My interests are in the area of ring theory of quantum function algebras, their underlying Poisson geometry and relations to cluster algebras.

The first family of quantum function algebras which I investigate are the quantum nilpotent algebras $\mathcal{U}_q^-(w)$ defined by De Concini, Kac and Procesi (associated to an arbitrary simple Lie algebra $\mathfrak{g}$ and a Weyl group element $w \in W$). In [6] I classified their torus invariant prime spectra$\mathcal{U}_q^-(w)$, described the inclusions between those ideals, and gave an explicit description of all such prime ideals in term of Demazure modules. A set theoretic classification of the torus invariant primes was also obtained by Mériaux and Cauchon [3]. Using [6] in a later paper I classified the torus invariant prime spectra of all partial flag varieties. Consequently, Geiß, Leclerc and Schröer [2] proved that in the case of simply laced $\mathfrak{g}$ the algebras $\mathcal{U}_q^-(w)$ are quantum cluster algebras. A celebrated (unpublished) theorem of Gabber establishes that the universal enveloping algebra of an arbitrary solvable Lie algebras is catenary. In [8] I proved that all algebras $\mathcal{U}_q^-(w)$ are catenary. My interests are how questions in cluster algebras and ring theory for $\mathcal{U}_q^-(w)$ interplay with each other. Zwicknagl's preprint [9] addresses this, but I believe that it has mistakes.

In a related direction I work on the spectra of quantum function algebras $R_q[G]$. In [7] I proved separation of variables type results for all Joseph's localizations [5] and the algebras $\mathcal{U}_q^-(w)$, and classified the maximal spectra of $R_q[G]$. The Joseph localizations are precisely the quantized coordinate rings of (all) double Bruhat cells in G. They play a key role in the study [5, 4] of the spectra of $R_q[G]$. Here ring theory relates once again to cluster algebras in connection with the Berenstein–Zelevinsky work on quantum double Bruhat cells and upper cluster algebras.

From ring theoretic perspective for both families of algebras $\mathcal{U}_q^-(w)$ and $R_q[G]$ there are many difficult open problems. The most important one is to describe the topology of their spectra.

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