BIRS Workshop: Combinatorics Meets Ergodic Theory
July 19 - 23, 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager
9:00–9:45	Vitaly Bergelson, *Some new results and open problems on cubic averages*
9:50–10:15	Coffee Break
10:15–11:00	Joel Moreira, *Partition regularity of polynomial configurations*
11:15–12:00	Freddie Manners, *The structural theory of nilspaces*
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:45	Lilian Matthiesen, *Multiplicative functions and nilsequences*
14:50	Group Photo; meet in foyer of TCPL
15:00–15:30	Coffee Break
15:30–16:15	Alexander Fish, *Recurrence, measure rigidity and characteristic polynomial patterns in difference sets of matrices*
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:45	Terence Tao, *Concatenation theorems for the Gowers uniformity norms, and applications*
10:00–10:30	Coffee Break
10:30–11:15	Yonatan Gutman, *Characterization of Host-Kra factors through a structural theorem for dynamical nilspaces*
11:30–12:15	Anush Tserunyan, *A general van der Corput lemma and underlying Ramsey theory*
12:15–13:30	Lunch
14:00–14:45	Donald Robertson, *Finite products sets and minimally almost periodic groups*
15:00–15:30	Coffee Break Free (Late) Afternoon
17:30–19:00	Dinner
19:00–20:30	Problem Session and Beer (run by Ben Green)

Wednesday

7:00–9:00	Breakfast
9:00–9:45	Tom Sanders, *Bounds in the Freiman-Ruzsa Theorem for Abelian groups of bounded exponent*
10:00–10:30	Coffee Break
10:30–11:15	Thomas Bloom, *Higher-order additive structure*
11:30–12:15	Pavel Zorin-Kranich, *Corners theorem over non-commutative groups*
12:15–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:45	Ben Krause, *A random pointwise ergodic theorem with Hardy field weights*
10:00–10:30	Coffee Break
10:30–11:15	Wenbo Sun, *Dynamical cubes and a criteria for systems having product extension*
11:30–12:15	Sebastian Donoso, *A pointwise cubic average for two commuting transformations*
12:15–13:30	Lunch
14:00–14:45	Isaac Goldbring, *High piecewise syndeticity of product sets in amenable groups*
15:00–15:30	Coffee Break
	Free (Late) Afternoon
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:45	Randall McCutcheon, *IP rich sets in* $\mathbb{Z}$
10:00–10:15	Coffee Break
10:15–11:00	Trevor Wooley, *Restriction theory and perturbations of Weyl sums*
11:30–13:30	Lunch

Checkout by
12 noon

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ABSTRACTS

Speaker: **Vitaly Bergelson** (Ohio State University)
Title: *Some new results and open problems on cubic averages*
Abstract: The term "cubic averages" refers to various (and sometimes far-reaching) multiple-parameter generalizations of the classical Khintchine's recurrence theorem and their combinatorial applications. We will survey various results on cubic averages, discuss the recent advancements and formulate some natural open problems.

Speaker: **Thomas Bloom** (University of Bristol)
Title: *Higher-order additive structure*
Abstract: A crucial part of the recent spectacular progress made in the cap-set problem by Bateman and Katz was a new result relating smooth methods to measure the additive properties of a set (e.g. the additive energy) with structural methods (e.g. density within a subgroup), in the spirit of Freiman's theorem and the Balog-Szemerédi-Gowers theorem. We discuss this result, a simplified proof, and various applications, including to the Gowers uniformity norms.

Speaker: **Sebastian Donoso** (University of Chile)
Title: *A pointwise cubic average for two commuting transformations*
Abstract: Huang, Shao and Ye recently studied pointwise multiple averages by using suitable topological models. Using a notion of dynamical cubes introduced by the authors, the Huang-Shao-Ye technique and the Host machinery of magic systems, we prove that for a system (X, μ, S, T) with commuting transformations S and T, the average

$$\frac{1}{N^2}\sum_{i,j=0}^{N-1} f_0(S^i x) f_1(T^j x) f_2(S^i T^j x)$$

converges a.e. as N goes to infinity for any $f_0, f_1, f_2 \in L^\infty(\mu)$. This is a joint work with Wenbo Sun.

Speaker: **Alexander Fish** (University of Sydney)
Title: *Recurrence, measure rigidity and characteristic polynomial patterns in difference sets of matrices*
Abstract: We present a new approach for establishing the recurrence of a set, through measure rigidity of associated action. Recall, that a subset S of integers (or of another amenable group G) is recurrent if for every set E in integers (in G) of positive density there exists a non-zero $s \in S$ such that the intersection of E and $E - s$ has positive density. By use of measure rigidity results of Benoist-Quint for algebraic actions on homogeneous spaces, we prove that for every set E of positive density inside traceless square matrices with integer values, there exists $k \geq 1$ such that the set of characteristic polynomials of matrices in $E - E$ contains ALL characteristic polynomials of traceless matrices divisible by k. This talk is based on a joint work with M. Bjorklund (Chalmers).

Speaker: **Isaac Goldbring** (University of Illinois at Chicago)
Title: *High piecewise syndeticity of product sets in amenable groups*
Abstract: In 2002, R. Jin proved that if A and B are subsets of the integers of positive Banach density, then A+B is piecewise syndetic. Later, Beiglbock, Bergelson, and Fish generalized Jins result to arbitrary countable amenable groups. In this talk, I will discuss a quantitative generalization of the amenable group version of Jins result by giving lower bounds for the density of the set of witnesses to the piecewise syndeticity of the product set BA. The new results in this talk are joint with M. Di Nasso, R. Jin, S. Leth,

M. Lupini, and K. Mahlburg and was partially completed during an American Institute of Mathematics SQUARES program.

Speaker: **Yonatan Gutman** (Instytut Matematyczny Polskiej Akademii Nauk)
Title: *Characterization of Host-Kra factors through a structural theorem for dynamical nilspaces*
Abstract: The factors introduced by Host & Kra (and by Ziegler using a different framework) for an ergodic $\mathbb{Z}$ -system are higher order analogues of the classical Kronecker factor. An important fact is that they are characteristic for various non-conventional ergodic averages including the one used by Furstenberg in order to establish Szemerdi's theorem. A key structural result implies that these factors are inverse limits of nilsystems. We give a new proof of this structural theorem in the context of finitely generated abelian group actions. The main idea is use a topological structure theorem joint with Freddie Manners and Péter Varjù which characterizes dynamical nilspaces as inverse limits of nilsystems (generalizing a theorem by Host, Kra and Maass). Nilspaces, which were introduced by Camarena and Szegedy are certain compact spaces which obey a relatively simple set of axioms. The proof consist of showing that the Host-Kra factors can be modelled as dynamical nilspaces to which the topological structural theorem applies.

Speaker: **Ben Krause** (University of California at Los Angeles)
Title: *A random pointwise ergodic theorem with Hardy field weights*
Abstract: Since Bourgain's work in the late eighties, random sequences have been used as a model for pointwise ergodic theorems. One indication at their amenability to analysis is LaVictoires L^1 random ergodic theorem:

Let $\{X_n\}$ be a sequence of independent $\{0,1\}$ valued random variables (on a probability space Ω) with expectations n^{-a}, $0 < a < 1/2$, and define the *counting function* $a_n(\omega)$ to be the smallest integer subject to the constraint

$$X_1(\omega) + \cdots + X_{a_n(\omega)}(\omega) = n.$$

LaVictoire proved the following theorem.

Theorem 1 (LaVictoire's Random Ergodic Theorem). *Almost surely: for each measure-preserving system (X, μ, T) and each $f \in L^1(X)$ the averages*

$$\frac{1}{N} \sum_{n=1}^{N} T^{a_n(\omega)} f$$

converge pointwise μ-almost everywhere.

The first aim of this talk will be to sketch LaVictoire's random ergodic theorem; we will then discuss a *Wiener-Wintner* variant of his result obtained in collaboration with Pavel Zorin-Kranich (University of Bonn):

Theorem 2. *In the above setting, let $0 < \epsilon < 1$ be arbitrary; almost surely, the following holds:*
For each measure-preserving system (X, μ, T) and each $f \in L^1(X)$ the averages

$$\frac{1}{N} \sum_{n=1}^{N} e(n^{1+\epsilon})\, T^{a_n(\omega)} f$$

converge to zero pointwise μ-almost everywhere.

Speaker: **Freddie Manners** (University of Oxford)
Title: *The structural theory of nilspaces*
Abstract: There are currently two approaches to the inverse theorem for the Gowers norms (often described as higher-order Fourier analysis): one by Green, Tao and Ziegler, and another by Szegedy, and Camarena and Szegedy, which builds on previous work by Host and Kra (and others).

The latter approach centres around axiomatic notions of "cubespaces" and "nilspaces", and in particular a powerful structural theorem for these objects. Roughly speaking, these objects consist of a topological space X equipped with some notion of when 2^k points of X form a "k-cube", subject to various axioms. The structural result says, again very roughly, that these objects are necessarily built out of nilmanifolds G/Γ.

I will discuss various aspects of these ideas, drawing on recent joint work with Yonatan Gutman and Péter Varjú.

Speaker: **Lilian Matthiesen** (Leibniz Universität Hannover)
Title: *Multiplicative functions and nilsequences*
Abstract: We discuss bounds on the correlation of multiplicative functions with polynomial nilsequences and their applications to evaluating linear correlations among themselves.

Speaker: **Randall McCutcheon** (University of Memphis)
Title: *IP rich sets in $\mathbb{Z}$*
Abstract: Some classical methods for establishing (multiple) recurrence in ergodic theory show that sets supporting Følner sequences (i.e. thick sets) qualify as sets of recurrence. IP methods, where successful, show more, namely that sets containing all finite sums of terms from an infinite sequence ("IP sets") qualify as sets of recurrence. Where IP methods fail, a hybrid method (often involving the notion of a "D set") has gained some traction in the past ten years. More recently, Bergelson-Leibman and Bergelson-Robertson have proved that a (broader than that of D-sets, as it turns out) class of "IP rich" sets are as good for recurrence in several settings where the corresponding results for IP sets are open. In this talk I will discuss tree-structure characterizations of both classes (IP rich sets and D sets) and sketch an example of an IP rich set that is not a D set.

Speaker: **Joel Moreira** (Ohio State university)
Title: *Partition regularity of polynomial configurations*
Abstract: A fundamental problem in Ramsey theory is to determine which configurations are partition regular. In 1933, R. Rado classified all linear configurations over the integers with this property, vastly improving upon van der Waerden's theorem on arithmetic progressions. The situation with polynomial configurations is very different, and partition regularity for some very simple looking polynomial configurations remains an open problem. I will describe some recent progress towards the partition regularity of the configuration $\{x+y, xy\}$. This is joint work with V. Bergelson.

Speaker: **Donald Robertson** (Ohio State University)
Title: *Finite products sets and minimally almost periodic groups*
Abstract: In this talk I will present joint work with V. Bergelson, C. Christopherson and P. Zorin-Kranich on a combinatorial characterization of minimally almost periodic, amenable groups based on examples, due to Strauss, of large sets of integers no shifts of which contain IP sets.

Speaker: **Tom Sanders** (Oxford University)
Title: *Bounds in the Freiman-Ruzsa Theorem for Abelian groups of bounded exponent*
Abstract: Ruzsa showed how covering lemmas could be used to prove Freimans theorem in $(\mathbb{Z}/2\mathbb{Z})^n$. Indeed, he showed that if $|A+A| \leq K|A|$ then A generated a group of size $\exp(O(K^4))|A|$ which up to the power is almost optimal. Since then Zohar, following work of Green and Tao, has established the exact largest size this group can be as a function of K and $|A|$ using compressions. These compression techniques have been extended to groups of the form $\mathbb{F}_p^n$, but they are harder to extend much more generally. On the other hand covering arguments extend very easily, so there is a natural question as to whether covering arguments can be used to get better versions of Freimans theorem. In this talk we shall discuss how to do this the results are slight, but perhaps the approach is interesting.

Speaker: **Wenbo Sun** (Northwestern University)
Title: *Dynamical cubes and a criteria for systems having product extension*
Abstract: For a minimal $\mathbb{Z}^2$-topological system, we introduce the cube structure and a generalization of the regionally proximal relation, which allow us to characterize product systems and their factors. We also introduce the concept of topological magic systems, which is the topological counterpart of measure theoretic magic systems introduced by Host in his study of multiple averages for commuting transformations. We give various applications of these structures, including the construction of some special factors in topological dynamics, and a computation of the automorphism group of tiling systems. This is joint work with Sebastian Donoso.

Speaker: **Terence Tao** (University of California at Los Angeles)
Title: *Concatenation theorems for the Gowers uniformity norms, and applications*
Abstract: A function $P(n,m)$ of two variables which is a polynomial of degree less than d_1 in the n variable, and a polynomial of degree less than d_2 in the m variable, is automatically a polynomial of degree less than d_1+d_2-1 jointly in the n,m variables. In joint work with Tamar Ziegler, we generalise this "concatenation of degrees" phenomenon to the Gowers unifomity norms; roughly speaking, we show that a function $f(n,m)$ which is "Gowers anti-uniform" of order d_1 in the n variable and Gowers anti-uniform of order d_2 in the m variable is automatically Gowers anti-uniform of order d_1+d_2-1 jointly in n,m. An analogous ergodic theory concatenation theorem is also obtained for the characteristic factors of the Gowers-Host-Kra seminorms.

As an application of this concatenation theorem, we can control certain "averaged local Gowers uniformity norms" by "global Gowers uniformity norms", which among other things yields asymptotics for the number of polynomial patterns in the primes.

Speaker: **Anush Tserunyan** (University of Illinois at Urbana-Champaign)
Title: *A general van der Corput lemma and underlying Ramsey theory*
Abstract: What often lies at the heart of multiple recurrence results is that for measure-preserving actions of semigroups, mixing along a suitable filter on the semigroup amplifies itself to multiple mixing along the same filter. This amplification is usually proved using a so-called van der Corput difference lemma. Instances of this lemma for specific filters have been proven before by Furstenberg, Bergelson-McCutcheon, and others, with a somewhat different proof in each case. We define a notion of differentiation for subsets of semigroups and isolate a class of filters that respect this notion. The filters in this class (call them ∂-filters) include all those for which the van der Corput lemma was known, and our main result is a van der Corput lemma for ∂-filters, which thus generalizes all its previous instances. This is done via proving a Ramsey theorem for graphs on the semigroup.

Speaker: **Trevor Wooley** (University of Bristol)
Title: *Restriction theory and perturbations of Weyl sums*
Abstract: We estimate mean values of exponential sums with arbitrary weights, deriving restriction theorems for t-term arguments of essentially optimal strength for moments of order s when $s \le t^2$. These estimates may be used to extend recent work of Smith, Keil and Henriot, concerning the solubility of certain translation invariant equations, from quadratic to higher degrees. In addition, these estimates yield strong bounds for weighted exponential sums on sets of full measure. The "efficient congruencing" method plays a pivotal role in this work.

Speaker: **Pavel Zorin-Kranich** (University of Bonn)
Title: *Corners theorem over non-commutative groups*
Abstract: The first viable non-commutative generalization of the corners theorem (a formulation of the multidimensional Szemeredi theorem) has been proposed by Bergelson, McCutcheon, and Zhang, who proved it in the case $k=2$. We discuss the lower bounds on the number of corners in the case $k=2$, some extensions to the case $k>2$, and uniform distribution of corner sizes over quasirandom groups.