Split Bregman Method for Minimization of Region-Scalable Fitting Energy for Image Segmentation

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Outline

- Review of Region-based Active Contour Models
  - Mumford Shah Model
  - CV Piecewise Constant Model
  - VC Piecewise Smooth Model
- Images with intensity inhomogeneity
- Region-Scalable Fitting Energy Model
- Split Bregman Method for Minimization of Region-Scalable Fitting Energy
- Experimental Results

Mumford Shah Model

Given an image I , find a contour C in Ω , and a piecewise smooth image u approximating the original image I which minimize the energy functional

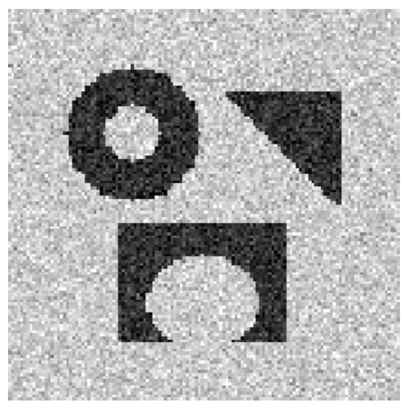
$$F^{MS}(u,C) = \int_{\Omega}(u-I)^2 + \mu\int_{\Omega\backslash C}|\nabla u|^2 + \nu|C|$$

where $|C|$ is the length of contour C .

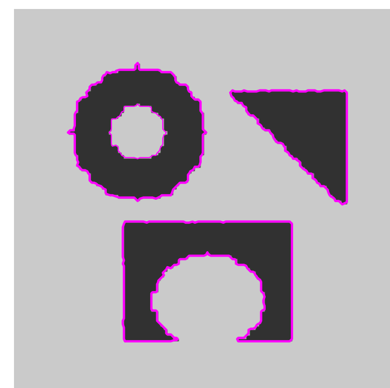
-- data fidelity

-- smooth approximation

-- contour compactness



$\Rightarrow$



Piecewise Constant Model

Chan & Vese: IEEE 2001 :

Assumption: Intensity are piecewise constant

inside and outside of the contour C

$$u(x) = \begin{cases} c_1, & x \in outside(C) \\ c_2, & x \in inside(C) \end{cases}$$

The model they proposed is to minimize the following energy:

$$F^{CV}(C, c_1, c_2) = \lambda_1 \int_{outside(C)} \left| I(x) - c_1 \right|^2 dx + \lambda_2 \int_{inside(C)} \left| I(x) - c_2 \right|^2 dx + \nu \left| C \right|$$

Where λ_1 **,** λ_2 **and** ν **are positive constants,** $outside(C)$ **and** $inside(C)$
Represent the regions outside and inside the contour C **, respectively.**

Piecewise Constant Model

Chan & Vese: IEEE 2001 :

Assumption: Intensity are piecewise constant

inside and outside of the contour,

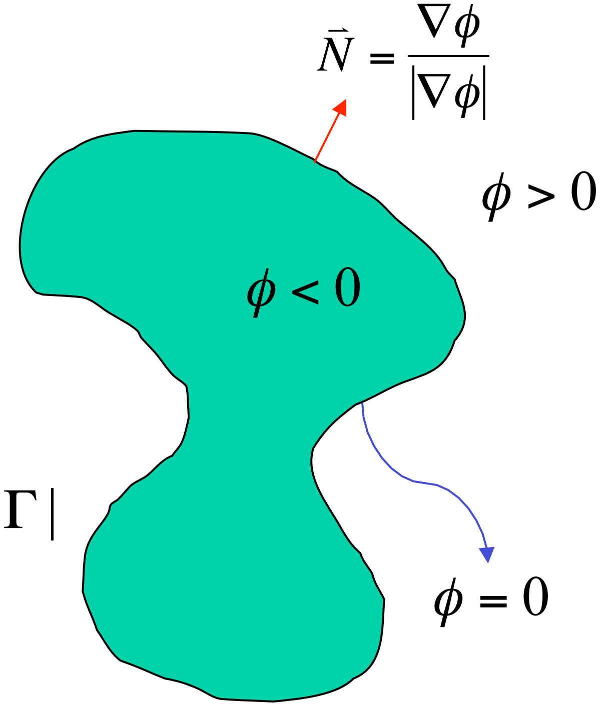
$$u(x,y) = c_1 H(\phi(x,y)) + c_2(1 - H(\phi(x,y))$$

Remind: Mumford and Shah functional

$$F^{MS}(u,\Gamma) = \alpha \int_{\Omega} (u - I)^2 dxdy + \beta \int_{\Omega \setminus \Gamma} |\nabla u|^2 \, dxdy + \mu |\Gamma|$$

Consider the following functional

$$E^{CVC}(c_1, c_2, \phi) = \alpha_1 \int_{\Omega} | c_1 - I |^2 \, H(\phi) dxdy + \alpha_2 \int_{\Omega} | c_2 - I |^2 \, (1 - H(\phi)) dxdy + \mu \int_{\Omega} \delta(\phi) | \nabla \phi | dxdy$$



Minimization Procedure

How can we minimize $F^{CVC}(c_1, c_2, \Gamma)$???

$$E^{CVC}(c_1, c_2, \phi) = \alpha_1 \int_\Omega | c_1 - I |^2 \, H(\phi) dxdy + \alpha_2 \int_\Omega | c_2 - I |^2 \, (1 - H(\phi)) dxdy + \mu \int_\Omega \delta(\phi) \, | \nabla \phi | dxdy$$

*** start from an initial guess for Γ**

*** morph Γ and update c_1 and c_2 in the descent direction of the functional until they reach the optimal solutions**

Keep the contour fixed and minimize the energy:

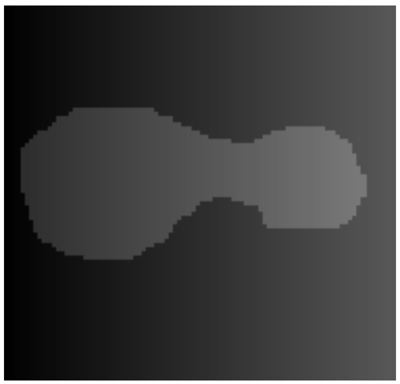
$$c_1(\phi) = \int_\Omega I(x,y) H(\phi) dxdy / \int_\Omega H(\phi) dxdy \; ; \; c_2 = \int_\Omega I(x,y)(1 - H(\phi)) dxdy / \int_\Omega (1 - H(\phi)) dxdy$$

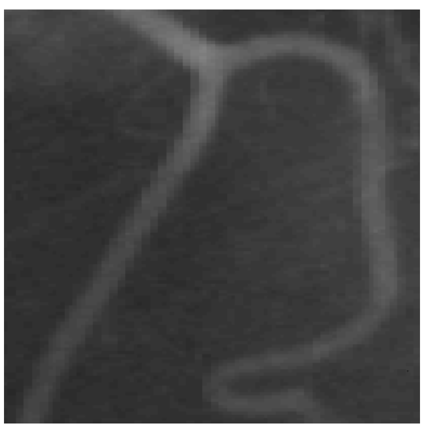
Keep c_1 and c_2 fixed and minimize w.r.t. ϕ

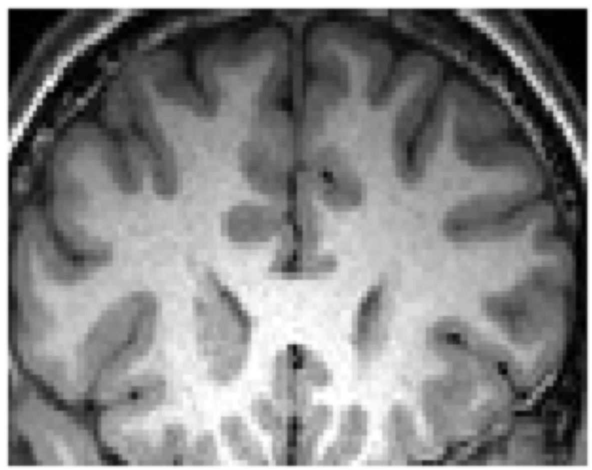
$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left[\mu_1 \nabla \cdot \left(\frac{\nabla \phi}{| \nabla \phi |} \right) - \mu_2 - \lambda_1 (I - c_1)^2 + \lambda_2 (I - c_2)^2 \right]$$

Images with intensity inhomogeneity

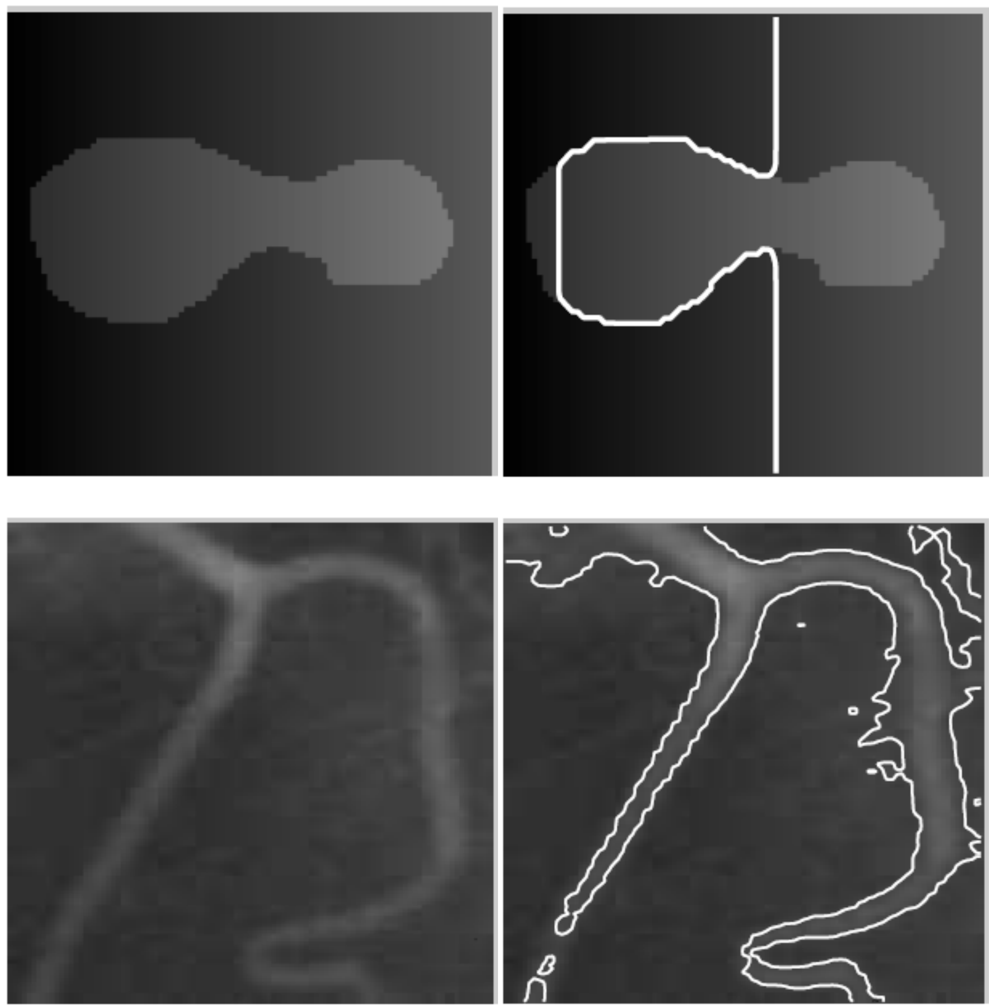
Intensity Images: gray scale images $I:\Omega \rightarrow R$ color images $I:\Omega \rightarrow R^3$







Numerical Results:
Difficulty for images with inhomogeneity



Piecewise Smooth Model

Vese & Chan : Int. J. Compute. Vis. 2002 :

Instead of considering piecewise constant inside and outside of the contour C ,

Introduce two functions u^+ and u^- such that:

$$u(x) = \begin{cases} u^+(x), & x \in outside(C) \\ u^-(x), & x \in inside(C) \end{cases}$$

Then the energy functional becomes:

$$F^{VCS}(u^+, u^-, C) = \int_{outside(C)} (u^+ - I)^2 + \int_{inside(C)} (u^- - I)^2 + \mu \int_{outside(C)} \left|\nabla u^+\right|^2 + \mu \int_{inside(C)} \left|\nabla u^-\right|^2 + \nu \left|C\right|$$

Minimization Procedure

Instead of considering piecewise constant inside and outside of the contour,

Introduce two functions u^+ and u^- such that

$$u(x,y) = u^+(x,y)H(\phi(x,y)) + u^-(x,y)(1 - H(\phi(x,y))$$

Remind: Mumford and Shah functional

$$F^{MS}(u,\Gamma) = \alpha \int_\Omega (u - I)^2 dxdy + \beta \int_{\Omega \backslash \Gamma} |\nabla u|^2 dxdy + \mu |\Gamma|$$

Consider the following functional

$$E^{VCS}(u^+,u^-,\phi) = \alpha \int_\Omega |u^+ - I|^2 H(\phi)dxdy + \alpha \int_\Omega |u^- - I|^2 (1 - H(\phi))dxdy$$
$$+ \beta \int_\Omega |\nabla u^+|^2 H(\phi)dxdy + \beta \int_\Omega |\nabla u^-|^2 (1 - H(\phi))dxdy + \mu \int_\Omega \delta(\phi) |\nabla \phi| dxdy$$

Piecewise Smooth Model

$$E^{VCS}(u^+,u^-,\phi) = \alpha\int_\Omega |u^+ - I|^2 H(\phi)dxdy + \alpha\int_\Omega |u^- - I|^2 (1-H(\phi))dxdy$$
$$+\beta\int_\Omega |\nabla u^+|^2 H(\phi)dxdy + \beta\int_\Omega |\nabla u^-|^2 (1-H(\phi))dxdy + \mu\int_\Omega \delta(\phi)|\nabla\phi| dxdy$$

Keep the contour fixed and minimize the energy: Euler-Lagrange equations

$$u^+ - I = \beta\Delta u^+ \ \ on \ \{\phi > 0\}, \quad \frac{\partial u^+}{\partial n} = 0 \ on \ \{\phi = 0\}$$
$$u^- - I = \beta\Delta u^- \ \ on \ \{\phi < 0\}, \quad \frac{\partial u^-}{\partial n} = 0 \ on \ \{\phi = 0\}$$

Keep u^+ and u^- fixed and minimize w.r.t. ϕ

$$\frac{\partial\phi}{\partial t} = \delta(\phi)\begin{bmatrix} \mu\nabla\cdot(\frac{\nabla\phi}{|\nabla\phi|}) - \alpha(u^+ - I)^2 + \alpha(u^- - I)^2 \\ -\beta|\nabla u^+|^2 + \beta|\nabla u^-|^2 \end{bmatrix}$$

Difficulity: At each iteration, 2 pde on irregular domains need to be solved

need to extend u^+ and u^-

Region-Scalable Fitting (RSF) Energy Model

Li et al. propose a region-scalable fitting energy model:

$$E(C, f_1(x), f_2(x)) = \sum_{i=1}^{2} \lambda_i \int \left[\int_{\Omega_i} K_\sigma(x-y) \left| I(y) - f_i(x) \right|^2 dy \right] dx + \nu \left| C \right|$$

The aim of the kernel function K_σ is to put heavier weights on points y which are close to the center point x. For simplicity, a Gaussian kernel with a scale parameter $\sigma > 0$ Was used:

$$K_\sigma(u) = \frac{1}{2\pi\sigma^2} e^{-|u|^2/2\sigma^2}$$

The level set formulation is:

$$E_\varepsilon\left(\phi, f_1(x), f_2(x)\right) = \sum_{i=1}^{2} \lambda_i \int \left(\int K_\sigma(x-y) \left| I(y) - f_i(x) \right|^2 M_i^\varepsilon\left(\phi(y)\right) dy \right) dx + \nu \int \left| \nabla H_\varepsilon\left(\phi(x)\right) \right| dx$$

where $M_1^\varepsilon(\phi) = H_\varepsilon(\phi)$ and $M_2^\varepsilon(\phi) = 1 - H_\varepsilon(\phi)$

A level set regularization term $P(\phi)$ is used to preserve the regularity of the level set function ϕ :

$$P(\phi) = \int \frac{1}{2} \left(\left| \nabla \phi(x) \right| - 1 \right)^2 dx$$

Gradient Descent Flow

There, the energy functional to minimize is:

$$F(\phi, f_1, f_2) = E_\varepsilon(\phi, f_1, f_2) + \mu P(\phi)$$

Keep ϕ fixed and minimize the energy:

$$f_i(x) = \frac{K_\sigma(x) * \left[M_i^\varepsilon(\phi(x)) I(x)\right]}{K_\sigma(x) * M_i^\varepsilon(\phi(x))}, \quad i = 1, 2$$

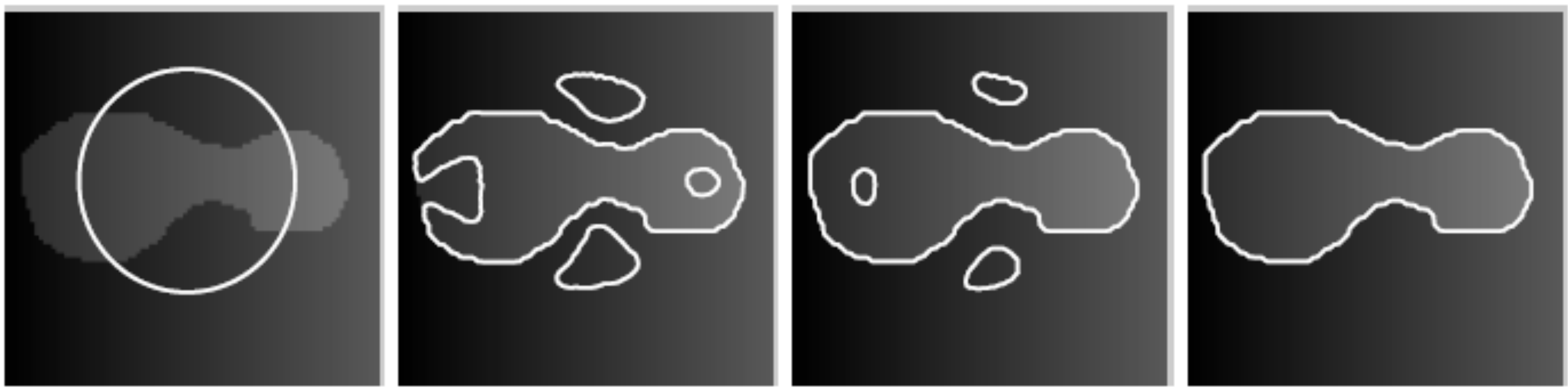
Keep f_1 and f_2 fixed and minimize w.r.t. ϕ :

$$\frac{\partial \phi}{\partial t} = -\delta_\varepsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \nu \delta_\varepsilon(\phi) div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) + \mu\left(\nabla^2 \phi - div\left(\frac{\nabla \phi}{|\nabla \phi|}\right)\right)$$

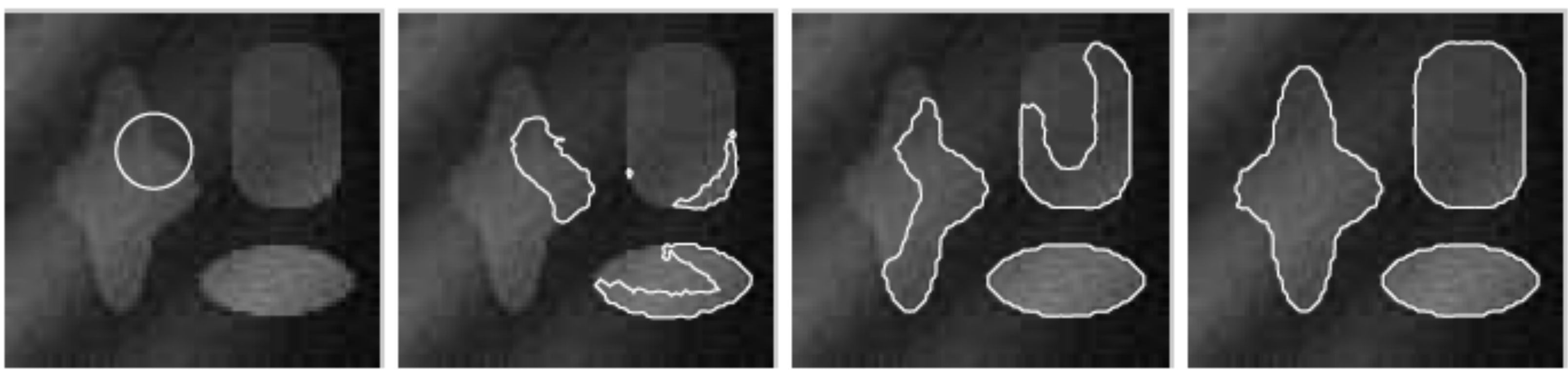
where δ_ε is the derivative of H_ε , and $e_i\,(i = 1 or 2)$ is defined as:

$$e_i(x) = \int K_\sigma(y - x)\left|I(x) - f_i(y)\right|^2 dy, \quad i = 1, 2$$

Some Results for RSF Model

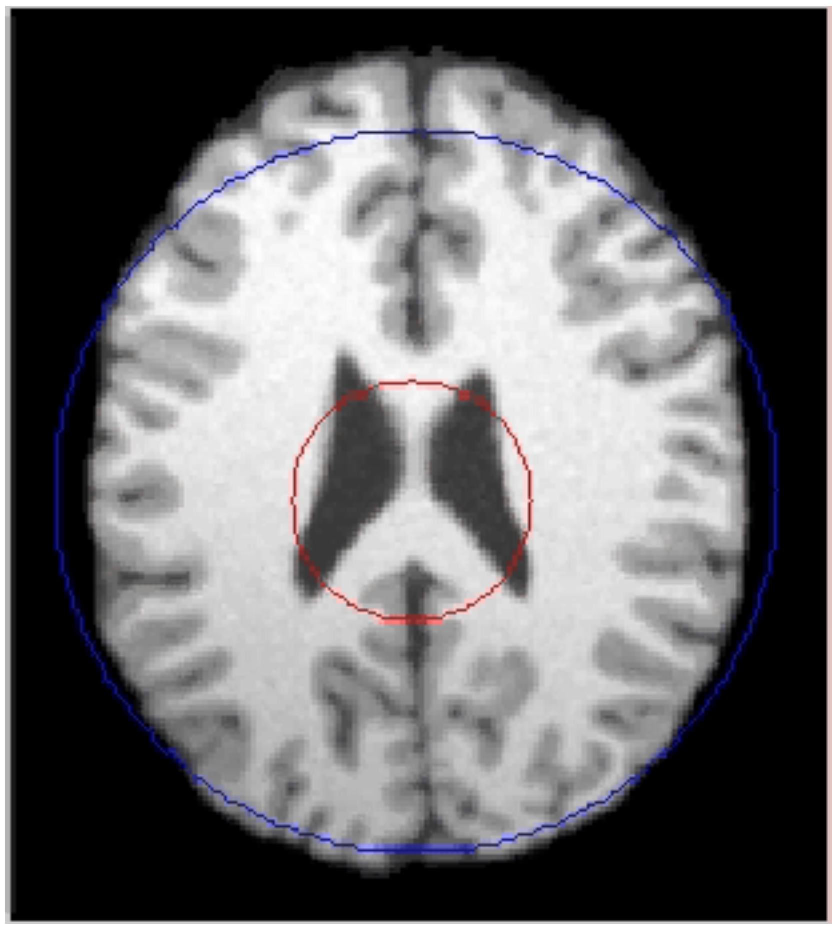


(a) Initial contour. (b) 10 iterations. (c) 20 iterations. (d) 50 iterations.



$\mu = 0.001 \times 255^2, \tau = 0.1, \nu = 1, \sigma = 3.0, \lambda_1 = \lambda_2 = 1.0$

Some Results for RSF Model



Split Bregman Method for Minimization of Region-Scalable Fitting Energy

Considering the gradient flow equation in the RSF model:

$$\frac{\partial \phi}{\partial t} = -\delta_\varepsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \nu \delta_\varepsilon(\phi) div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) + \mu\left(\nabla^2 \phi - div\left(\frac{\nabla \phi}{|\nabla \phi|}\right)\right)$$

Drop the last term and take $\nu = 1$:

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left((-\lambda_1 e_1 + \lambda_2 e_2) + div\left(\frac{\nabla \phi}{|\nabla \phi|}\right)\right)$$

Following the idea from Chan et al. , the stationary solution of the above equation coincides with the stationary solution of:

$$\frac{\partial \phi}{\partial t} = \left((-\lambda_1 e_1 + \lambda_2 e_2) + div\left(\frac{\nabla \phi}{|\nabla \phi|}\right)\right)$$

This simplified flow represents the gradient descent for minimization problem

$$\min_{a_0 \le \phi \le b_0} E(\phi) = \min_{a_0 \le \phi \le b_0} |\nabla \phi|_1 + \langle \phi, r \rangle$$

where the restriction $a_0 \le \phi \le b_0$ is to guarantee a unique global minimizer and $r = \lambda_1 e_1 - \lambda_2 e_2$

Then the segmented region can be found for some $\alpha \in (a_0, b_0)$:

$$\Omega_1 = \{x : \phi(x) > \alpha\}$$

The new proposed
Region-Scalable Fitting Energy

Replace the standard TV norm $TV(\phi) = \int |\nabla \phi| = |\nabla \phi|_1$ with the weighted version:

$TV_g(\phi) = \int g|\nabla \phi| = |\nabla \phi|_g$, where

$$g(\xi) = \frac{1}{1 + \beta |\xi|^2}$$

is the non-negative edge detector function.

Then the minimization problem becomes:

$$\min_{a_0 \le \phi \le b_0} E(\phi) = \min_{a_0 \le \phi \le b_0} |\nabla \phi|_g + \langle \phi, r \rangle$$

To apply the Split Bregman approach, an auxillary variable $\vec{d} \leftarrow \nabla \phi$ is introduced.

Apply Bregman iteration to strictly enforce the constraint $\vec{d} = \nabla \phi$, the resulting sequence of optimization problems is:

$$\begin{cases} \left(\phi^{k+1}, \vec{d}^{k+1}\right) = \arg\min_{a_0 \le \phi \le b_0} \left|\vec{d}\right|_g + \langle \phi, r \rangle + \frac{\lambda}{2} \left\| \vec{d} - \nabla \phi - \vec{b}^k \right\|^2 \\ \vec{b}^{k+1} = \vec{b}^k + \nabla \phi^{k+1} - \vec{d}^{k+1} \end{cases}$$

Apply Split Bregman Method for Minimization

For fixed $\vec{d}$, minimize w.r.t. ϕ :

$$\Delta\phi = \frac{r}{\lambda} + \nabla \cdot \left(\vec{d} - \vec{b} \right), \; a_0 < \phi < b_0$$

Using central discretization for Laplace operator and backward difference for divergence operator, the numerical scheme is:

$$\begin{cases} \alpha_{i,j} = d^x_{i-1,j} - d^x_{i,j} + d^y_{i,j-1} - d^y_{i,j} - \left(b^x_{i-1,j} - b^x_{i,j} + b^y_{i,j-1} - b^y_{i,j} \right) \\ \beta_{i,j} = \frac{1}{4}\left(\phi_{i-1,j} + \phi_{i+1,j} + \phi_{i,j-1} + \phi_{i,j+1} - \frac{r}{\lambda} + \alpha_{i,j} \right) \\ \phi_{i,j} = \max\left\{ \min\left\{ \beta_{i,j}, b_0 \right\}, a_0 \right\} \end{cases}$$

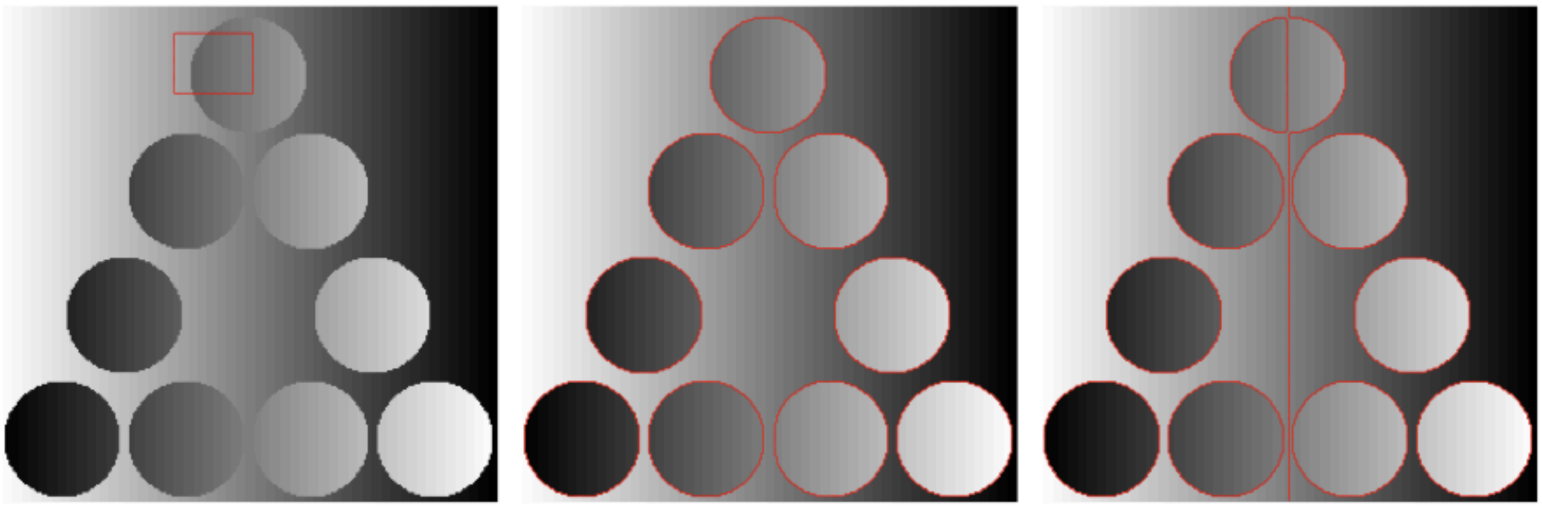
For fixed ϕ , minimize w.r.t. $\vec{d}$:

$$\vec{d}^{k+1} = shrink_g\left(\vec{b}^k + \nabla\phi^{k+1}, \frac{1}{\lambda} \right) = shrink\left(\vec{b}^k + \nabla\phi^{k+1}, \frac{g}{\lambda} \right)$$

where

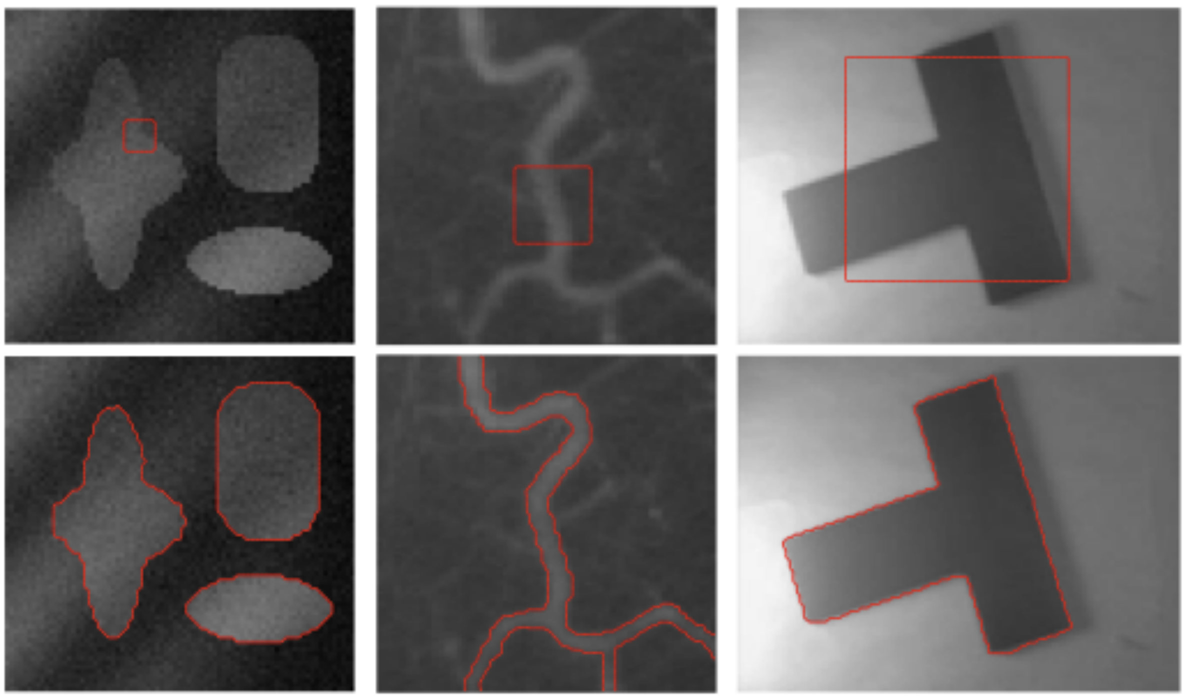
$$shrink(x, r) = \frac{x}{|x|} \max\left(|x| - r, 0 \right)$$

Experimental Results (1):
Segmentation of a synthetic image



- **Comparison** between the proposed method and split Bregman on PC model
- Column 1: the original image and the initial contour
- Column 2: the result of our proposed method
- Column 3: the result of the split Bregman on PC model

Experimental Results (2): Boundary extraction for four challenging inhomogeneous images



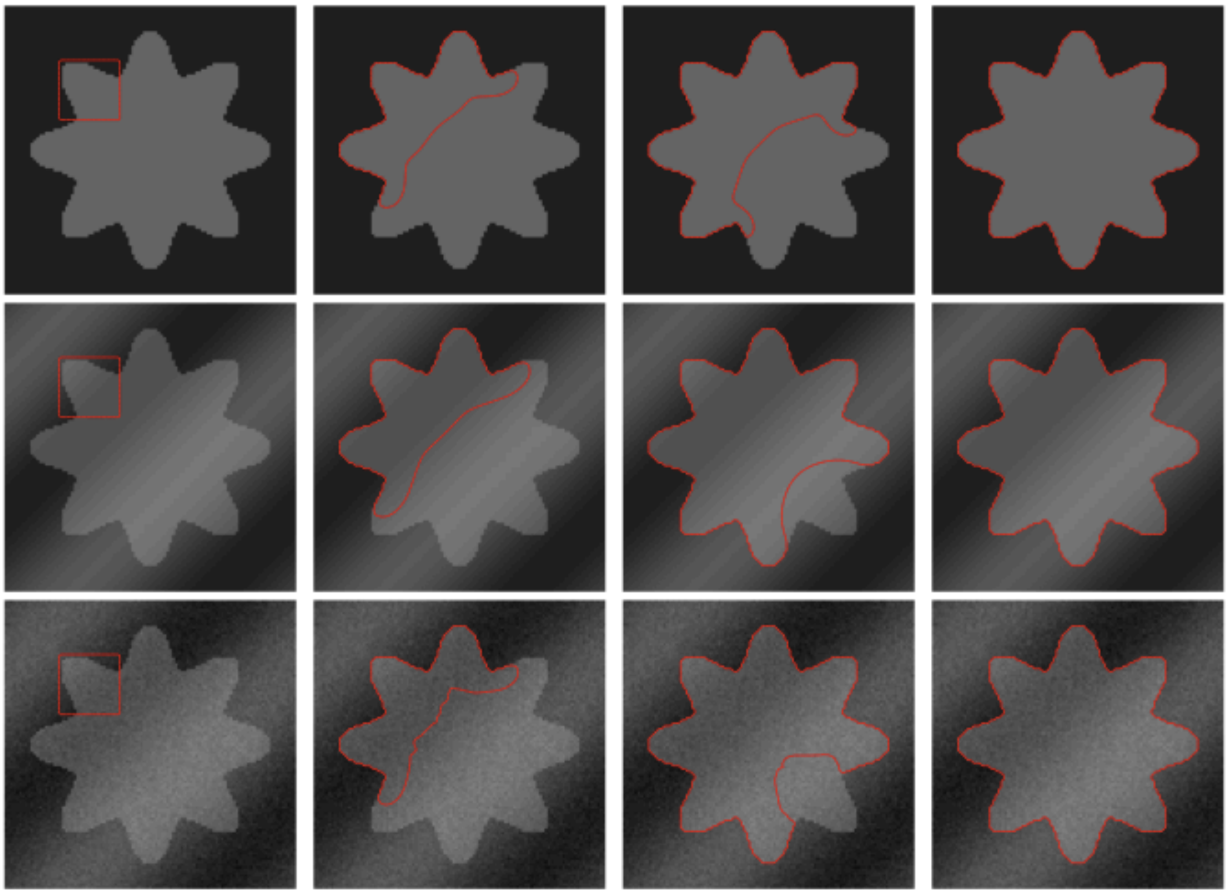
- Top row: original images with initial contours
- Bottom row: segmentation results with final contours

Efficiency demonstrated by comparing the iteration number and computation time with the original RSF model

	Image 1	Image 2	Image 3	Image 4
Our model	32(0.33)	67(1.13)	26(0.49)	48(0.70)
RSF model	200(1.40)	150(1.74)	300(3.72)	300(3.01)

- **From this table, it is clear that our method is more efficient than the RSF model because we apply the split Bregman approach to the optimization problem.**

Experimental Results (3):
Segmentation of three synthetic flower images with different distribution of intensities



- Row 1: piecewise constant image
- Row 2: inhomogeneous clean image
- Row 3: inhomogeneous image with noise

Experimental Results (4):

Detect boundary for a color image of flower

- The curve evolution process from the initial contour to the final contour is shown above.

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5. Split Bregman Method for Minimization of region-Scalable Fitting Energy for Image Segmentation by Yunyun Yang, Chunming Li, Chiu-Yen Kao, Stanley Osher, Advances in Visual Computing, volume 6454 of Lecture Notes in Computer Sciences, pages 117-128, 2010

The End

Thank you for your attention!!

Questions??