

Focussed Research Group on Geophysical Viscoplastic Flows

Neil J. Balmforth (Department of Mathematics, UBC)

October 25 – November 1, 2015

1 Preamble

Mud, lava and ice are examples of complex geophysical fluids, in the sense that they have a complex constitution at the microscopic scale that controls their macroscopic material properties and complicates their flow behaviour in comparison to simple viscous fluids like air and water. Under the right physical conditions, all three geophysical materials can collapse and flow, creating natural hazards and triggering environmental and economic disasters. The unifying feature of the flow behaviour of these three materials is that they possess a yield stress and nonlinear rheology - the hallmarks of viscoplastic fluids.

A recent example relevant locally to Banff was the failure of the Mount Polley mine tailings dam. In August 2014, this failure released millions of cubic metres of water and toxic mine waste products in a few hours, prompting the government to declare a local state of emergency and leaving a devastated area in which the long-term environmental impacts remain unclear. Fluid mechanics is key to understanding several aspects of the disaster, ranging from the catastrophic erosional incision of the dam breach at the initiation of the release, to the sediment plume entering Quesnel lake at its terminus. More relevantly to viscoplastic fluid mechanics, the outburst likely took the form of a concentrated mud flow over part of its duration. Such flows are similar to dense snow avalanches and other types of landslides and debris flows, but the detailed mathematical modelling of such phenomenon remains in its infancy.

In October of 2015, BIRS hosted the *Sixth Workshop on Viscoplastic Fluids: from Theory to Experiment*. The first meeting of this series was held at Banff in 2005, and the sixth installment returned to BIRS a decade later. The Focussed Research Group ran concurrently with this workshop to gain a topical perspective on the status of the subject and to consider in particular the mathematical modelling of geophysical applications of viscoplastic fluid mechanics. The focussed group remained together beyond the 5-day workshop, dedicating more time to existing collaborations, to develop new ones, and, overall, to discuss a range of more specific problems in more depth than were presented the workshop.

2 Group members

- Neil Balmforth, University of British Columbia
 - Richard Craster, Imperial College
 - Duncan Hewitt, University of Cambridge
 - Sarah Hormozi, University of Ohio
 - Andy Hogg, University of Bristol
 - Jim McElwaine, University of Durham
 - David Pritchard University of Strathclyde
 - Anthony Wachs, University of British Columbia

The group members all had experience in the modelling of complex geophysical flows. However, they also had a spectrum of expertise ranging from numerical computation to asymptotic analysis and analogue experimentation. Several had already forged existing collaborations; we built upon these and established more, exploiting the different, but complementary approaches taken by different group members

Existing collaborations between the group members include some fostered by some of the earlier workshops on viscoplastic fluids. From the perspective of techniques, numerical expertise is provided by Hormozi, Hewitt, McElwaine and Wachs, asymptotic analysis by Balmforth, Craster, Hogg and Pritchard, experimental expertise by McElwaine. In terms of areas of application, Hormozi, Balmforth and Wachs are experts in viscoplastic fluids, McElwaine is leading researchers in granular materials, Hormozi, Pritchard and Wachs have research experience in engineering problems, and Balmforth, Hewitt and Hogg have expertise in complex geophysical fluids.

3 Agenda

The original mandate of the group members was focussed on the mathematical modelling of environmental hazards like mud flows, outbursts from tailing ponds and granular avalanches. However, as the 5-day workshop progressed, it became clear what other topical and interesting problems might be profitably discussed by the FRG over the days that followed. The major step forward was to identify and outline some specific problems to look at, either collectively as a whole group, or in smaller collaborations of group members. Since the meeting, progress has already been made on some of these problems. For the future, our intent is to maintain the FRG as a productive collective and to try to meet regularly on similar occasions to the Banff meeting.

Specific topics that were identified and prompted in depth discussions:

Stopping times: Owing to the nature of the fluid, and specifically the yield stress, viscoplastic fluids are often observed to approach a state of rest in finite time when the driving forces are insufficient to sustain motion. By contrast, asymptotic theories for free surfaces flows, of the kind typical in complex geophysical gravity currents, suggest that the stopping time is infinite (as shown by Hogg, Balmforth and Craster and others). Computations by some of the group members of the full flow problem, with any asymptotic approximation, do in fact suggest the stopping time is finite. Evidently, there is some feature of the asymptotics that is incorrect. One possibility is that the asymptotic approximation is not uniform in time and breaks down as the fluid comes to rest, demanding a more refined treatment, a more faithful perturbation theory.

The effect of thixotropy: A recent recognition in viscoplastic fluid mechanics is that the material structure of real fluids is rather more complicated than captured by standard models. Notably, ageing effects resulting from the relatively swift destruction of the microstructure, but its much lengthier healing, result in a persistent time-dependent rheology that is poorly captured by current models which assume an instantaneous relation between stress and strain rate. Many clays and other geophysical materials are thixotropic, but a systematic analysis of how this might affect their flow behaviour in environmental settings has yet to be provided (Balmforth, Hewitt and Pritchard have made initial steps in this direction). From cruder perspective, the FRG speculated on whether related ideas might be relevant to more varied problems including the surges seen on ice-covered rives and lakes, and to the mechanism of mucus clearance during a cough.

Failure mechanisms: Problems in geotechnical engineering are sometimes posed in terms of the calculation of critical factors dictating the onset of flow, or failure. In the collapse of a vertical embankment, for example, one requires the critical height or cohesive strength of the soil. Such problems are often posed using the limit-point analysis of plasticity theory. In fact, the ideal plastic material is closely related to the perfect viscoplastic fluid, which the two potentially behaving identically near the onset of flow. Thus the failure criteria plasticity theory may be relevant in viscoplastic fluid mechanics. Despite this, viscoplastic fluids often appear to fail due to the appearance of narrow viscoplastic boundary layers which are not necessarily features of the plasticity problem (as computations by Balmforth, Hormozi and Wachs have shown). Classical discussion of such boundary layers dates back to Oldroyd in 1947. However, his boundary layer equations are complicated nonlinear partial differential equations that have never been solved. A major question is whether they do indeed characterize the boundary layer, or whether they do not actually have a solution and a different theory is needed. There is also an interesting connection with granular mechanics (the failure of

sand piles and so forth) which McEwaine and Hogg have studied extensively, both experimentally and with discrete element simulations. A key question is whether the analysis of the granular problem can inform the viscoplastic theory, or vice versa. For all these contexts, the FRG discussed the idea that significant progress might be made by looking at specific, idealized flow problems. For such problem, one can provide numerical computations of the full flow problem to compare with boundary-layer asymptotics. Such computations require detailed care in view of the small but finite velocities that characterize failure near the critical conditions (the limit point), and there is a distinct possibility that new tools of numerical analysis are needed to deal accurately with the situation.

Suspension mechanics: Recent studies of concentrated suspension have proposed different rheological characterizations than the traditional viscoplastic constitutive laws. In part, these are based on the granular nature of the solid constituents, opting for a two-phase approach rather than the single-phase theories common in viscoplasticity. Nevertheless, both approaches lead to constitutive laws that describe fluid yield stress and nonlinear viscous behaviour. This raises the question of how to relate the two theories, and in what situations one might be preferable over the other. The applications include the flow of slurries through cracks in hydrofracture and mud hydrology, or the disposal of mine tailings. Also related is the compressional rheology of mineral slurries, which is used to characterize these materials and is necessary to understand their de-watering characteristics during disposal. De-watering is essential in other types of suspensions, such as wood pulp (as recently explored by Hewitt and others), with potentially significant industrial impact. Geophysical currents are usually dominated by shear stresses, whereas de-watering problems more typically involve compression. This raises the interesting question of whether the material behaves differently under compression than in shear, and if it does, how to characterize that. A critical effect that poses many natural hazards and which results from suspension rheology is liquefaction. As far as the FRG was aware, the modelling of this process was rudimentary, and much could be gained from improving this situation. The FRG considered approaching the problem from the theoretical side. In particular, we discussed how to formulate model flow problems that could be explored using existing empirical constitutive models. The results can be compared with solutions from computational simulations at the particle level (as pioneered by Wachs and Hormozi), thereby providing a demanding test of the empirical models.

Slip: Recent experimental work has demonstrated conclusively that many viscoplastic materials slip over smooth surfaces. This is not real slip, but a manifestation of the development of spatial inhomogeneity within the fluid: the microstructure which dictates the non-Newtonian behaviour becomes depleted near the surface due to the migration of the constituents that form the microstructure out of a thin layer adjacent to the surface. This apparent slip is well known for polymer solutions, and has now been demonstrated clearly for many viscoplastic fluids. Despite this, slip is often ignored in theoretical modelling, primarily because one needs a characterization of the slip; *i.e.* a slip law. Until recently, little information on such laws was available. However, careful experiments and micromechanical models have now proposed a number of realistic, calibrated slip laws that can be incorporated into theoretical flow models. This now places us in a position where we can model wall slip and revise many of the flow solutions to problems in which this phenomenon should be taken into account and previously been incorrectly modelled by its omission. The slip issue certainly applies to geophysical gravity currents for which the slip over the underlying surface may well be a controlling factor in the flow dynamics, including when materials become mobile and when they return to rest. There is also the interesting question of whether the modelling in comparison to observations or experiments could inform the slip model in situations in which the calibration or the slip law itself is not known.

4 Bibliography

The best sources of references for recent work in viscoplastic fluid dynamics is the series of proceedings that have followed the bi-annual workshops:

- Visoplastic Fluid Mechanics: from theory to application
- I: *Journal of Non-Newtonian Fluid Mechanics*, bf vol. 142
- II: *Journal of Non-Newtonian Fluid Mechanics*, bf vol. 158
- III: *Rheologica Acta*, bf vol. 50

IV: *Journal of Non-Newtonian Fluid Mechanics*, bf vol. 193

V: *Journal of Non-Newtonian Fluid Mechanics*, bf vol. 220

A further issue of the *Journal of Non-Newtonian Fluid Mechanics* has been set aside for the upcoming sixth proceedings.