Transport w/o quasiparticles

Good metals, bad metals and insulators

Sean Hartnoll (Stanford) -- BIRS, Feb. 2013

Based on:

1201.3917 w/ Diego Hofman

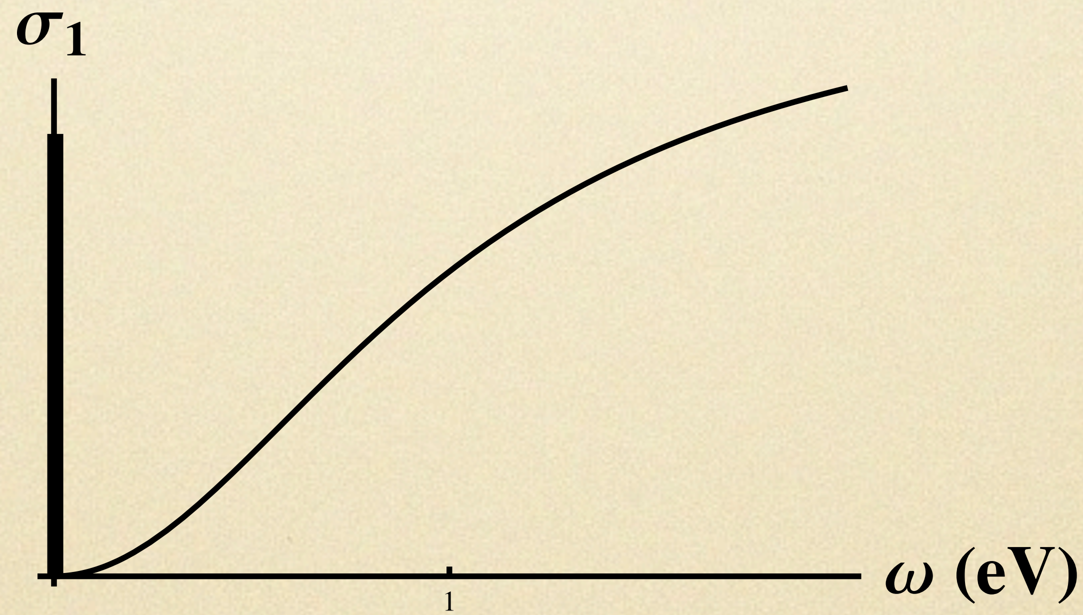
1212.2998 w/ Aristos Donos

(also work in progress with Barkeshli & Mahajan)

Finite density transport

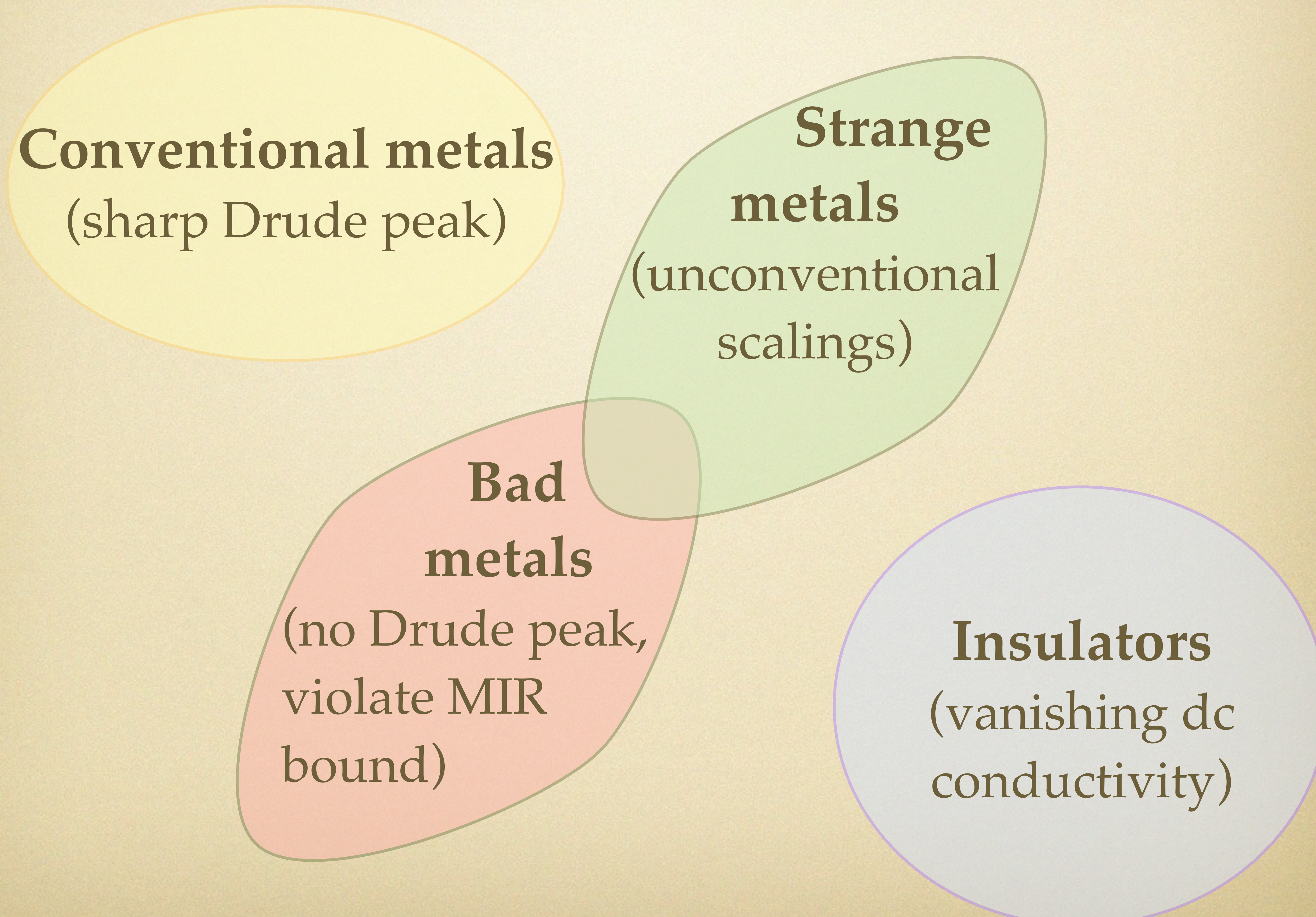
- If the total momentum (or any other operator that overlaps with the total current) is conserved, the d.c. conductivity is infinite.

- The **optical conductivity** of a perfect metal:

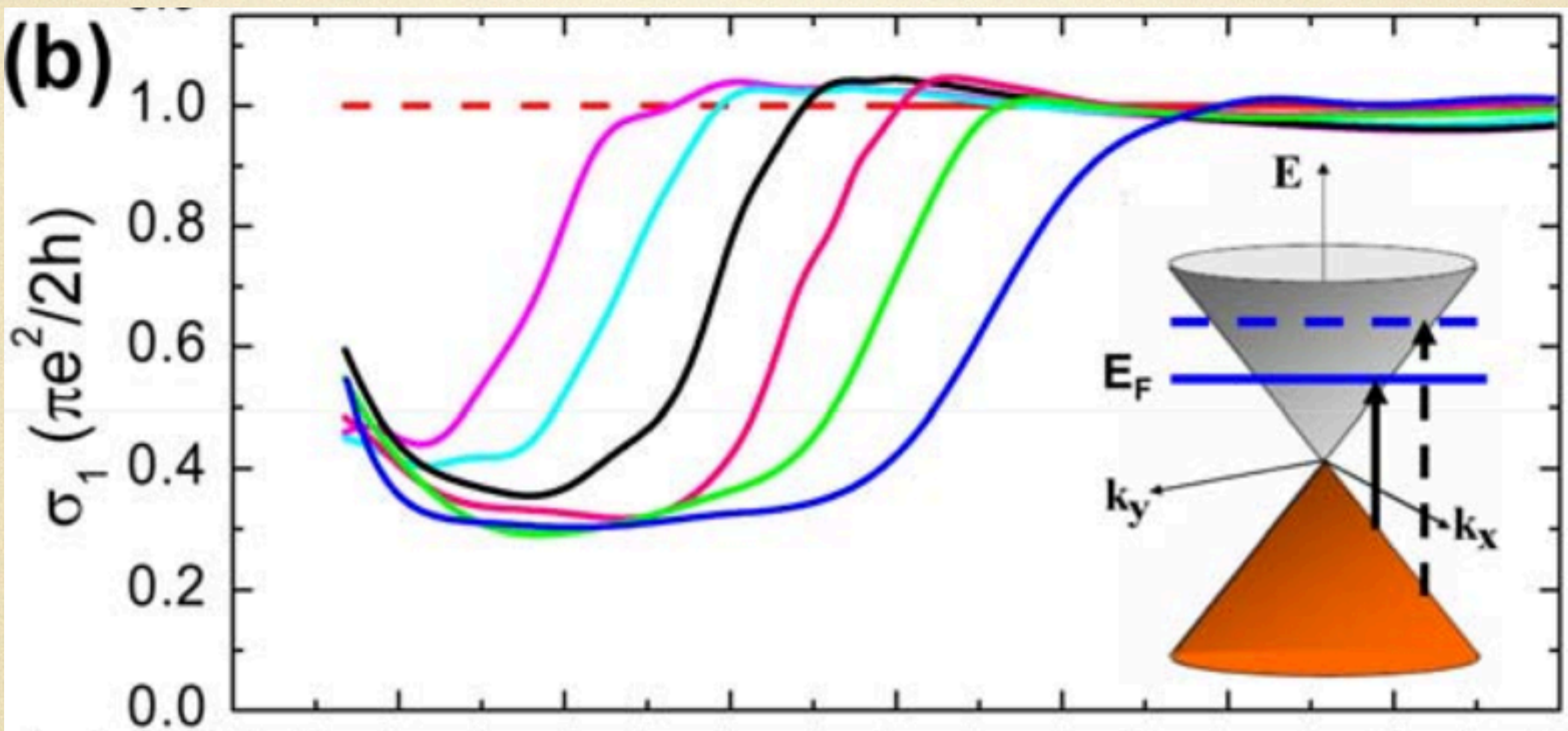


- **Only** alternative to breaking momentum conservation is to **dilute** the charge carriers. Makes spectral weight of delta function small: $\frac{\chi_{JP}^2}{\chi_{JJ}\chi_{PP}} \ll 1$

Observed behaviors



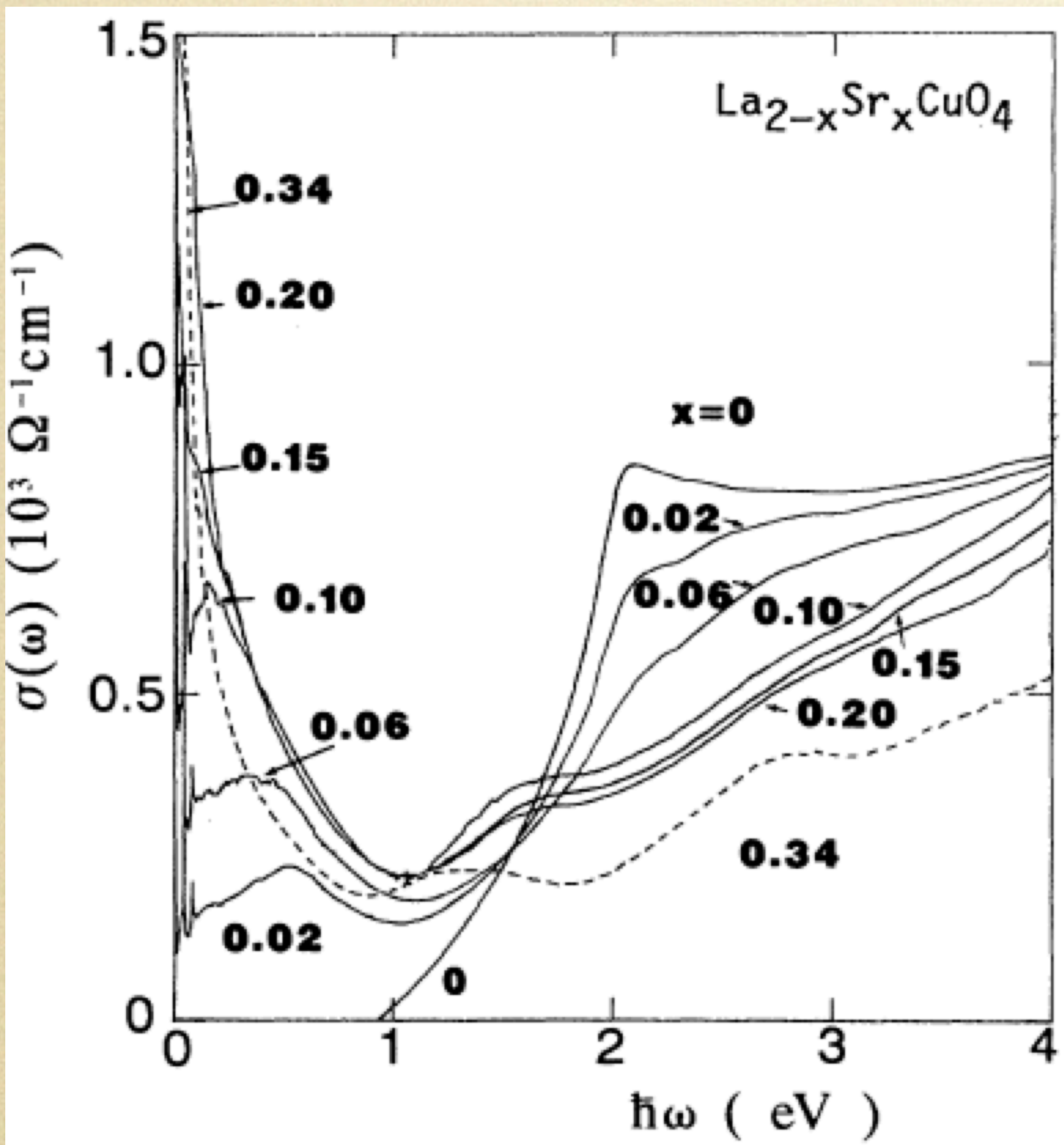
A conventional metal



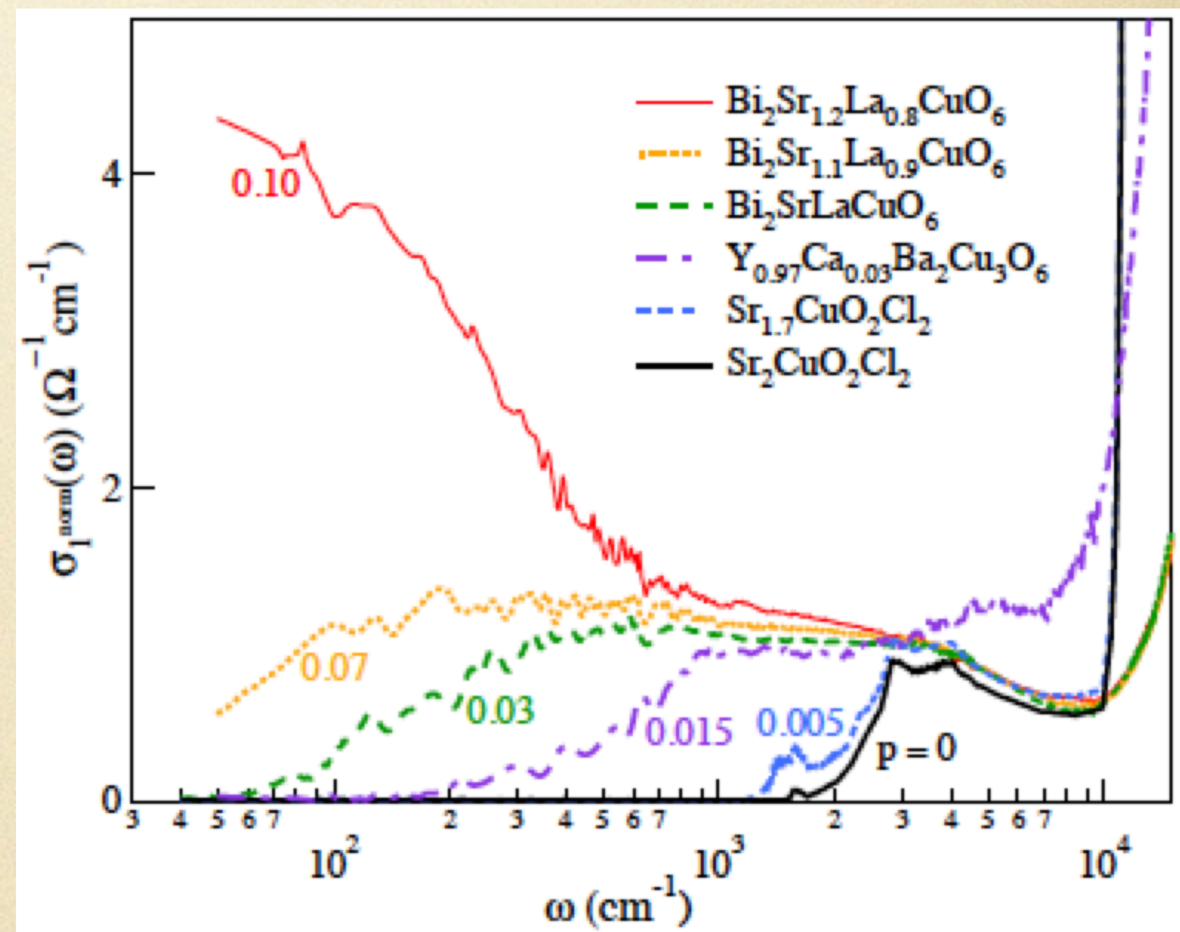
Optical conductivity in graphene by Li et al. 0807.3780

Metal-insulator transitions

Dramatic spectral weight transfer from Drude peak to interband scales: **Itinerant** to **localized** charge



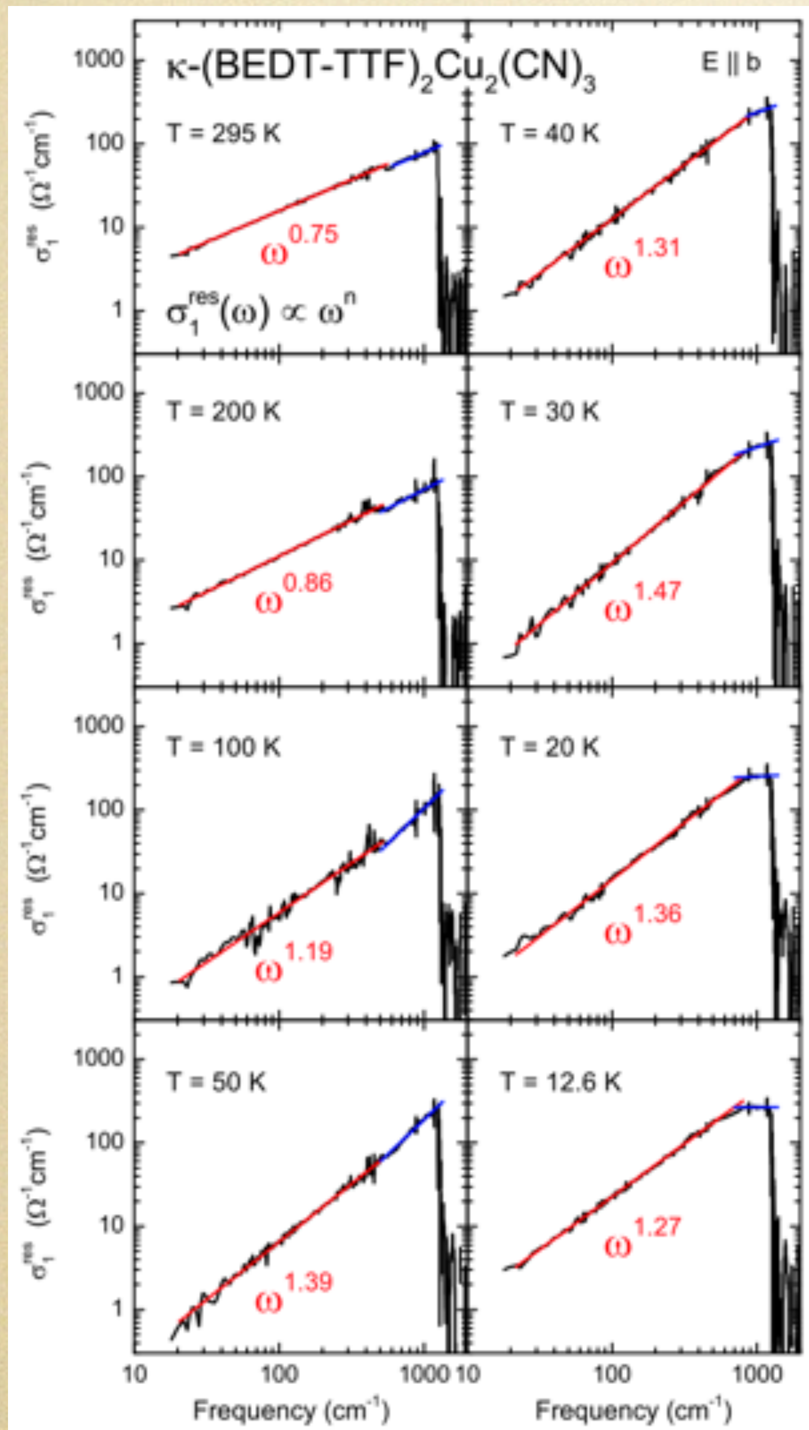
Uchida et al. ’91



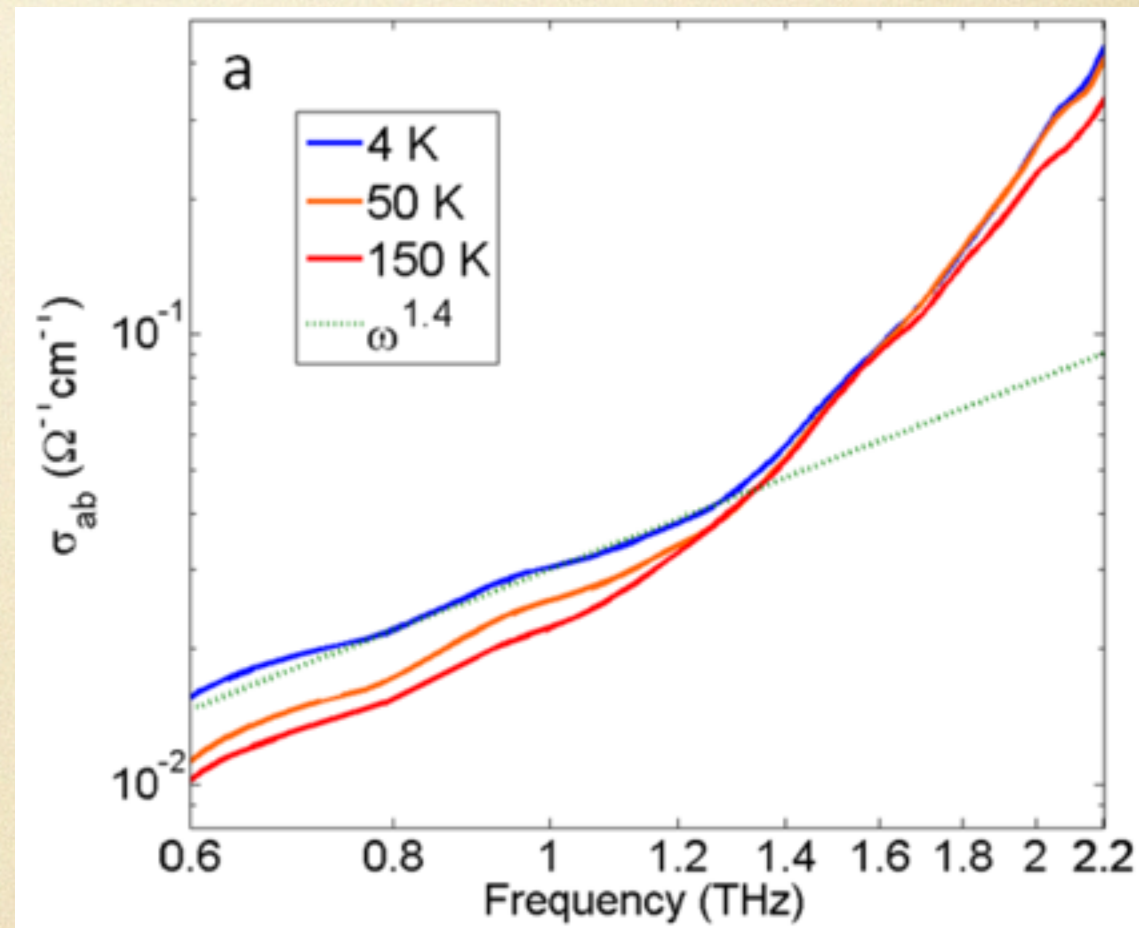
Nicoletti et al ’11

Gapless insulators

Quantum spin liquid candidates show a power law 'soft' gap in the optical conductivity



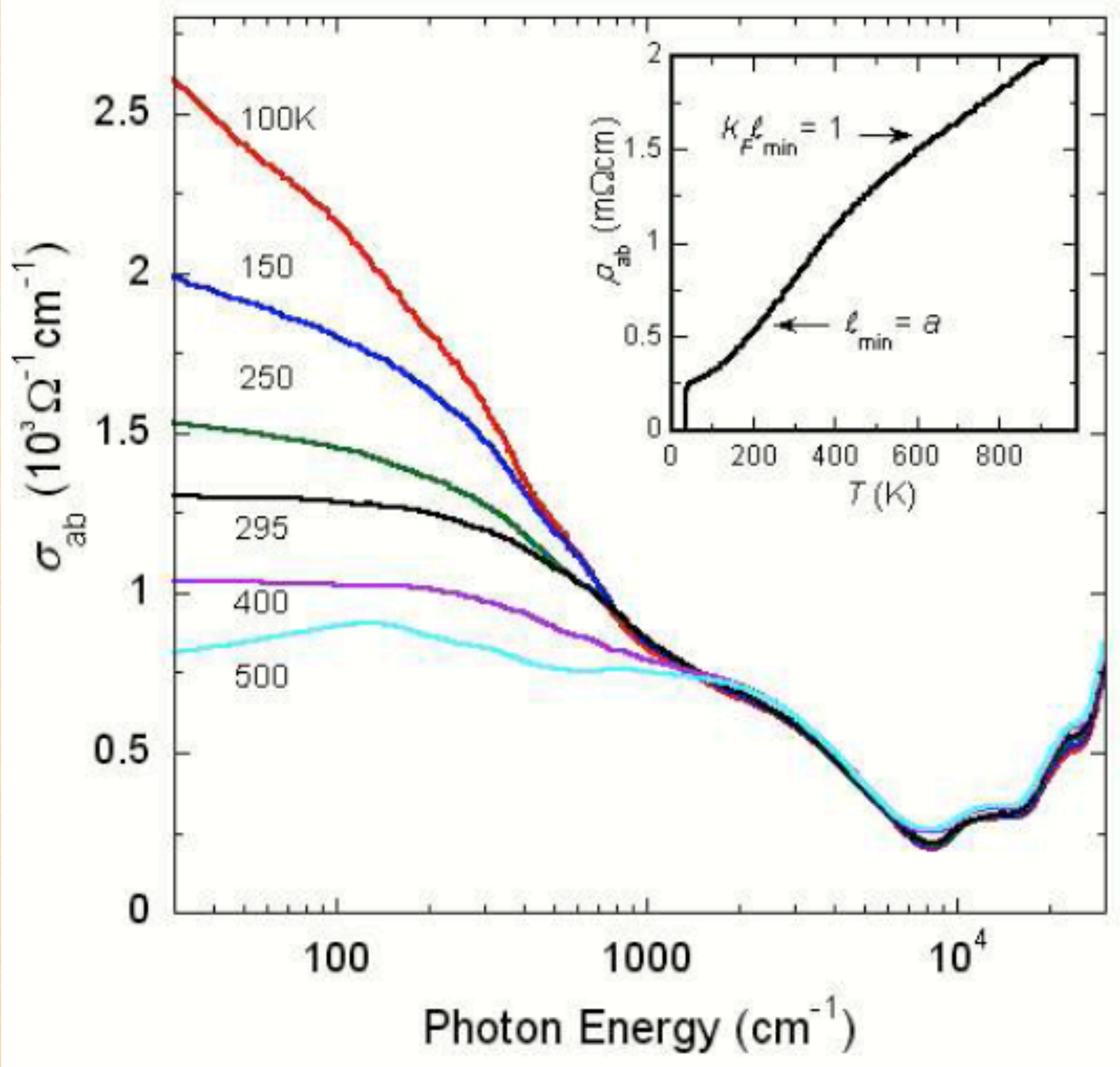
Elsässer et al. 1208.1664



Herbertsmithite

by Pilon et al 1301.3501

A bad metal



$La_{1.9}Sr_{0.1}CuO_4$ by Takenaka et al.'03, from Hussey et al. '04

Theory of sharp Drude peaks

- Sharp Drude peak
  $\Leftrightarrow$ Momentum relaxation rate Γ small

- Can treat momentum-nonconserving operators (remnant of UV lattice) as perturbations of a translationally invariant effective IR theory.

- E.g. Umklapp scattering in a Fermi liquid:

$$\mathcal{O}(k_L) = \int \left(\prod_{i=1}^{4} d\omega_i d^2 k_i \right) \psi^\dagger(k_1)\psi^\dagger(k_2)\psi(k_3)\psi(k_4)\delta(k_1 + k_2 - k_3 - k_4 - k_L)$$

- In holographic models, often least irrelevant operator is:

$$J^t(k_L)$$

Theory of sharp Drude peaks

- Framework to treat Γ perturbatively:

  Memory matrix formalism. (cf. Rosch and Andrei, 1+1)

$$\sigma(\omega) = \frac{1}{-i\omega + M(\omega)\chi^{-1}}\chi\,,$$

$$M(\omega) = \int_0^{1/T} d\lambda \left\langle \dot{A}(0)\mathcal{Q}\frac{i}{\omega - \mathcal{Q}L\mathcal{Q}}\mathcal{Q}\dot{B}(i\lambda)\right\rangle\,.$$

- For case of scattering by a lattice

$$\Gamma = \frac{g^2 k_L^2}{\chi_{\vec{P}\vec{P}}} \lim_{\omega\to 0} \left.\frac{\operatorname{Im} G^R_{\mathcal{O}\mathcal{O}}(\omega, k_L)}{\omega}\right|_{g=0}\,.$$

(Hartnoll and Hofman, 1201.3917)

Semi-local criticality

(cf. Iqbal, Liu, Mezei)

- Common in holography that IR geometries have $z = \infty$ (with or without ground state entropy).
- Scaling of time but not space $\Rightarrow$ efficient low energy dissipation in momentum-violating processes.
- Find e.g. dc resistivity

$$r(T) \sim T^{2\Delta(k_L)}$$

(Hartnoll and Hofman, 1201.3917)

(Confirmed numerically by Horowitz, Santos, Tong)

- Theories with $z < \infty$ **do not** dissipate efficiently in momentum-violating processes.

Making the lattice relevant

- Claim: (at least some) metal-insulator transitions are described by momentum-nonconserving operators becoming relevant in the effective low energy theory.

(cf. Emery, Luther, Peschel, 1+1)

- We found a holographic realization of this mechanism.

(Donos and Hartnoll, 1212.2998)

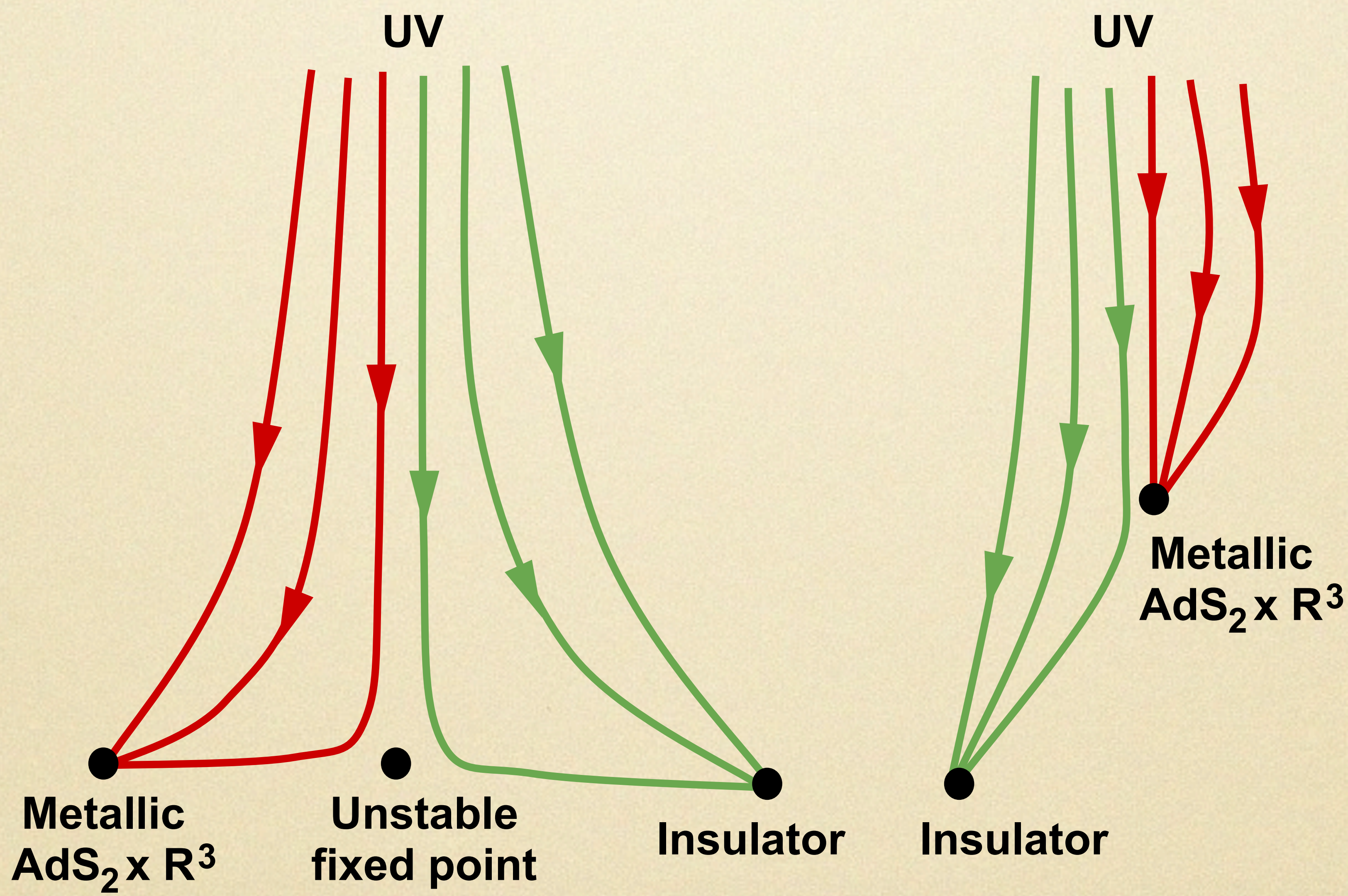
- Simple theory

$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{4}W_{ab}W^{ab} - \frac{m^2}{2}B_aB^a\right) - \frac{\kappa}{2}\int B \wedge F \wedge W\,.$$

(Chern-Simons term not essential but helps to find the IR geometries)

RG flow scenarios

The theory has ($T=0$) IR geometries both with and without translation invariance



(Donos and Hartnoll, 1212.2998)

A technical simplification

- To capture the physics without solving PDEs we use a lattice that breaks translation invariance while retaining homogeneity:

$$B^{(0)} = \lambda\, \omega_2$$

$$\omega_2 + i\omega_3 = e^{ipx_1}\,(dx_2 + i dx_3)$$

(Invariant under Bianchi VII_0 algebra,
cf. Nakamura-Ooguri-Park, Donos-Gauntlett, Kachru-Trivedi-....)

- Beyond a simplification, realization of smectic metal phases due to strong yet anisotropic lattice scattering in the IR.

(cf. Emery, Fradkin, Kivelson, Lubensky;
Vishwanath, Carpentier)

Metal-insulator transition

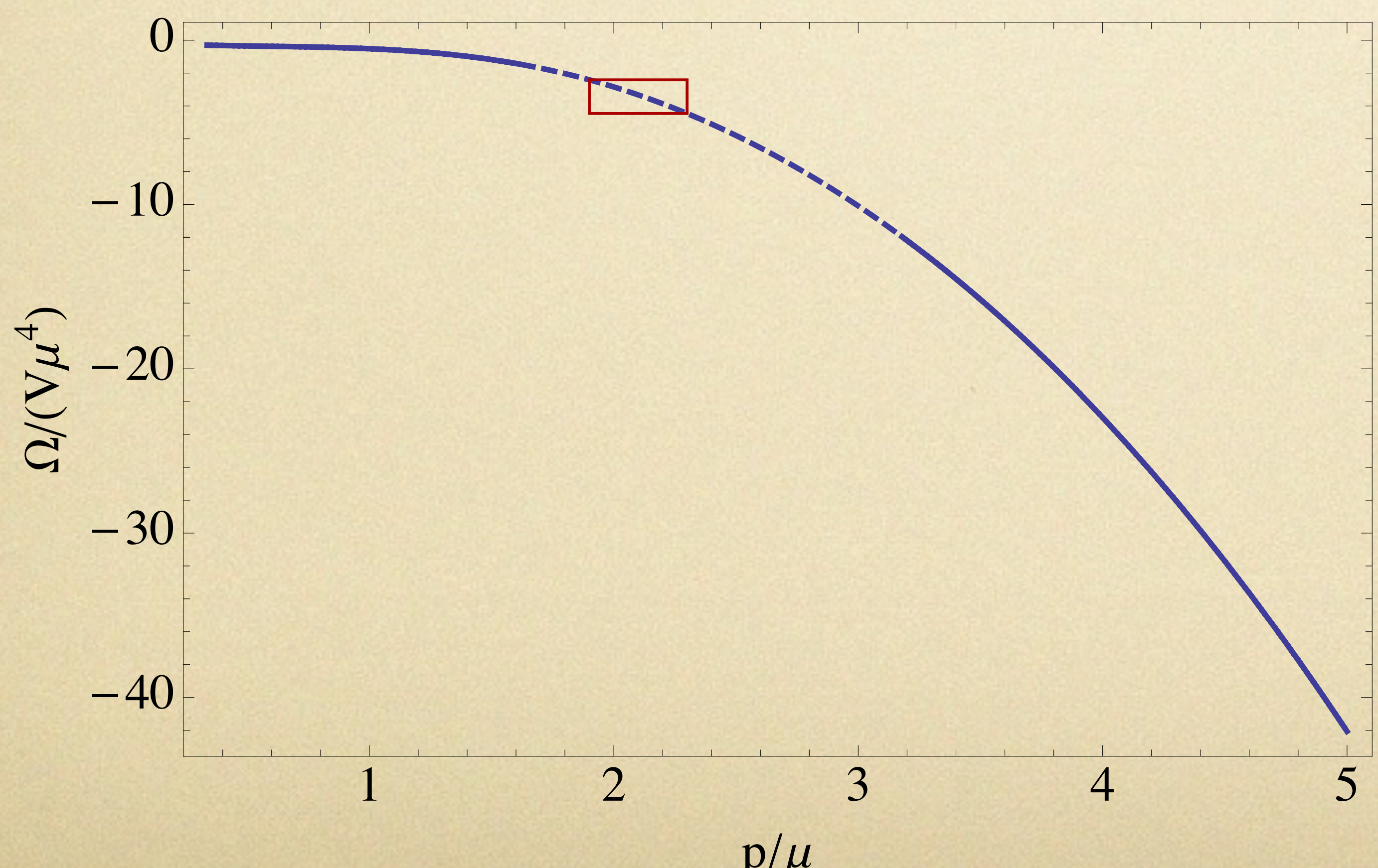
(Donos and Hartnoll, 1212.2998)

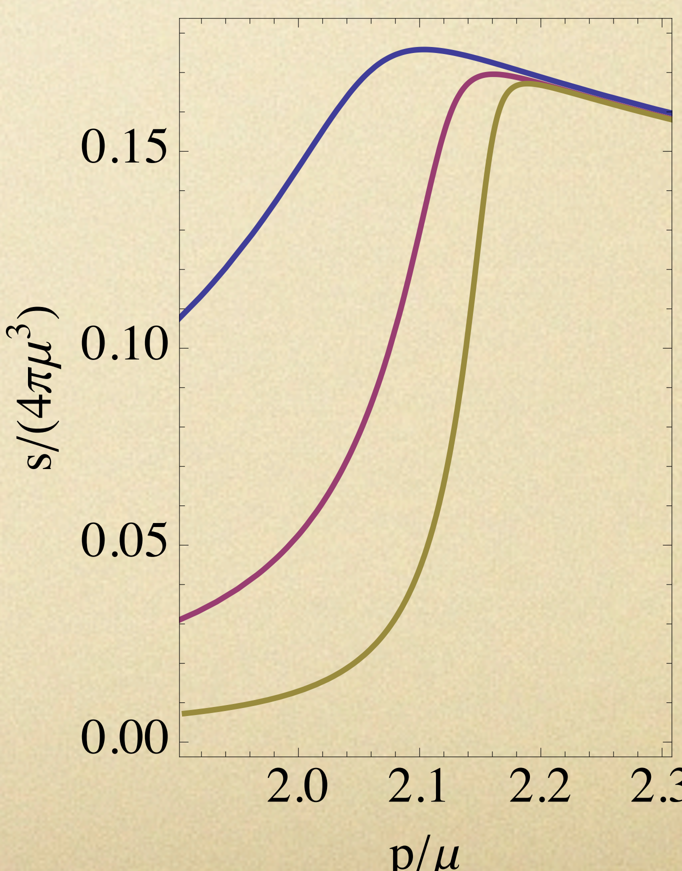
- The metallic and insulating IR geometries are:

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + dx^2_{\mathbb{R}^3}\,, \quad A = 2\sqrt{6}\, r\, dt\,, \quad B = 0\,.$$

$$ds^2 = -cr^2 dt^2 + \frac{dr^2}{cr^2} + \frac{dx_1^2}{r^{1/3}} + r^{2/3}\omega_2^2 + r^{1/3}\omega_3^2\,, \quad A = 0\,, \quad B = b\,\omega_2\,.$$

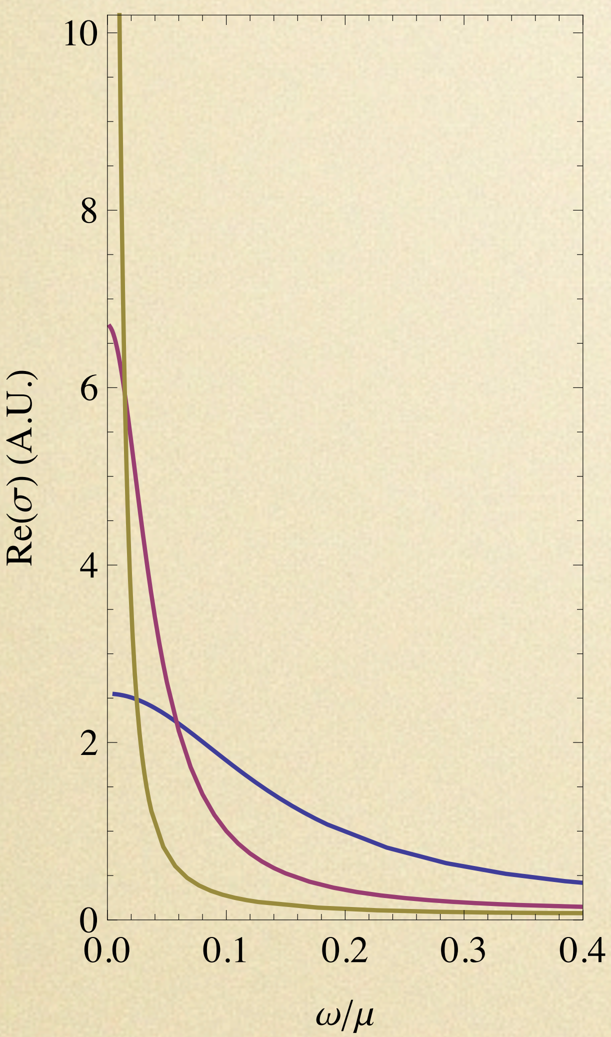
(cf. D'Hoker and Kraus)

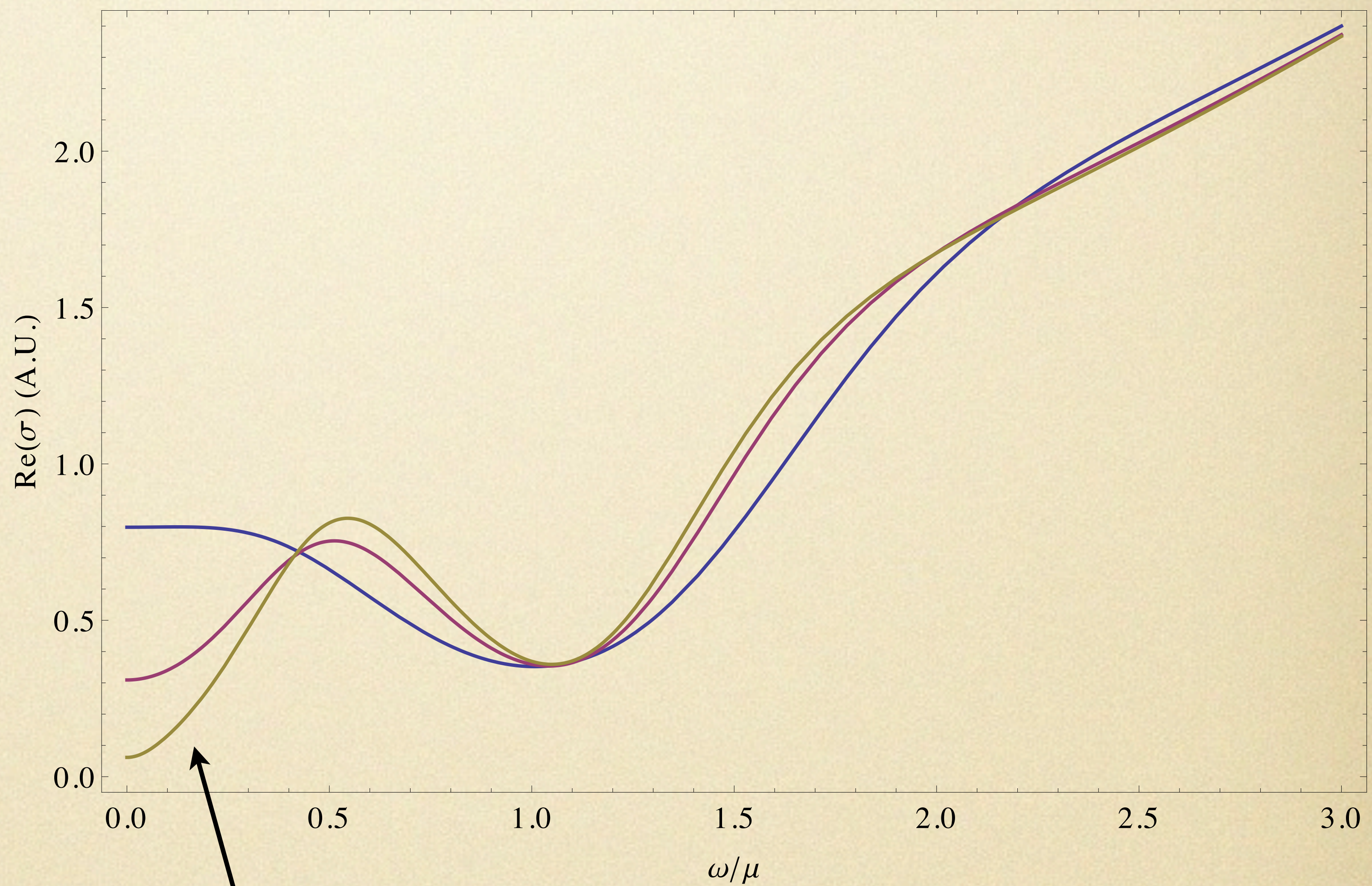




Spectral weight transfer

(Donos and Hartnoll, 1212.2998)





Metal

Insulator

$$\sigma(\omega) \sim \omega^{4/3} \quad \text{at } T = 0$$

Further comments

- Can compute d.c. conductivities analytically:

$$\begin{aligned} \text{metal:} \quad & \sigma(T) \sim T^{-2\Delta(k_L)} \,, \\ \text{insulator:} \quad & \sigma(T) \sim T^{4/3} \,. \end{aligned}$$

- In between the metallic and insulating phases we found bad metals with no Drude peak and large resistivities.

- ‘Mid-infrared peak’ in insulating phase.

Take home messages

- Objective: non-quasiparticle language for transport.
- Good metal: Effective low energy theory translation invariant up to perturbative effects of momentum-nonconserving operators.
- Effects become relevant: metal-insulator transition.
- Holography precisely realizes this scenario.
- Simple model exhibits experimental features that are difficult to otherwise describe in a controlled way: major spectral weight transfer, bad metals, insulators with power law gaps.