Orbifolds and cosets via invariant theory

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Based partly on joint work with T. Arakawa, T. Creutzig, and K. Kawasetsu

1. Orbifolds and cosets

Let $\mathcal{V}$ be a vertex algebra.

$G \subset \text{Aut}(\mathcal{V})$ a finite-dimensional, reductive group. Define *orbifold*

$$\mathcal{V}^G = \{v \in \mathcal{V} | \ gv = v, \quad \forall g \in G\}.$$

$\mathcal{A} \subset \mathcal{V}$ a vertex subalgebra. Define *coset*

$$\text{Com}(\mathcal{A}, \mathcal{V}) = \{v \in \mathcal{V} | \ [a(z), v(w)] = 0, \quad \forall a \in \mathcal{A}\}.$$

(Frenkel-Zhu, 1992).

Suppose $\mathcal{V}$ has a nice property, such as strong finite generation, C_2-cofiniteness, or rationality.

Problem: Do $\mathcal{V}^G$ and $\text{Com}(\mathcal{A}, \mathcal{V})$ inherit this property?

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2. Classical invariant theory

G a finite-dimensional reductive group.

V a finite-dimensional G-module (over $\mathbb{C}$).

$\mathbb{C}[V]$ ring of polynomial functions on V.

$\mathbb{C}[V]^G$ ring of G-invariant polynomials.

Fundamental problem: Find generators and relations for $\mathbb{C}[V]^G$.

Thm: (Hilbert, 1893) $\mathbb{C}[V]^G$ is finitely generated for any G and V.

Basis theorem, Nullstellensatz, and syzygy theorem were all introduced by Hilbert in connection with this problem.

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3. First and second fundamental theorems

Let V be a G-module. For $j \geq 0$, let $V_j \cong V$. Let

$$R = \mathbb{C}[\oplus_{j \geq 0} V_j]^G.$$

First fundamental theorem (FFT) for (G, V) is a set of generators for R.

Second fundamental theorem (SFT) for (G, V) is a set of generators for the ideal of relations in R.

Some known examples:

- Standard representations of classical groups (Weyl, 1939)
- Adjoint representations of classical groups (Procesi, 1976),
- 7-dimensional respresentation of G_2 (Schwarz, 1988).

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4. Example: $G = \mathbb{Z}/2\mathbb{Z}$ and $V = \mathbb{C}$

Generator $\theta \in \mathbb{Z}/2\mathbb{Z}$ acts on V by -1.

x_j a basis for V_j^* for $j \geq 0$.

$\theta(x_j) = -x_j$.

$R = \mathbb{C}[\oplus_{j \geq 0} V_j]^{\mathbb{Z}/2\mathbb{Z}} = \mathbb{C}[x_0, x_1, x_2, \dots]^{\mathbb{Z}/2\mathbb{Z}}$ is the subalgebra of even degree.

FFT: R has quadratic generators $q_{i,j} = x_i x_j, \quad i \leq j$.

SFT: Relations are $q_{i,j} q_{k,l} - q_{i,k} q_{j,l}$.

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5. Example (Dong-Nagatomo, 1999)

Heisenberg vertex algebra $\mathcal{H}$ has generator $b(z)$ satisfying

$$b(z)b(w) \sim (z-w)^{-2}.$$

Basis $\{: \partial^{k_1} b \cdots \partial^{k_r} b : \mid 0 \leq k_1 \leq \cdots \leq k_r\}$.

$\mathrm{Aut}(\mathcal{H}) \cong \mathbb{Z}/2\mathbb{Z}$, generator $\theta : \mathcal{H} \to \mathcal{H}$ acts by $\theta(b) = -b$.

$\mathcal{H}$ is linearly isomorphic to $\mathbb{C}[x_0, x_1, x_2, \dots]$ where $x_j \leftrightarrow \partial^j b$.

Derivation $\partial(x_j) = x_{j+1}$.

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6. Example, cont'd

$R = \mathbb{C}[x_0, x_1, x_2, \dots]^{\mathbb{Z}/2\mathbb{Z}}$ has generators

$$q_{i,j} = x_i x_j, \qquad 0 \leq i \leq j.$$

Relations are $q_{i,j}q_{k,l} - q_{i,k}q_{j,l}$.

$R \cong \mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$, and $q_{i,j}$ correspond to strong generators for $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$:

$$\omega_{i,j} = :\partial^i b \partial^j b:, \qquad 0 \leq i \leq j.$$

Recall $\partial(q_{i,j}) = q_{i+1,j} + q_{i,j+1}$.

$\{q_{0,2k} | k \geq 0\}$ minimal generating set for R as a *differential algebra*.

$\{\omega_{0,2k} | k \geq 0\}$ strongly generates $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$. But this is *not* minimal!

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Thm: (Dong-Nagatomo, 1999) $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$ has minimal strong generating set $\{\omega_{0,0}, \omega_{0,2}\}$, and is of type $\mathcal{W}(2,4)$.

For all $k \geq 2$, we have a *decoupling relation* $\omega_{0,2k} = P(\omega_{0,0}, \omega_{0,2})$.

Ex :
$$\omega_{0,4} = -\frac{2}{5} :\omega_{0,0}\partial^2\omega_{0,0}: + \frac{4}{5} :\omega_{0,0}\omega_{0,2}: + \frac{1}{5} :\partial\omega_{0,0}\partial\omega_{0,0}: + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

Alternatively, this can be written in the form
$$\omega_{0,4} = -\frac{4}{5}(:\omega_{0,0}\omega_{1,1}: - :\omega_{0,1}\omega_{0,1}:) + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

This is a *quantum correction* of the analogous classical relation $q_{0,0}q_{1,1} - q_{0,1}q_{0,1}$.

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Thm: (Dong-Nagatomo, 1999) $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$ has minimal strong generating set $\{\omega_{0,0}, \omega_{0,2}\}$, and is of type $\mathcal{W}(2,4)$.

For all $k \geq 2$, we have a *decoupling relation* $\omega_{0,2k} = P(\omega_{0,0}, \omega_{0,2})$.

Ex : $$\omega_{0,4} = -\frac{2}{5} :\omega_{0,0}\partial^2\omega_{0,0}: + \frac{4}{5} :\omega_{0,0}\omega_{0,2}: + \frac{1}{5} :\partial\omega_{0,0}\partial\omega_{0,0}: + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

Alternatively, this can be written in the form

$$\omega_{0,4} = -\frac{4}{5}(:\omega_{0,0}\omega_{1,1}: - :\omega_{0,1}\omega_{0,1}:) + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

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8. Free field algebras

Heisenberg algebra $\mathcal{H}(n)$: even generators $b^i, i = 1, \dots, n$,

$$b^i(z)b^j(w) \sim \delta_{i,j}(z-w)^{-2}.$$

Free fermion algebra $\mathcal{F}(n)$: odd generators $\phi^i, i = 1, \dots, n$,

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$\beta\gamma$-system $\mathcal{S}(n)$: even generators $\beta^i, \gamma^i, i = 1, \dots, n$,

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Symplectic fermion algebra $\mathcal{A}(n)$: odd generators $e^i, f^i, i = 1, \dots, n$,

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9. Orbifolds of free field algebras

$\mathcal{H}(n)$ and $\mathcal{F}(n)$ have full automorphism group $O(n)$.

$\mathcal{S}(n)$ and $\mathcal{A}(n)$ have full automorphism group $Sp(2n)$.

Thm: (L, 2012) $\mathcal{S}(n)^{Sp(2n)}$ is of type $\mathcal{W}(2, 4, \dots, 2n^2 + 4n)$.

Thm: (L, 2012) $\mathcal{F}(n)^{O(n)}$ is of type $\mathcal{W}(2, 4, \dots, 2n)$.

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Thm: (L, 2012) This conjecture holds for $1 \leq n \leq 6$. For all n, $\mathcal{H}(n)^{O(n)}$ is strongly finitely generated (SFG).

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10. Orbifolds of free field algebras, cont'd

Thm: (L, 2012) Let $\mathcal{V}$ be either $\mathcal{H}(n)$, $\mathcal{F}(n)$, $\mathcal{S}(n)$, or $\mathcal{A}(n)$. For any reductive $G \subset \text{Aut}(\mathcal{V})$, $\mathcal{V}^G$ is SFG.

Sketch of proof: For any reductive $G \subset \text{Aut}(\mathcal{V})$, $\mathcal{V}^G$ is a module over $\mathcal{V}^{\text{Aut}(\mathcal{V})}$.

By a theorem of Dong-Li-Mason (1996), $\mathcal{V}$ has a decomposition

$$\mathcal{V} = \bigoplus_{\nu \in S} L_\nu \otimes M_\nu.$$

L_ν ranges over all irreducible, finite-dimensional $\text{Aut}(\mathcal{V})$-modules.

M_ν are inequivalent, irreducible $\mathcal{V}^{\text{Aut}(\mathcal{V})}$-modules.

Zhu algebra of $\mathcal{V}^{\text{Aut}(\mathcal{V})}$ is abelian, so each M_ν is highest-weight.

Using SFG property of $\mathcal{V}^{\text{Aut}(\mathcal{V})}$, each M_ν is C_1-cofinite.

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11. Orbifolds of free field algebras, cont'd

$\mathcal{V}^G$ is also a direct sum of irreducible $\mathcal{V}^{\text{Aut}(\mathcal{V})}$-modules.

$\mathcal{V}^G$ has a generating set that lies in the direct sum of *finitely many* of these modules.

SFG property of $\mathcal{V}^G$ follows from these observations.

Let $\mathcal{V} = \mathcal{H}(n) \otimes \mathcal{F}(m) \otimes \mathcal{S}(r) \otimes \mathcal{A}(s)$ be a general free field algebra.

Let $G \subset \text{Aut}(\mathcal{V})$ be any reductive group preserving the tensor factors, i.e, $G \subset O(n) \times O(m) \times Sp(2r) \times Sp(2s)$.

Cor: $\mathcal{V}^G$ is SFG.

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12. Deformable families

$K \subset \mathbb{C}$ a subset which is at most countable.

F_K the $\mathbb{C}$-algebra of rational functions

$$\frac{p(\kappa)}{q(\kappa)}, \qquad \deg(p) \leq \deg(q),$$

such that the roots of q lie in K.

A *deformable family* $\mathcal{B}$ is a vertex algebra defined over F_K.

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12. **Deformable families**

$K \subset \mathbb{C}$ a subset which is at most countable.

F_K the $\mathbb{C}$-algebra of rational functions

$$\frac{p(\kappa)}{q(\kappa)}, \qquad \deg(p) \leq \deg(q),$$

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13. **Example**

$V^k(\mathfrak{g})$ universal affine VA of a simple Lie algebra $\mathfrak{g}$.

Take $\xi_1, \dots, \xi_n$ orthonormal basis of $\mathfrak{g}$ with respect to $\langle , \rangle$.

Generators of $V^k(\mathfrak{g})$ are X^{ξ_i}, satisfying

$$X^{\xi_i}(z)X^{\xi_j}(w) \sim k\delta_{i,j}(z-w)^{-2} + X^{[\xi_i,\xi_j]}(w)(z-w)^{-1}.$$

Let $Y^{\xi_i} = \frac{1}{\sqrt{k}}X^{\xi_i}$. These satisfy

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κ a formal variable satisfying $\kappa^2 = k$, and let $K = \{0\}$.

Deformable family $\mathcal{V}$ generated by $\{Y^{\xi_i}\}$ satisfies
$\mathcal{V}_k = \mathcal{V}/(\kappa^2 - k) \cong V^k(\mathfrak{g})$ for $k \neq 0$.

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14. Example, cont'd

Let $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ be a finite-dimensional Lie superalgebra, with $\dim(\mathfrak{g}_0) = n$ and $\dim(\mathfrak{g}_1) = 2m$.

Suppose $\mathfrak{g}$ has a nondegenerate form $\langle , \rangle$.

Then there exists a deformable family $\mathcal{V}$ with $K = \{0\}$ such that $\mathcal{V}_k \cong V^k(\mathfrak{g})$ for $k \neq 0$, and

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Let $\mathcal{W}$ be any tensor product of free field and affine VAs.

Let $G \subset \mathrm{Aut}(\mathcal{W})$ be a reductive group preserving the tensor factors.

Cor: (Creutzig-L, 2014) $\mathcal{W}^G$ is SFG for generic values of k.

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15. Cosets of affine VAs inside larger structures

Let $\mathfrak{g}$ be a reductive Lie algebra with a nondegenerate form $\langle , \rangle$, and $V^k(\mathfrak{g})$ the corresponding affine VA.

Let $\mathcal{B}^k$ be a vertex algebra with structure constants depending continuously on k admitting a map $V^k(\mathfrak{g}) \to \mathcal{B}^k$.

Need some technical assumptions, including:

• Action of $\mathfrak{g}$ integrates to action of a connected Lie group G on $\mathcal{B}^k$ with $\mathfrak{g} = \text{Lie}(G)$.

• $\lim_{k\to\infty} \mathcal{B}^k = \mathcal{H}(n) \otimes \tilde{\mathcal{B}}$ where $n = \dim(\mathfrak{g})$.

Thm: (Creutzig-L, 2014) Let $\mathcal{C}^k = \text{Com}(V^k(\mathfrak{g}), \mathcal{B}^k)$. Then G acts on $\tilde{\mathcal{B}}$ and

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16. **Example**

$\mathfrak{g}$ a simply-laced, simple Lie algebra, $d = \dim(\mathfrak{g})$, $V^k(\mathfrak{g})$ universal affine VA, $V_k(\mathfrak{g})$ simple quotient.

Diagonal map $V^k(\mathfrak{g}) \to V^{k-1}(\mathfrak{g}) \otimes V_1(\mathfrak{g})$.

It is believed that

$$\mathcal{C}^k = \mathrm{Com}(V^k(\mathfrak{g}), V^{k-1}(\mathfrak{g}) \otimes V_1(\mathfrak{g}))$$

is isomorphic to universal principal $\mathcal{W}(\mathfrak{g})$. Known for $\mathfrak{g} = \mathfrak{sl}_2$.

We have

$$\lim_{k\to\infty} V^{k-1}(\mathfrak{g}) \otimes V_1(\mathfrak{g}) = \mathcal{H}(d) \otimes V_1(\mathfrak{g}), \qquad \lim_{k\to\infty} \mathcal{C}^k \cong V_1(\mathfrak{g})^G.$$

It is is known that $V_1(\mathfrak{g})^G \cong \mathcal{W}(\mathfrak{g})$ with $c = \mathrm{rank}(\mathfrak{g})$.

Cor: For generic values of k, $\mathcal{C}^k$ has a minimal strong generating set in the same weights as $\mathcal{W}(\mathfrak{g})$.

For $\mathfrak{g} = \mathfrak{sl}_3$, we have verified $\mathcal{C}^k \cong \mathcal{W}(\mathfrak{sl}_3)$ by computer.

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$\mathfrak{g}$ a simply-laced, simple Lie algebra, $d = \dim(\mathfrak{g})$, $V^k(\mathfrak{g})$ universal affine VA, $V_k(\mathfrak{g})$ simple quotient.

Diagonal map $V^k(\mathfrak{g}) \to V^{k-1}(\mathfrak{g}) \otimes V_1(\mathfrak{g})$.

It is believed that

$$\mathcal{C}^k = \mathrm{Com}(V^k(\mathfrak{g}), V^{k-1}(\mathfrak{g}) \otimes V_1(\mathfrak{g}))$$

is isomorphic to universal principal $\mathcal{W}(\mathfrak{g})$. Known for $\mathfrak{g} = \mathfrak{sl}_2$.

We have

$$\lim_{k\to\infty} V^{k-1}(\mathfrak{g}) \otimes V_1(\mathfrak{g}) = \mathcal{H}(d) \otimes V_1(\mathfrak{g}), \qquad \lim_{k\to\infty} \mathcal{C}^k \cong V_1(\mathfrak{g})^G.$$

It is is known that $V_1(\mathfrak{g})^G \cong \mathcal{W}(\mathfrak{g})$ with $c = \mathrm{rank}(\mathfrak{g})$.

Cor: For generic values of k, $\mathcal{C}^k$ has a minimal strong generating set in the same weights as $\mathcal{W}(\mathfrak{g})$.

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17. Another example

For $n \geq 3$, recall minimal $\mathcal{W}$-algebra $\mathcal{W}^k(\mathfrak{sl}_n, f_\theta)$. We have

$$V^{k+1}(\mathfrak{gl}_{n-2}) \to \mathcal{W}^k(\mathfrak{sl}_n, f_\theta).$$

Let $\mathcal{C}^k = \text{Com}(V^{k+1}(\mathfrak{gl}_{n-2}), \mathcal{W}^k(\mathfrak{sl}_n, f_\theta))$.

Thm: (Arakawa, Creutzig, Kawasetsu, L, 2016) For generic values of k, $\mathcal{C}^k$ is of type $\mathcal{W}(2, 3, \dots, n^2 - 2)$.

For $n \geq 2$, recall minimal $\mathcal{W}$-algebra $\mathcal{W}^k(\mathfrak{sp}_{2n}, f_\theta)$. We have

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18. Structure of the nongeneric set

Given a coset $\mathcal{C}^k = \text{Com}(V^k(\mathfrak{g}), \mathcal{B}^k)$, let S be a strong generating set for $\mathcal{C}^k$ that works for generic values of k.

Call $k \in \mathbb{C}$ *nongeneric* if S does not strongly generate $\mathcal{C}^k$.

Example: $\mathcal{C}^k = \text{Com}(\mathcal{H}, \mathcal{W}^k(\mathfrak{sl}_3, f_\theta))$ is of type $\mathcal{W}(2,3,4,5,6,7)$ for generic values of k. Here $\mathcal{H} = V^{k+1}(\mathfrak{gl}_1)$ is a Heisenberg VA.

Thm: (Arakawa, Creutzig, L, 2015) In this case, the nongeneric set is $\{-1, -3/2\}$.

Idea: First, find an infinite strong generating set $\omega_2, \omega_3, \dots$ that works for all k, where ω_n has weight n. Then find relations

$$\lambda(n,k)\omega_n = P(\omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7), \qquad n \geq 8,$$

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$$\lambda(n,k) = (-1)^{n+1}\frac{n(n-7)(n-5)!}{4!(n-3)!}(k+1)(2k+3).$$

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19. Why is the nongeneric set important?

Suppose we have $V^k(\mathfrak{g}) \to \mathcal{B}^k$. Suppose k is a value such that $\mathcal{B}^k$ is not simple.

Let $\mathcal{B}_k$ be the simple quotient and suppose we have induced map

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Let

$$\mathcal{C}^k = \text{Com}(V^k(\mathfrak{g}), \mathcal{B}^k), \qquad \mathcal{C}_k = \text{Com}(V_k(\mathfrak{g}), \mathcal{B}_k).$$

Always get map $\pi : \mathcal{C}^k \to \mathcal{C}_k$. Under fairly general conditions, π is surjective if k is a *positive real number*.

If π is surjective, strong generators for $\mathcal{C}^k$ descend to strong generators for $\mathcal{C}_k$.

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20. Example: Rational Bershadsky-Polyakov algebras

Thm: (Arakawa, 2010) For $p = 5, 7, \dots$, $\mathcal{W}_{p/2-3}(\mathfrak{sl}_3, f_\theta)$ is C_2-cofinite and rational.

Heisenberg algebra $\mathcal{H} \subset \mathcal{W}_{p/2-3}(\mathfrak{sl}_3, f_\theta)$ is part of a lattice vertex algebra V_L for $L = \sqrt{3p-9}\mathbb{Z}$.

We have

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Cor: This $\mathcal{W}(\mathfrak{sl}_{p-3})$ is of type $\mathcal{W}(2,3,4,5,6,7)$ for all $p \geq 11$ even though the *universal* $\mathcal{W}(\mathfrak{sl}_{p-3})$-algebra is of type $\mathcal{W}(2,3,\dots,p-3)$.

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$$\mathcal{C}_{p/2-3} = \mathrm{Com}(\mathcal{H}, \mathcal{W}_{p/2-3}) = \mathrm{Com}(V_L, \mathcal{W}_{p/2-3}).$$

Thm: (Arakawa, Creutzig, L, 2015) $\mathcal{C}_{p/2-3}$ is isomorphic to the principal, rational $\mathcal{W}(\mathfrak{sl}_{p-3})$-algebra with $c = -\frac{3}{p}(p-4)^2$.

$\mathcal{W}_{p/2-3}(\mathfrak{sl}_3, f_\theta)$ is a simple current extension of $V_L \otimes \mathcal{W}(\mathfrak{sl}_{p-3})$.

Cor: This $\mathcal{W}(\mathfrak{sl}_{p-3})$ is of type $\mathcal{W}(2,3,4,5,6,7)$ for all $p \geq 11$ even though the *universal* $\mathcal{W}(\mathfrak{sl}_{p-3})$-algebra is of type $\mathcal{W}(2,3,\dots,p-3)$.

21. Example: $\mathcal{W}_k(\mathfrak{sl}_4, f_\theta)$ for $k \in \mathbb{N}$

Recall: $\mathcal{C}^k = \text{Com}(V^{k+1}(\mathfrak{gl}_2), \mathcal{W}^k(\mathfrak{sl}_4, f_\theta))$ is generically of type $\mathcal{W}(2, 3, \dots, 14)$.

For $k \in \mathbb{N}$, we have $V_{k+1}(\mathfrak{gl}_2) \to \mathcal{W}_k(\mathfrak{sl}_4, f_\theta)$. Let

$$\mathcal{C}_k = \text{Com}(V_{k+1}(\mathfrak{gl}_2), \mathcal{W}_k(\mathfrak{sl}_4, f_\theta)),$$

which has $c = -\frac{6k^3+31k^2+49k+24}{(k+3)(k+4)}$.

Thm: (Arakawa, Creutzig, Kawasetsu, L, 2016)

- $\mathcal{C}_0$ is isomorphic to the simple $\mathcal{W}(\mathfrak{sl}_3)$-algebra with $c = -2$.
- $\mathcal{C}_1$ is isomorphic to the simple parafermion algebra $\mathcal{N}_{-6/5}(\mathfrak{sl}_2) = \text{Com}(\mathcal{H}, V_{-6/5}(\mathfrak{sl}_2))$.

Conj: For $k \in \mathbb{N}$, $\mathcal{C}_k$ is isomorphic $\text{Com}(\mathcal{H}, \mathcal{W}^{(2)}_{k+1})$. Here $\mathcal{W}^{(2)}_{k+1}$ is the Feigin-Semikhatov algebra.

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22. **Example:** $\mathcal{W}_k(\mathfrak{sp}_4, f_\theta)$ **for** $k \in 1/2 + \mathbb{N}$

Recall: $\mathcal{C}^k = \text{Com}(V^{k+1/2}(\mathfrak{sp}_2), \mathcal{W}^k(\mathfrak{sp}_4, f_\theta))$ is generically of type $\mathcal{W}(2,4,6,8,10)$.

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Conj: $\mathcal{C}_k$ is isomorphic to principal, rational $\mathcal{W}(\mathfrak{sp}_{2k+1})$ with $c = -\frac{6(2+k)^2(1+2k)}{(3+k)(5+2k)}$.

If true, this family of rational $\mathcal{W}(\mathfrak{sp}_{2k+1})$-algebras is of type $\mathcal{W}(2,4,6,8,10)$ for all but finitely many values of k.

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