Banff International Research Station
for Mathematical Innovation and Discovery

Theoretical and Applied Aspects of Nonnegative Matrices (12w2156)
July 27—29, 2012

SCHEDULE

Friday

16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours) Lecture rooms available after 16:00.

18:00 Dinner **(Note: Meals throughout the weekend are not covered by BIRS)**

19:30 Overview and Survey Lectures – 201 TCPL:

19:30-20:00: Combinatorial – Bryan Shader
20:00-20:30: Classical Algebraic – Leslie Hogben
20:30-21:00: Numerical – Chun-Hua Guo

Informal gathering in 2nd floor lounge, Corbett Hall (if desired). Beverages and a small assortment of snacks are available in the lounge on a cash honor system.

Saturday

7:00-9:00 Breakfast

9:00 Lectures: Talks of 30 minutes plus 10 minutes discussion including challenges, open questions, and future directions for the area. (201 TCBL)

9:00-9:40 Wayne Barret
9:40-10:20 Naomi Shaked-Monderer
20 min coffee break (foyer TCBL)
10:40-11:20 Craig Erickson
11:20-12:20 One hour discussion

Lunch (hike/walk for those interested – to be organized on site)

15:00-15:20 Coffee break
15:20-16:00 Daniel Szyld
20 min break for discussion
16:20-17:00 Colin Garnett
17:00-18:00 Discussion

Dinner (to be organized on site)

Sunday

7:00-9:00 Breakfast

9:00 Lectures: Talks of 30 minutes plus 10 minutes discussion including challenges, open questions, and future directions for the area. (201 TCBL)

9:00-9:40 Pierre Marechal
9:40-10:20 Shawn Wang
10:20-11:30 Group discussions (including coffee break 10:20-10:40am)
11:30 Wrap up all together

Sunday afternoon will be devoted to informal small group discussions based on the challenges brought by the speakers – depending on available time…

Checkout by 12 noon.

** 2-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, TCPL, Reading Room) until 15:00 on Sunday, although participants are still required to checkout of the guest rooms by 12 noon. There is no coffee break service on Sunday afternoon, but self-serve coffee and tea are always available in the 2nd floor lounge, Corbett Hall. **

MEALS

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL) *(included in workshop)*

For meal options at the Banff Centre, there are food outlets on The Banff Centre campus such as Vistas Main Dining Room on the 4th floor of Sally Borden Building (breakfast: 7:00-9:30am; lunch: 11:30am-1:30pm; dinner: 5:30-7:30pm), Le Cafe (ground floor, Sally Borden Building) and the Maclab Bistro (Kinnear Centre). You will also find a good selection of restaurants in the town of Banff which is a 10-15 minute walk from Corbett Hall.

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: Wayne Barrett (Brigham Young University)
Title: Bounds for the Largest Eigenvalue of a graph

Abstract: The largest eigenvalue λ_1 of the adjacency matrix of a graph *G* has been studied intensively and there are both lower and upper bounds in terms of well-known graph parameters. Let *n*: the number of vertices in *G*; *m*: the number of edges in *G*; $d_1, d_2, \ldots, d_n$: the degrees of the vertices of *G*; □*(G)*: the maximum degree of *G*; □*(G)*: the number of vertices in a largest clique in *G*.

Some upper bounds on λ_1 are:

□ $n\left(\frac{\omega(G)-1}{\omega(G)}\right)$ $\sqrt{2m\frac{\omega(G)-1}{\omega(G)}}$ $\underset{i\sim j}{max}\sqrt{d_i d_j}$ $\frac{1}{2}\left(\sqrt{1+8m}-1\right)$ $\sqrt{2m-n+1}$

(the last one requires that G be connected), while some lower bounds are:

$\frac{2m}{n}$ □*(G)-1*

Most of the upper bounds look independent of each other, but, surprisingly, there are inequalities between several pairs, and lower bounds can be used to show that some upper bounds are better than others. Can even stronger bounds be found using similar arguments?

Speaker: Minerva Catral (Xavier University)
Title: Reduced rank nonnegative matrix factorization for symmetric nonnegative matrices

Abstract: Let *V* be an $m \times n$ nonnegative matrix. The *nonnegative matrix factorization* (NNMF) problem consists of finding nonnegative matrix factors *W* of size $m \times r$ and *H* of size $r \times n$ such that $V \approx WH$. In [M. Catral, L. Han, M. Neumann and R.J. Plemmons, On Reduced Rank Nonnegative Matrix Factorization for Symmetric Nonnegative Matrices. Linear Algebra and Its Applications, 393:107-126, 2004] we considered the algorithm provided by Lee and Seung [D. D. Lee and H. S. Seung. Algorithms for non-negative matrix factorization, Advances in Neural Information Processing, 2000] which finds nonnegative *W* and *H* such that $\|V - WH\|_F$ is minimized. This algorithm is based on the following formulation of the NNMF problem: Find *W* and *H* which solve

(1) min *f(W,H)* subject to $W \geq O$ and $H \geq O$,

where $f(W,H) := \frac{1}{2}\|V - WH\|_F^2$ and where *W* is of size $m \times r$ and *H* is of size $r \times n$ and with $r \leq \min\{m,n\}$. For the case $r=1$, a complete characterization of the solution is known. For the case $m=n$ and in which *V* is symmetric, we focus on questions concerning when the best approximate factorization results in the product *WH* being symmetric and on cases in which the best approximation cannot be a symmetric matrix. It is also shown that any positive semidefinite symmetric nonnegative matrix *V* generated by a Soules basis admits, for every $1 \le r \le \mathrm{rank}(V)$, a nonnegative factorization *WH* which coincides with the best approximation in the Frobenius norm to *V* of rank not exceeding *r*. In the paper [CHNP] we studied theoretical properties of the solutions to (1) which are attained at a stationary point (i.e., the gradients of *f* in *H* and *W* satisfy $\nabla_W f = \mathbf{0}$,$\nabla_H f = \mathbf{0}$) for nonnegative symmetric matrices *V*. In particular we have shown that the solutions do occur at a stationary point when $r=1$ and when *V* is generated via a Soules basis. An interesting question which deserves further research is under what conditions on a nonnegative symmetric matrix *V* and a value of *r* satisfying $1 \le r \le \mathrm{rank}(V)$, problem (1) has solutions which occur at a stationary point.

Speaker: Craig Erickson (Iowa State University)
Title: *Sign patterns that allow strong eventual nonnegativity*

Abstract: A sign pattern is potentially eventually nonnegative (potentially strongly eventually nonnegative) if there is a matrix with this sign pattern that is eventually nonnegative (eventually nonnegative and has some power that is both nonnegative and irreducible). The study of potentially eventually nonnegative sign patterns is hindered by nilpotent matrices and matrices which have powers that are reducible with a nilpotent diagonal block. This talk presents results about potentially strongly eventually nonnegative sign patterns.

Speaker: Colin Garnett (University of Victoria)
Title: *The international standard book number: an introduction to Smith Normal Form and its use in analyzing integer lattices.*

Abstract: The international standard book number (ISBN) is one of the first error correcting codes introduced in an abstract algebra course. By treating the ISBN as an quotient lattice we are able to use Smith Normal Form to describe a basis for the lattice of all valid ISBNs. This talk will describe the problem of determining a lattice quotient by sampling points in the lattice. We will also discuss the probability that a given number of lattice points generate a basis for the lattice.

Speaker: Chun-Hua Guo (University of Regina)
Title: Theory and Numerics of M-matrix Algebraic Riccati Equations

Abstract: M-matrix algebraic Riccati equations are matrix equations of the form *XCX-XD-AX+B=O*, where A and D are square matrices and the matrix
$\begin{bmatrix} D & -C \\ -B & A \end{bmatrix}$ is a nonsingular M-matrix or an irreducible singular M-matrix. Such equations have applications in transport theory and Markov models, and have been studied extensively for more than 10 years. The solution of practical interest is the minimal nonnegative solution.

I will give an overview of some basic theoretical results about the matrix equation and some efficient numerical methods for finding the minimal solution. The theory of nonnegative matrices (the Perron--Frobenius theorem in particular) has played a major role in these

developments. I will also mention a few unsolved theoretical problems related to the design and analysis of numerical algorithms for this special class of nonlinear matrix equations.

Speaker: Leslie Hogben (Iowa State University)
Title: *Perron-Frobenius theory of nonnegative matrices and their generalizations*

Abstract: This talk will survey classical Perron-Frobenius theory of positive and nonnegative matrices, and how it extends to generalizations such as eventually positive matrices, eventually nonnegative matrices, eventually exponentially positive matrices, etc.

Speaker: Pierre Marechal (University of Victoria)
Title: K-optimal design via SDP and entropy optimization

Abstract: In this talk, we consider the problem of optimal design of experiments. A two step inference strategy is proposed. The first step consists in minimizing the condition number of the so-called information matrix. This step can be turned into an SDP problem. The second step is more classical, and it entails the minimization of a convex integral functional under linear constraints. This step is formulated in some infinite-dimensional space and is solved by means of a dual approach. Numerical simulations will show the relevance of our approach.

Speaker: Naomi Shaked-Monderer (Emek Yezreel College)
Title: *Boundary completely positive matrices and their cp-ranks*

Abstract: While it is long known that the maximum cp-rank of $n \times n$ completely positive matrices is obtained on the interior of the completely positive cone, it has not been known whether the maximum is also attained on the boundary of the cone. We answer this question in the affirmative for any n. This may be useful in the study of upper bounds on the cp-rank, in particular the DJL conjecture. Drew, Johnson and Loewy [Linear and Multilinear Algebra 37 (1994) 304] conjectured that for n at least 4, the cp-rank of every $n \times n$ completely positive matrix is at most $\left\lfloor \frac{n^2}{4} \right\rfloor$. Loewy and Tam [Linear Algebra and its Applications 363 (2003) 161-176] showed that the conjectured bound does hold for 5×5 completely matrices with at least one zero. We use the new result to complete the proof of the DJL conjecture in the 5×5 case.

For treating the $n = 5$ case, results on (extreme) copositive matrices (in particular, on the zero sets of such matrices) are used. It seems that this approach can be useful also in dealing with the general problem of finding a sharp upper bound on the cp-rank of $n \times n$ completely positive matrices.

Speaker: Bryan Shader (University of Wyoming)
Title: Combinatorics & Nonnegative

Abstract: This talk will provide a forward-looking survey of some of the ways that combinatorics is using nonnegative matrix theory, as well as how problems associated with nonnegative matrix theory are being attacked by combinatorial means. Open problems and new research frontiers will be discussed.

Speaker: Daniel Szyld (Temple University)
Title: Matrices with Perron-Frobenius Properties

Abstract: General matrices with a Perron-Frobenius property are studied, i.e., matrices which have a positive dominant eigenvalue, with the corresponding eigenvector being positive or non-negative. We concentrate on matrices which are not necessarily non-negative, and whose powers are not necessarily non-negative. Several characterizations of matrices having Perron-Frobenius properties are presented, including some depending on spectral, combinatorial, and geometric characteristics. We also study generalizations of M-matrices, i.e., matrices of the form $sI-B$ with B having a Perron-Frobenius property, and $\rho(B) \leq s$.
(joint work with Abed Elhashash)

Speaker: Shawn Wang (University of British Columbia: Okanagan)
Title: *Moving Average: Its Connection to A Fixed Point Iterative Scheme Using Nonnegative Matrices and Extensions*

Abstract: We show that the moving average is closely connected to a Gauss-Seidel type fixed point method studied by Bauschke, Wang and Wylie, which is observed converging numerically but without a proof. Their algorithm relies on a product of nonnegative matrices. Our analysis establishes a rigorous proof of convergence of their algorithm in a special case and identifies the limit explicitly. Moving averages in Banach spaces and generalized *f*-means are studied.

List of Participants:

Barrett, Wayne
Bodine, Elizabeth
Catral, Minerva
Cavers, Michael
Elhashash, Abed
Erickson, Craig
Fallat, Shaun
Garnett, Colin
Guo, Chun-Hua
Hogben, Leslie
McDonald, Judi
Marechal, Pierre
Nasserasr, Shahla
Olesky, Dale
Shader, Bryan
Shaked-Monderer, Naomi
Stuart, Jeff
Szyld, Daniel
Tarazaga, Pablo
Tsatsomeros, Michael
van den Driessche, Pauline
Wang, Shawn
Wilson, Ulrica