Self Adjoint Extensions and Singularity Resolution in String Theory and Quantum Gravity (BIRS Workshop 11w5080)
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Organizers:
Robert Brandenberger (McGill University)
Walter Craig (McMaster University)
Thomas Thiemann (Albert Einstein Institute, Golm)
Neil Turok (Perimeter Institute)
Mark Van Raamsdonk (University of British Columbia)

Final report: Fundamental cosmological theories describing the origin of structure in the early universe include the standard big bang model (SBB), and the inflationary cosmology which is the current paradigm; the latter was introduced to avoid problems of causality which would otherwise occur given the uniformity of the cosmic microwave background (CMB). However despite its successes, current models of inflation suffer from several drawbacks, which has led to the effort to find alternatives. The most serious problem is the fact that these cosmological models all contain an initial space-like singularity. In the present state of theoretical cosmology, there are a number of competing approaches to resolve this cosmological singularity. Some of the competing theories that have the most currency include on one hand holographic cosmologies making use of the anti-deSitter/conformal field theory (AdS/CFT) correspondence (which emerges from string theory) [1 - 4], and on the other loop quantum gravity [5,6]. Similar mathematical challenges appear in both approaches, e.g. the need to extend the dynamics via self-adjoint extensions. In addition to the demands of making sense mathematically, such theories are held to the high standard of corresponding to observational data, of which there is now an increasingly large quantity. It is a requirement of each of these theories to resolve the singularities of general relativity, either at the big bang, at a bounce, or at the formation of a black hole. Furthermore, some of the current theories have specific problems associated with boundary conditions, for example the ekpyrotic scenario envisions that one must impose boundary conditions at a time-like singularity, while the AdS/CFT scenario involves a five dimensional space-time, with boundary conditions on a four dimensional boundary which possesses its own Lorentzian metric.

The objective of our workshop was to bring together physicists and mathematicians who are interested in problems of physical importance in quantum mechanics, quantum field theories, and general relativity. In particular we sought out experts in the problem of self-adjoint extensions in operator theory, in order to promote discussions between physicists and mathematicians related to the goal of resolving cosmological singularities, and ideally to stimulate new collaborations that would be of benefit to both disciplines. The physicists that we invited to the workshop included string theorists, experts in loop quantum gravity, and cosmologists who are involved in the AdS/CFT approach to resolving cosmological singularities. While our first goal was to stimulate interactions between mathematicians and physicists, a secondary goal of this workshop was to encourage more discussions and collaborations between the relevant research communities in theoretical physics itself.

Participants. There were 16 participants, who can be roughly classified into the following groups or topics:

Loop quantum gravity.
Martin Bojowald (Pennsylvania State University)
Bianca Dittrich (Albert Einstein Institute/Perimeter Institute)
Wojciech Kaminski (Albert Einstein Institute - Max Planck)
Tomasz Pawlowski (University of New Brunswick)

String theory.
David Lowe (Brown University)
Vaidya Sachindeo (Indian Institute of Science)
Sumit Das (University of Kentucky)
Misha Smolkin (Perimeter Institute)
Omid Saremi (University of California - Berkeley)

Cosmology.
Robert Brandenberger (McGill University)
Yi Wang (McGill University)
Elisa Ferreira (University of Sao Paolo/McGill University)

Mathematics.
Walter Craig (McMaster University)
Niky Kamran (McGill University)
Jared Wunsch (Northwestern University)
Jakob Yngvason (Universität Wien)

Details of the workshop. The structure of the workshop was atraditional, in that several key participants were assigned by the organizers to give tutorials rather than more traditional research presentations, not all participants spoke (in the Oberwolfach style of meeting), and furthermore the lectures and their order of presentation, other than those on the first day, were decided by public acclaim in a discussion at the end of the first afternoon session. In particular the mathematics talks were essentially all pre-assigned tutorials, and they were distributed through the workshop as punctuation to the physics lectures.

The workshop opened with an summary lecture by Robert Brandenberger, who set the tone and stated the challenges of current theoretical cosmology. Essentially the goal of cosmology is to understand the history and the dynamics of the early universe, drawing information from theoretical physcs, and to make connections with the wealth of data on the large-scale structure of the universe. At present there is a lot of data, both on the homogeneous character of the universe as well as on its inhomogeneities. Key information comes from the cosmic microwave background (CMB) - specifically from the size and structure of its anisotropies, but this is not the sole source. Standard cosmologies are based on classical physics, with space-time evolving according to Einstein's general relativity and matter assumed to be an ensemble of ideal gasses. There is a well known Lagrangian, an action principle, and equations of motion, and their homogeneous solutions which admit a singularity (the big bang), the standard from which the typical physicist works, are the Friedmann – Robertson – Walker space-times. Already this basis presents problems, as for one there is no information in a completely homogeneous singlarity. Furthermore, the degree of homogeneity of the CMB violates the principle of causality in the standard big bang model. The current paradigm by which one escapes this latter difficulty is to posit a period of exponential inflation. This is a theory that has been a phenomenological success, whose predictions have been verified with great accuracy. Nonetheless, inflation has its own difficulties. It is still based on an initial space-time singularity which is governed by standard cosmology, and it is not clear that classical physics is valid at length scales less than the Planck length which arise in inflationary cosmology since current data which we are measuring eminates from sub- (or trans-) Planck scales which have grown to cosmological scales through the inflation process. Hence the first question before the workshop has to do with resolution of such a space-time singularity. One such possibility is a 'bounce', a cosmology which begins in a contracting phase and makes a smooth transition to an expanding period at a nonzero local minimum of the scale factor which is sufficiently large to avoid trans-Planckian length scales. Cosmological bounces occur in several recent early universe scenarios based either on string theory or on loop quantum gravity. Other scenarios are also possible, in particular an "emergent" scenario in which the universe begins in a quasi-static phase, and which can be realized in the context of "string gas cosmology", and which leads to measurable, and thus verifiable consequences (slight red shifts vs slight blue shifts in the gravitational wave spectrum). It remains to understand the content of the new physics that would go into such proposals.

The main body of the physics lectures of the workshop were essentially on the theme of proposals for the new physics with which to understand either the non-classical physics on sub-Planck scales, or a theory of quantum mechanics and gravitation that will give rise to one of the alternate scenarios to a singularity. Omid Saremi gave a tutorial on the core material of the AdS/CFT scenario, which works with a five dimensional space-time (the bulk manifold) with a high degree of gauge symmetries, which has a four dimensional boundary with its own Lorentzian metric. Classical fields in the bulk manifold give rise to quantum fields and their observables on the boundary manifold. This is a beautiful picture geometrically, and the correspondence between bulk fields and boundary fields is through the self-adjoint boundary conditions of the title of the workshop. However the quantum theory of this sceario suffers on the rigorous mathematical level from the physical assumption that the Euclidian version of the theory (that is, one in which the metric signature is switched to being Riemannian) gives the same answers as the original Lorentzian metric. Martin Bojowald gave a series of talks, essentially a tutorial, on the foundations of loop quantum gravity. This theory also starts out very geometrically, describing the receipe for quantizing classical fields, the standard approach (as yet unsuccessful) to quantize the classical theory of gravity, and then the ideas behind a quantization of a theory based on closed curves (loops) and their dual objects. We understood from his lectures that the structures behind the quantum algebras that are constructed in this field are very complicated, and that the necessary algebraic manipulations are nontrivial. To a mathematician, this program gave the impression that, although the basic objects of the theory are simple and geometrically beautiful, the direction taken in developing this field is a receipe that is strongly guided by a close analogy with classical quantization. David Lowe gave the principal presentation from the string theory point of view, starting with a classical description of space-time in conformally compact coordinates, onto which he introduced a potential and the string theory landscape. He gave the he estimate that there are roughly 10^4 six-dimensional Calabi-Yau manifolds, which after around 100 cycles of a bouncing universe, gives rise to the possibility of 10^{500} possible vacuua, definitely a problem of non-uniqueness. He then went on to outline some selection principles, old and new, which if successful would partial resolve this problem.

There were four mathematics tutorial talks for the workshop. Walter Craig gave a talk on the classical result of Weyl's limit point/limit circle theory, which is principle for selection of self-adjoint boundary conditions for ordinary differential operators. Jakob Yngvason completed this discussion of self-adjoint boundary conditions with the theory of deficiency indices, and their extension to partial differential operators in the form of a Fredholm condition. Niky Kamran spoke on the theory of stability of space-times, including the wel developed theory (by now) of the nonlinear stability of Minkowski space-time, and the newly developing theory of the stability of Schwarzschild space-times. This made a good impression on many of our physicist colleagues, and will we believe have an impact on their future point of view with regard to the topic of classical general relativity. Finally, Jared Wunsch spoke on the method of energy estimates, from which, when microlocalized, one may derive theorems of energy transport for both linear and nonlinear hyperbolic systems. He also mentioned versions of this form of energy transport for quantum mechanical systems, as well as commutator estimates in general and Morawetz estimates in particular.

Several of the participants formed a working group to study an extension of an approach to AdS/CFT cosmology proposed by Sumit Das and his collaborators [4]. The goal of the participants of the working group is to extend the analysis to the case of inhomogeneous universes (with small amplitude inhomogeneities which correspond to the observed cosmological fluctuations). The members of the working group met at informal evening discussion sessions. Key members of the working group were Brandenberger, Das, Ferreira, Kamran and Wang. The work of this group has continued after the workshop. It is being explored whether the new mathematics results related to the initial and boundary value problems in AdS space-time which Kamran and collaborators have recently developed can be applied to construct a model in which the inhomogeneities corresponding to cosmological fluctuations can be carried forwards from the initial contracting phase of a bouncing cosmology to the expanding phase.

Definition of success. In the back of our mind was the possibility that this mix of cosmologists on the forefront of their research discipline, and mathematical analysts would hit upon a solution to a major problem in the discipline, or at least make a significant step in the direction of a solution of a major problem, to which subsequent collaboration would progress (as was achieved in a different context in [7][8]). Our BIRS program has indeed led to new collaborations being formed with the goal of solving the challenge from cosmology which motivated the workshop. More generally, it was agreed by all participants that the week at BIRS was very intensive, with well prepared short courses both on the mathematical side and on the physics content, that were very well received. It seems as though the several disciplines of theoretical cosmology that were represented were able to benefit from the opportunity to present their work to each other, something which, we learned, seems to be unfortunately rarer than it should be. We came away from the workshop with a better appreciation of the rigor of an analytical argument and the scope of some of the theorems of general relativity that have been proved over the last decade or so. We also came away with a deeper appreciation of the demands that observations make on cosmologists, that is, one must not only construct a conceptially rigorous theory, but it is also demanded to be quantitative and predictive. That is, it must make specific predictions that have the possibility to be verified or falsified within a forseeable future.

References.

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