BIRS Workshop 13w5037: Interactions of gauge theory with contact and symplectic topology in dimensions 3 and 4
March 24–29, 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:55	**Matt Hedden**: *Taut foliations, left-orderability, and L-spaces*
10:00–10:30	Coffee Break, TCPL
10:30–11:25	**Ina Petkova**: *An absolute $\mathbb{Z}/2$ grading on bordered Floer homology*
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required)
14:10–15:05	**Paolo Lisca**: *Stein fillable contact 3-manifolds and positive open books of genus one*
15:05–15:30	Coffee Break, TCPL
15:30–16:25	**Cagatay Kutluhan**: *Holonomy filtration and knots*
16:35–17:30	**R. Inanc Baykur**: *Topological complexity of symplectic 4-manifolds and Stein fillings*
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:55	**Vincent Colin**: *An extension of Heegaard Floer homology to higher dimensions*
10:00–10:30	Coffee Break, TCPL
10:30–11:25	**Andriy Haydys**: *Fukaya-Seidel category and gauge theory*
11:30–13:30	Lunch
13:30–14:15	**Lenny Ng**: *Topological strings and knot contact homology I*
14:20–15:05	**Tobias Ekholm**: *Topological strings and knot contact homology II*
15:05–15:30	Coffee Break, TCPL
15:30–16:15	**Jonathan Bloom**: *A bordered monopole Floer theory I*
16:30–17:15	**John Baldwin**: *A bordered monopole Floer theory II*
17:30–19:30	Dinner

Wednesday

7:00–8:30	Breakfast
8:30–9:25	**David Gay**: *Morse 2-functions on and trisections of 4-manifolds*
9:35–10:30	**Michael Hutchings**: *Embedded contact homology as a (symplectic) field theory*
10:30–11:00	Coffee Break, TCPL
11:00-11:55	**Olga Plamenevskaya**: *Looking for flexibility in higher-dimensional contact manifolds*
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:55	**Katrin Wehrheim**: *How to construct 2+1+1 topological field theories via the symplectic category*
10:00–10:30	Coffee Break, TCPL
10:30–11:25	**Stefano Vidussi**: *On the topology of SCY 4-manifolds*
11:30–13:30	Lunch
14:00–14:55	**Daniel Ruberman**: *Embeddings of non-orientable surfaces in* $M^3 \times I$
15:00–15:30	Coffee Break, TCPL
15:30–16:25	**Liam Watson**: *Bordered Heegaard Floer homology and the Alexander module*
16:35–17:30	**Thomas Mark**: *Floer homology and the fractional Dehn twist coefficient*
17:30–19:30	Dinner

Friday

7:00–8:30	Breakfast
8:30–9:25	**Nikolai Saveliev**: *Index theory of the de Rham complex on manifolds with periodic ends*
9:35–10:30	**Chris Herald**: *The pillowcase and perturbations of traceless representations of knot groups*
10:30–11:00	Coffee Break, TCPL
11:00-11:55	**Eric Harper**: *Instanton homology of corks* W_n
12:00–13:30	Lunch

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Speaker: **John Baldwin** (Boston College)
Title: *A bordered monopole Floer theory II* [see J. Bloom for part I]
Abstract: I'll pick up where Jon's talk left off, focusing more on the topological aspects of our construction. In particular, I'll describe a finite set of bordered handlebodies which "generate" our category. The morphism spaces for this generating set define a finitely generated A-infinity algebra. A bordered 3-manifold then gives rise to a module over this algebra, and a cobordism of bordered 3-manifolds defines a map of modules. I'll describe how one proves a pairing theorem and how the homology of our algebra is related to Khovanov's H^n algebra.

Speaker: **R. Inanc Baykur** (MPIM Bonn)
Title: *Topological complexity of symplectic 4-manifolds and Stein fillings*
Abstract: Following the ground-breaking works of Donaldson and Giroux, Lefschetz pencils and open books have become central tools in the study of symplectic 4-manifolds and contact 3-manifolds. An open question at the heart of this relationship is whether or not there exists an a priori bound on the topological complexity of a symplectic 4-manifold, coming from the genus of a compatible Lefschetz pencil on it, and a similar question inquires if there is such a bound on any Stein filling of a fixed contact 3-manifold, coming from the genus of a compatible open book. We will present our solutions to both questions, making heroic use of positive factorizations in surface mapping class groups of various flavors. This is joint work with J. Van Horn-Morris.

Speaker: **Jonathan Bloom** (MIT)
Title: *A bordered monopole Floer theory I* [see J. Baldwin for part II]
Abstract: I'll discuss work-in-progress with John Baldwin toward constructing a gauge theoretic analogue of the Fukaya category and monopole Floer theoretic invariants of bordered 3-manifolds. Our construction associates an A-infinity category to a surface, an A-infinity functor to a bordered 3-manifold, and an A-infinity natural transformation to a 4-dimensional cobordism of bordered 3-manifolds. In this talk, I'll describe the basic ideas that go into constructing these categories, functors, and natural transformations.

Speaker: **Vincent Colin** (Univ. Nantes)
Title: *An extension of Heegaard Floer homology to higher dimensions*
Abstract: In dimension three, Heegaard Floer homology can be computed from the page and the monodromy of an open book decomposition supporting a contact structure. In a joint work in progress with Yasha Eliashberg and Ko Honda, we explain how to extend the definition of $\widehat{\mathrm{HF}}$ to contact manifolds of arbitrary odd dimension.

Speaker: **Tobias Ekholm** (Uppsala)
Title: *Topological strings and knot contact homology II* [see L. Ng for part I]
Abstract: After a very brief description of some aspects of topological string theory and its relation to Chern-Simons theory, we discuss possible geometric explanations of the recently observed relation between knot contact homology and open topological strings. A key role is played by Lagrangian fillings of the conormal lift of a link that we will use in several ways. For example, we discuss a certain class of non-exact fillings that when equipped with a flat U(1)-connection induce augmentations and thereby parameterize branches of the augmentation variety that agree with branches of the corresponding variety defined through topological string theory.

Speaker: **David Gay** (U. Georgia)
Title: *Morse 2-functions on and trisections of 4-manifolds*
Abstract: I'll outline how to use Morse 2-functions (stable maps to dimension 2) to get a perfect analogue in dimension 4 of the existence and uniqueness of Heegaard splittings in dimension 3. The analogue of a Heegaard splitting of a 3-manifold is a trisection of a 4-manifold, a splitting into three boundary connect sums of $S^1 \times B^3$'s suitably glued together. The parallels with the Heegaard theory is striking enough, but there is also a strong connection with open book decompositions of 3-manifolds. In fact, extending the definition of a trisection to dimension 3 in the obvious way, a trisection of a 3-manifold is precisely an open book decomposition. There is also an obvious definition of a k-section of an n-manifold, with the most interesting cases being $k = n-1$ and $k = n$. Sadly, we have no higher dimensional existence or uniqueness results, but the idea is still intriguing. This is joint work with Rob Kirby.

Speaker: **Eric Harper** (McMaster)
Title: *Instanton homology of corks W_n*
Abstract: The corks W_n recently constructed by Akbulut and Yasui give rise to exotic structures on smooth 4-manifolds via an involution of their boundary homology 3-sphere. The cork W_1, also known as Akbulut's cork, gave rise to the first example of a diffeomorphism acting non-trivially on the instanton homology of an irreducible homology 3-sphere. In this talk, we will show that each cork W_n has a diffeomorphism on the boundary that acts non-trivially on the instanton homology and we will compute the instanton homology groups in the cases $n = 2, 3$.

Speaker: **Andriy Haydys** (U. Bielefeld)
Title: *Fukaya-Seidel category and gauge theory*
Abstract: In this talk I will outline a new construction of the Fukaya-Seidel category, which is associated to a symplectic manifold equipped with a compatible almost complex structure J and a J-holomorphic Morse function. For the infinite dimensional case of the holomorphic Chern-Simons functional this construction conjecturally associates a Fukaya-Seidel-type category to a smooth three-manifold.

Speaker: **Matt Hedden** (Michigan State)
Title: *Taut foliations, left-orderability, and L-spaces*
Abstract: I will discuss joint work with Adam Levine on the Floer homology of manifolds obtained by gluing together knot complements. In the case that the knots are torus knots, we obtain certain graph manifolds for which the questions of existence of left-orderings of the fundamental group and of taut foliations can be dealt with explicitly.

Speaker: **Chris Herald** (U. Nevada Reno)
Title: *The pillowcase and perturbations of traceless representations of knot groups*
Abstract: I'll discuss joint work with Matt Hedden and Paul Kirk which aims to explicitly understand the generating set for Kronheimer and Mrowka's singular instanton chain complex for a knot. A priori, the set of generators is not clear, since the relevant flat moduli space contains positive dimensional components. I'll show how to explicitly perturb the Chern-Simons functional, which is Bott-Morse but not Morse, to obtain a Morse function, ensuring a finite set of generators for the complex, whilst preserving the desirable feature of being able to identify generators in terms of particular representations of the (augmented) knot group. Along the way, Ill describe two different surfaces along which we can decompose the complement of a knot with the earring, as in the KM singular instanton configuration space. Each gives rise to a Lagrangian intersection picture in a pillowcase, i.e., a fiber product descriptions, for the flat moduli space of singular connections and sheds light on the chain complex generators.

Speaker: **Michael Hutchings** (UC Berkeley)
Title: *Embedded contact homology as a (symplectic) field theory*
Abstract: We explain how ECH can be extended to a kind of TQFT which recovers the Seiberg-Witten

invariants of closed symplectic four-manifolds cut along contact type hypersurfaces. While the construction uses Seiberg-Witten theory, it has applications to contact geometry. For example, one corollary is that the contact invariant (in ECH, HM, or HF) is functorial under strong symplectic cobordisms.

Speaker: **Cagatay Kutluhan** (Harvard)
Title: *Holonomy filtration and knots*
Abstract: I will describe a $\mathbb{Z} \oplus \mathbb{Z}$-filtered monopole knot homology isomorphic to Ozsvath-Szabo's knot Floer homology.

Speaker: **Paolo Lisca** (Pisa)
Title: *Stein fillable contact 3-manifolds and positive open books of genus one*
Abstract: A two-dimensional open book (S, h) determines a closed, oriented three-manifold $Y(S, h)$ and a contact structure $C(S, h)$ on $Y(S, h)$. The contact structure $C(S, h)$ is Stein fillable if h is positive, i.e. h can be written as a product of right-handed Dehn twists. Work of Wendl implies that when S has genus zero the converse statement holds, that is if $C(S, h)$ is Stein fillable then h is positive. On the other hand, Wand as well as Baker, Etnyre and Van Horn-Morris constructed counterexamples to the converse statement with S of genus two. In this talk I will present a proof of the converse statement under the assumption that S is a one-holed torus and $Y(S, h)$ is a Heegaard Floer L-space. If time permits I will describe a (still conjectural) classification up to diffeomorphisms of the Stein fillings of $(Y(S, h), C(S, h))$, where S is a one-holed torus, h is positive and $Y(S, h)$ is a Heegaard Floer L-space.

Speaker: **Thomas Mark** (Virginia)
Title: *Floer homology and the fractional Dehn twist coefficient*
Abstract: An open book decomposition of a 3-manifold Y is equivalent to a choice of fibered link embedded in Y. For a fibered knot (i.e. a one-component link), the monodromy of the fibration on the complement gives rise to a rational number called the fractional Dehn twist coefficient. This number measures the twisting of the monodromy around the boundary of the fiber surface. I will describe how the Heegaard Floer homology of a 3-manifold Y provides bounds for the fractional Dehn twist coefficient of any open book decomposition of Y with connected binding, and discuss applications to knot theory and contact topology. One consequence is the existence of an a priori upper bound on the absolute value of the twist coefficient of any fibered knot in a given 3-manifold. This is joint work with Matthew Hedden.

Speaker: **Lenny Ng** (Duke)
Title: *Topological strings and knot contact homology I* [see T. Ekholm for part II]
Abstract: Knot contact homology is an invariant of topological knots and links given by the Legendrian contact homology of the conormal lift. Recently, knot contact homology, or more specifically a subsidiary invariant called the augmentation polynomial, has been connected in surprising ways to other areas of knot theory and even string theory. I will introduce the augmentation polynomial (and its link generalization, the augmentation variety) and present some of its properties. These include an interpretation as a "stable A-polynomial" derived from certain representations of the knot group, and a conjectural interpretation as a recurrence relation for colored HOMFLY polynomials.

Speaker: **Ina Petkova** (Rice)
Title: *An absolute $\mathbb{Z}/2$ grading on bordered Floer homology*
Abstract: Bordered Floer homology is a TQFT-type generalization of Heegaard Floer homology to 3-manifolds with boundary. It is related to HF homology via a gluing formula. In its original form, this formula only recovers the homological HF grading as a relative grading, and within a choice of spin^c structures for the bordered manifolds.
After a brief description of bordered Floer homology and its original grading by sets with a Heisenberg group action, I will discuss how to obtain an absolute $\mathbb{Z}/2$ grading, simultaneously for all spin^c structures, that recovers the absolute $\mathbb{Z}/2$ grading after gluing. As a motivating example, with this $\mathbb{Z}/2$ grading,

bordered Floer homology categorifies the kernel of the homology map induced by the inclusion of the boundary into the 3-manifold.

Speaker: **Olga Plamenevskaya** (Stony Brook)
Title: *Looking for flexibility in higher-dimensional contact manifolds*
Abstract: By a classical result of Eliashberg, contact manifolds in dimension 3 come in two flavors: tight (rigid) and overtwisted (flexible). In higher dimensions, a class of flexible contact structures is yet to be found. However, some attempts to generalize the notion of an overtwisted disk have been made. One such object is a "plastikstufe" introduced by Niederkruger following some ideas of Gromov. We show that under certain conditions, the presence of plastikstufe leads to the following flexibility phenomena: 1) Legendrian knots become "loose", i.e. satisfy an h-principle, and 2) non-isotopic contact structures become isotopic after connect-summing with a manifold containing a plastikstufe. This is based on previous work of Murphy and Cieliebak-Eliashberg. (Joint with E. Murphy, K. Niederkruger, and A. Stipsicz.)

Speaker: **Daniel Ruberman** (Brandeis)
Title: *Embeddings of non-orientable surfaces in $M^3 \times I$*
Abstract: We use the correction terms from Heegaard Floer homology to obtain obstructions to embedding closed, non-orientable surfaces in 3- and 4-dimensional manifolds, focusing on embeddings in a lens space or the product of a lens space and an interval. For instance, we show that if the projective plane or the Klein bottle embeds nontrivially in $L(p,q) \times I$, then it must also embed in $L(p,q)$; it is reasonable to conjecture that the same is true for nonorientable surfaces of any genus. This is joint work with Adam Levine and Saso Strle.

Speaker: **Nikolai Saveliev** (Univ. Miami)
Title: *Index theory of the de Rham complex on manifolds with periodic ends*
Abstract: The analytic index of the de Rham complex on a compact orientable manifold is known to equal its Euler characteristic; the same holds for manifolds with product ends, for a properly understood L^2 index. We show that this is no longer true for more general manifolds with periodic ends, by providing an explicit formula for the difference between the L^2 index of the de Rham complex and the Euler characteristic of the manifold in terms of topology of the end. This research is a continuation of the joint project with T. Mrowka and D. Ruberman studying index theory on manifolds with periodic ends and its applications in low-dimensional topology and gauge theory.

Speaker: **Stefano Vidussi** (UC Riverside)
Title: *On the topology of SCY 4-manifolds*
Abstract: I will discuss some new (or gently used) results and conjectures on the structure of symplectic 4-manifolds with trivial canonical class, focusing on those with positive first Betti number.

Speaker: **Liam Watson** (UCLA)
Title: *Bordered Heegaard Floer homology and the Alexander module*
Abstract: The Alexander module is an invariant for knots arising from the module structure of the universal abelian cover of the knot exterior. This is a natural setting in which to define the Alexander polynomial, and as such it is natural to ask how the Alexander module arises in Heegaard Floer theory. Bordered Heegaard Floer homology provide the right tool to answer this question. This talk will explain how bordered invariants determine the Alexander module, categorifying the Seifert form along the way. In particular, appealing to some joint work with Matt Hedden, we'll show that there are examples of knots with identical Alexander module and identical knot Floer homology, that are distinguished by the relevant bordered invariants. This is part of a joint project with Jen Hom, Sam Lewallen and Tye Lidman.

Speaker: **Katrin Wehrheim** (MIT)
Title: *How to construct 2+1+1 topological field theories via the symplectic category*

Abstract: In previous work with Chris Woodward, we axiomatized and constructed 2+1 (connected) topological field theories that factor through the (monotone) symplectic category. Based on a representation of 4-cobordisms by Morse 2-functions and a reduction of the Gay-Kirby moves to "strip shrinking moves", I can now argue that the extension of such 2+1 theories to 2+1+1 dimensions hinges on a single "quilt axiom" – an identity of pseudoholomorphic quilt invariants associated to crossings in the Morse diagram. In the case of 2+1 theories arising from certain representation spaces resp. symmetric products, this yields conjectural symplectic constructions of Donaldson resp. Seiberg-Witten invariants, and in particular an approach to proving invariance of Perutz' Lagrangian matching 'invariants'.