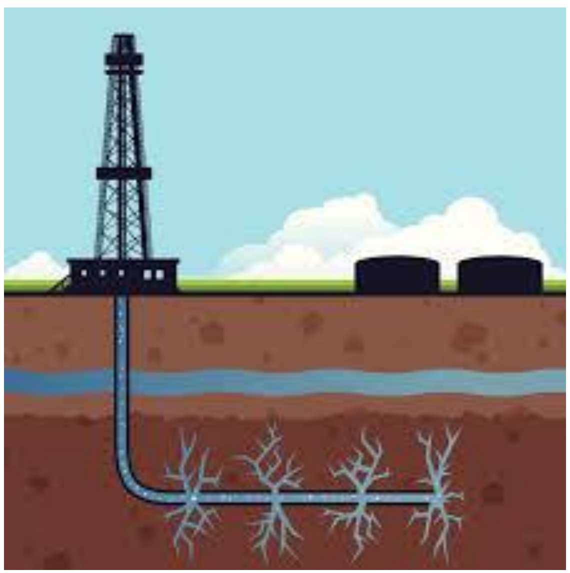

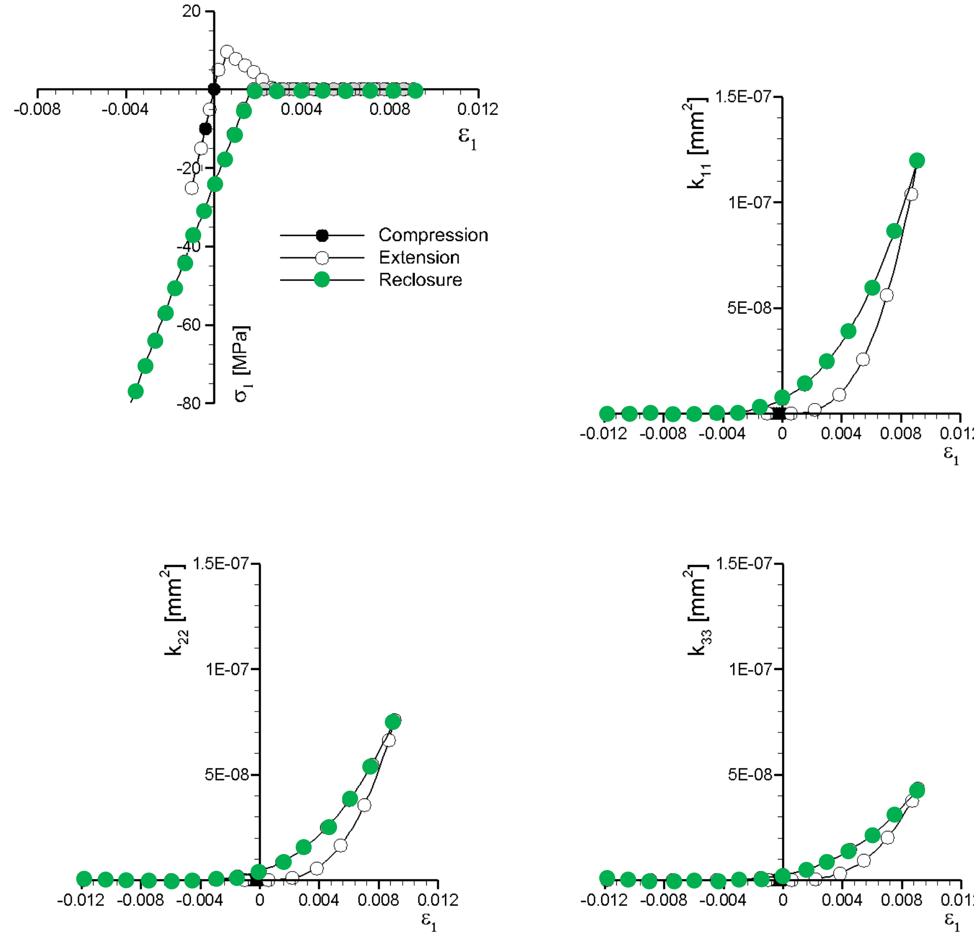

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