# Mitigating Interference with Integer-Forcing Architectures

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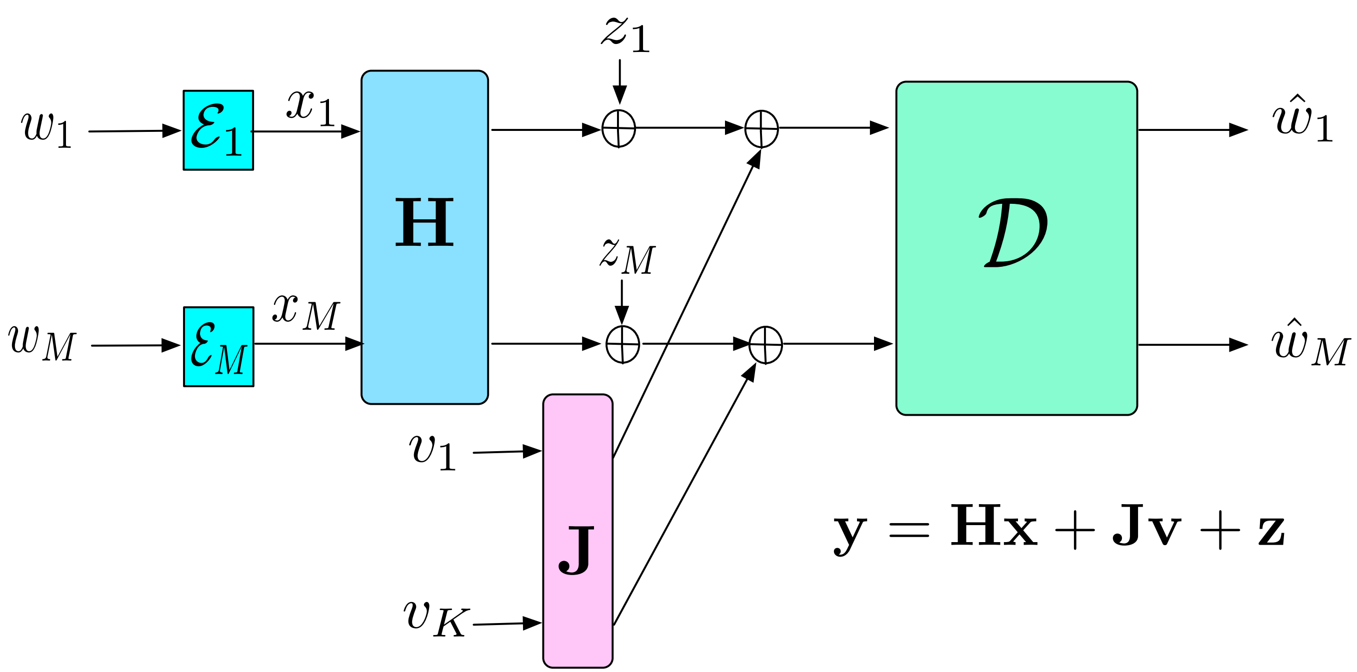
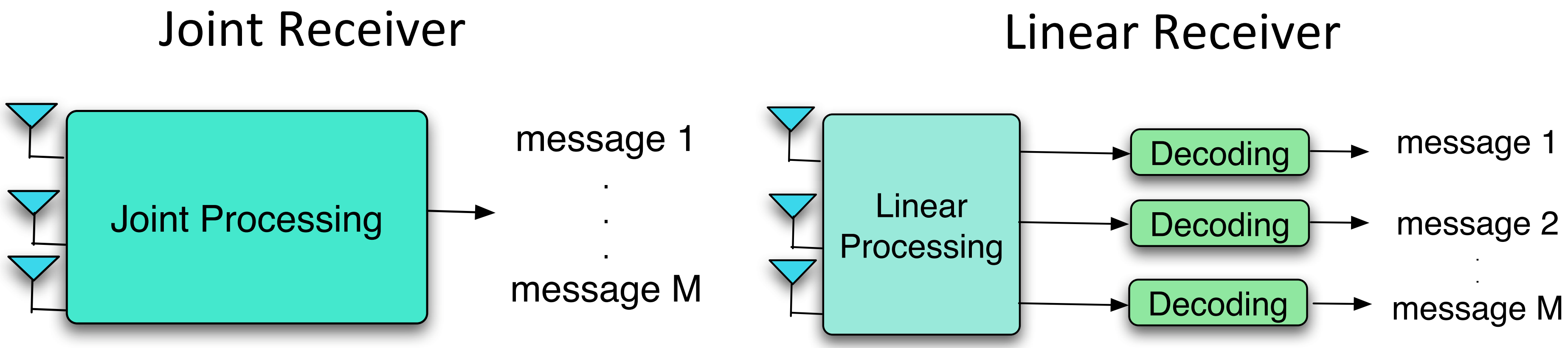
# MIMO Channel with External Interference




$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{J}\mathbf{v} + \mathbf{z}$$

- M x M MIMO with K dimensional external interference
- Each antenna encodes an independent message (or data stream) at rate R/M using power P
- Channel state information is known only at the receiver

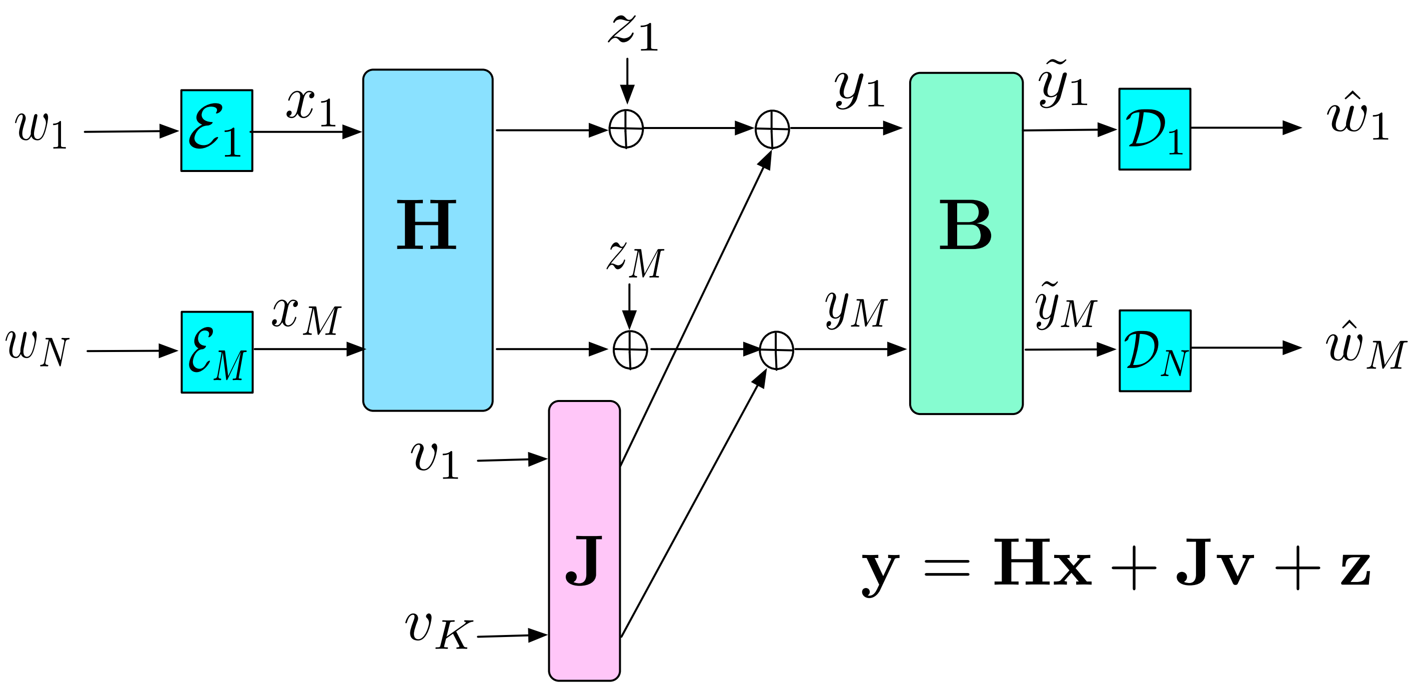
# Receiver Architectures




- Previously proposed Integer-Forcing Linear Receiver for MIMO channels without external interference [Zhan-Nazer-Erez-Gastpar ISIT ’10]

- Show that It can be used to mitigated external interference

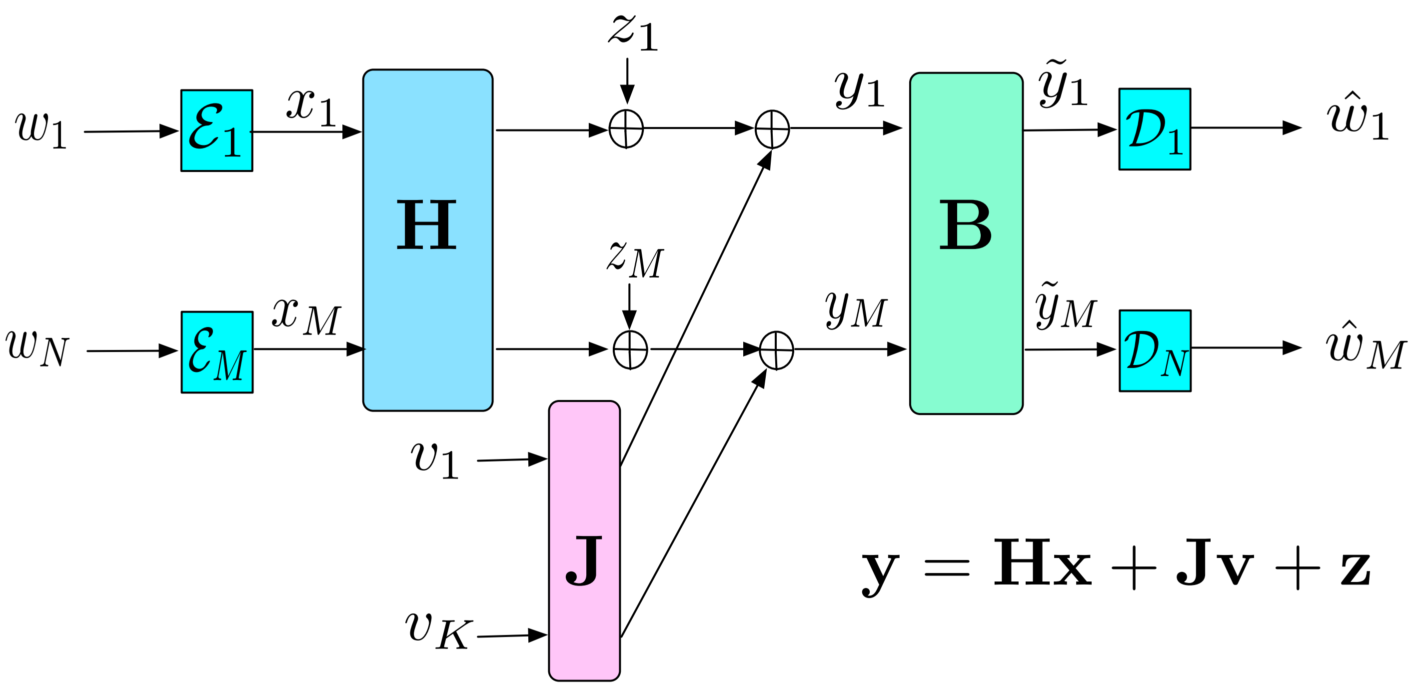
# Traditional Linear Receivers




$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{J}\mathbf{v} + \mathbf{z}$$

- Project received signal: $\tilde{\mathbf{y}} = \mathbf{B}\mathbf{y}$
- Each stream $\tilde{y}_m$ is fed into a separate decoder that attempts to recover a message $\mathbf{w}_m$

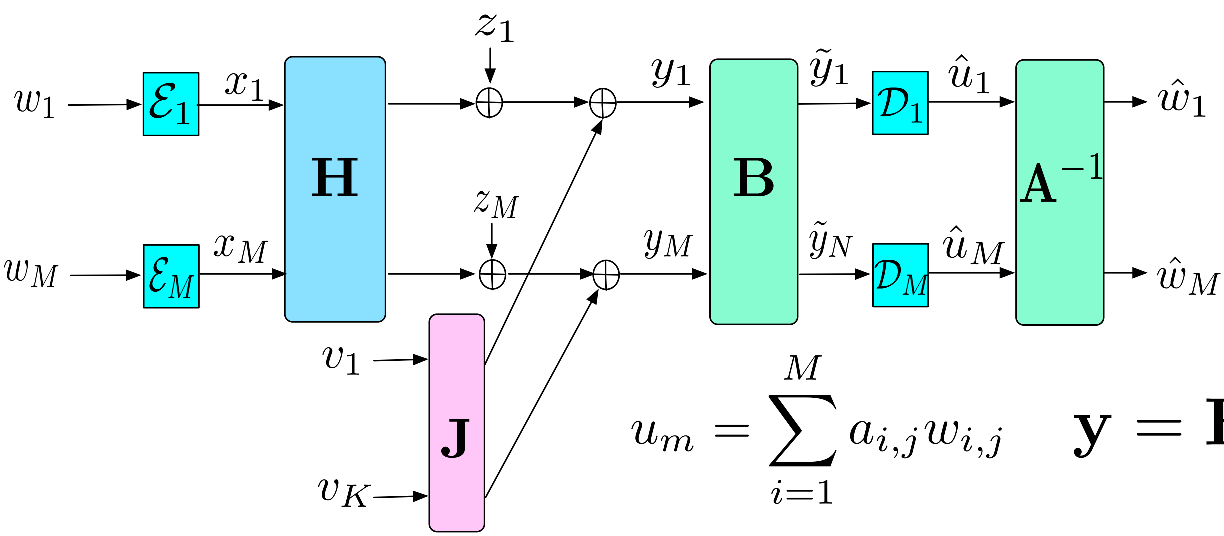
# Traditional Linear Receivers




$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{J}\mathbf{v} + \mathbf{z}$$

- Null Interference: Transmit M-K data streams and then project output unto space orthogonal to external interference
- Treat Interference as Noise: Transmit M streams and then use Decorrelator or linear MMSE estimator

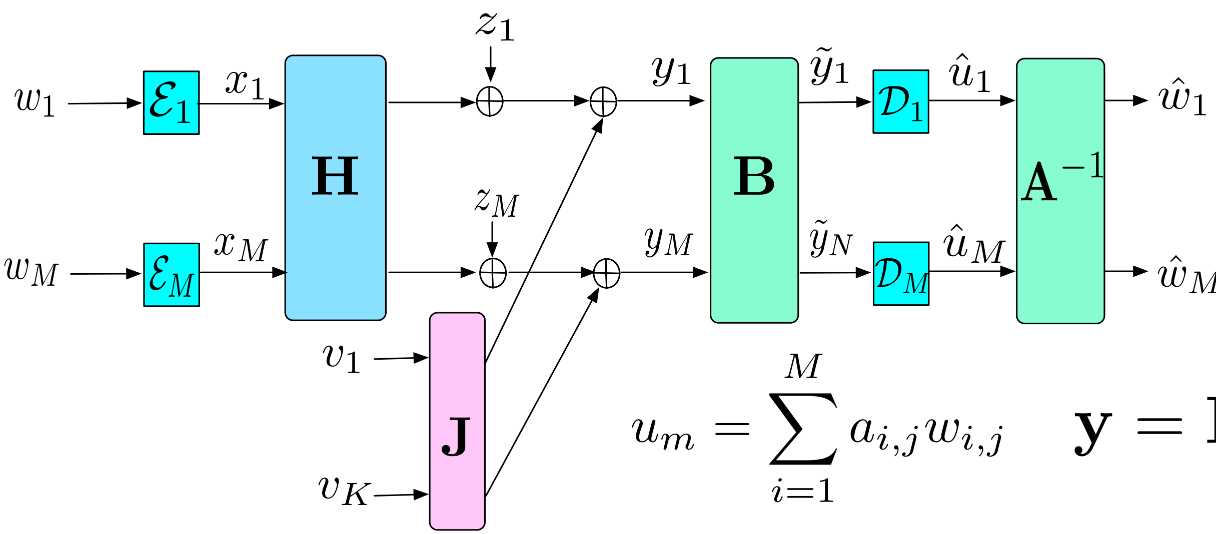
# Integer-Forcing Architecture




$$u_m = \sum_{i=1}^{M} a_{i,j} w_{i,j} \qquad \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{J}\mathbf{v} + \mathbf{z}$$

- Project received signal: $\tilde{\mathbf{y}} = \mathbf{B}\mathbf{y}$
- Each stream $\tilde{y}_m$ is fed into a separate decoder that attempts to recover an equation $\mathbf{u}_m$
- Equations can be digitally solved for the original messages
- Compute-and-Forward is used to decode equations [Nazer-Gastpar IT ’11]

# Integer-Forcing Architecture

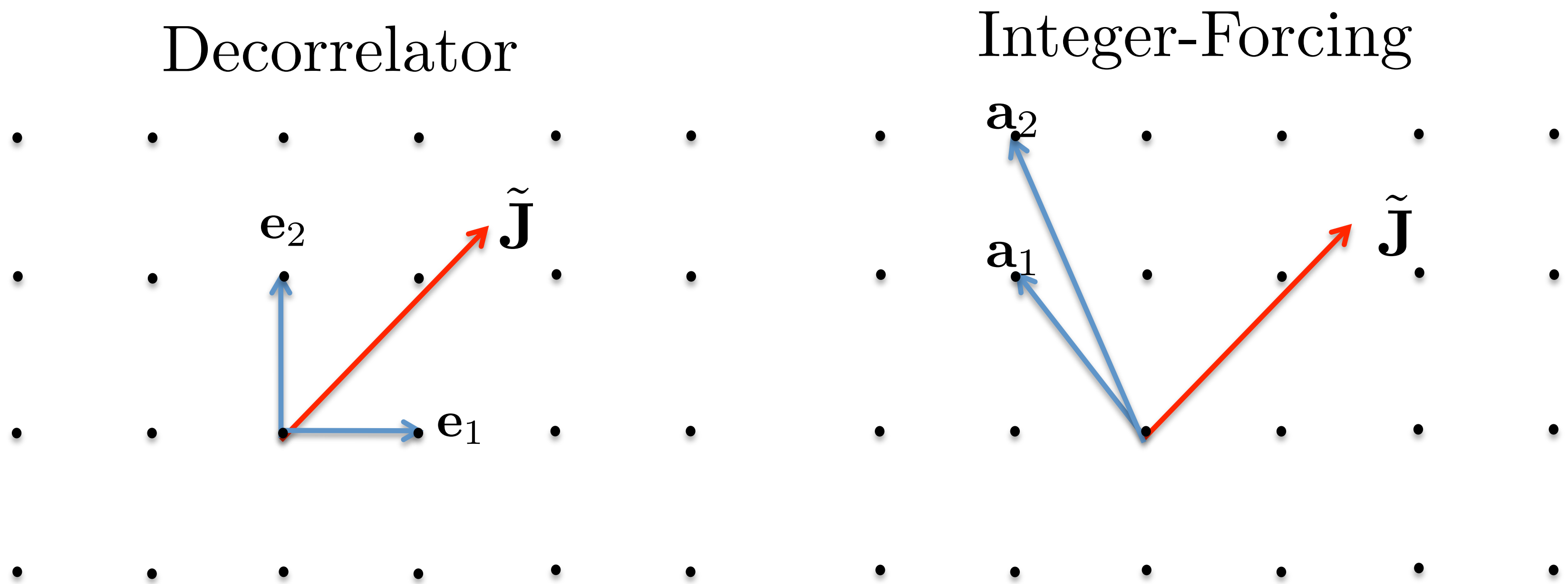



$$u_m = \sum_{i=1}^{M} a_{i,j} w_{i,j} \qquad \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{J}\mathbf{v} + \mathbf{z}$$

- Freedom to choose $\mathbf{A}$ to be any full rank integer matrix
- Choosing $\mathbf{A} = \mathbf{I}$ reduces to traditional linear receivers

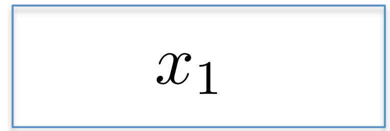
# Choosing Equations

$$\sigma_{\text{EFFECTIVE,m}} \leq \lambda_{\text{MAX}}^2 \|\mathbf{a}_m\|^2 + \|\tilde{\mathbf{J}}^T \mathbf{a}_m\|^2 INR$$



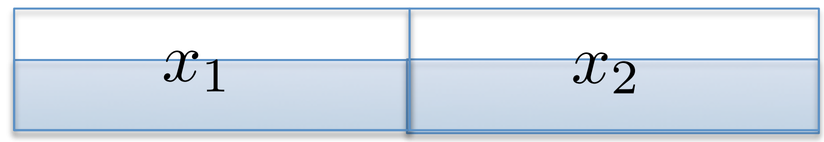
# Mitigating Interference

After Preprocessing:

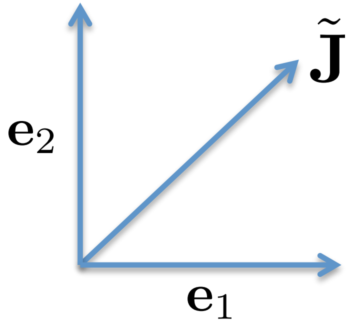



Null Interference

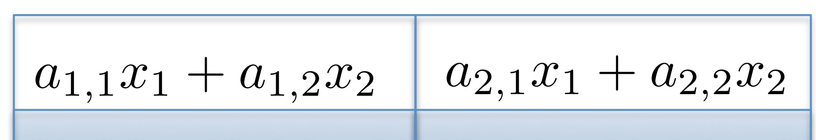

- One Data Stream
- No External Interference after preprocessing



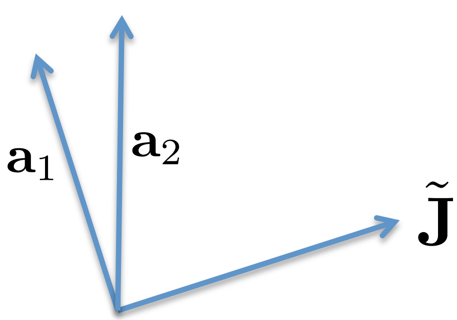

Treat Interference as Noise




- Two Data Streams
- External Interference Mostly Unmitigated

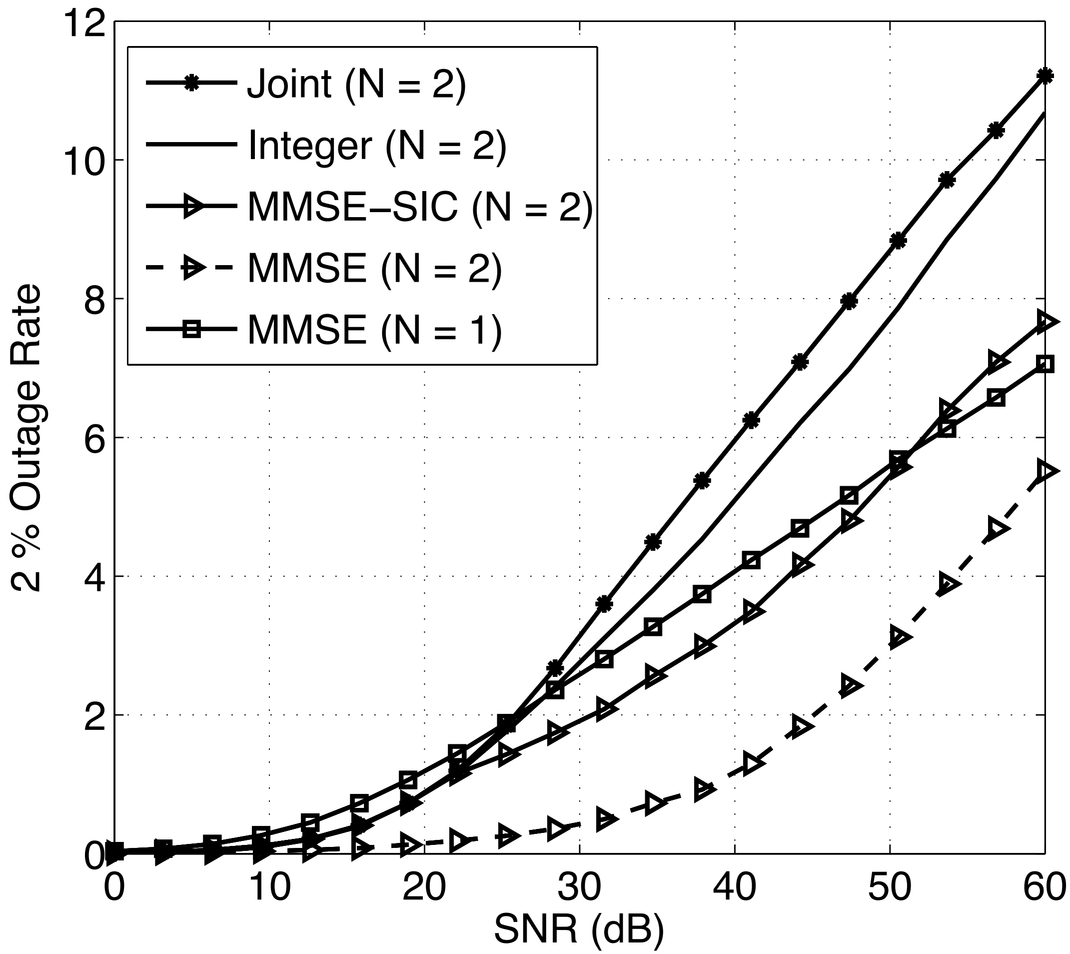



Integer-Forcing




- Two Linear combinations of Data Streams
- External Interference Mostly Mitigated

# 2% Outage Rate




$$\alpha = \frac{\log INR}{\log SNR}$$

$\alpha = 0.2$

# Generalized Degrees of Freedom

**Theorem**

Consider the M x M MIMO channel with K dimensional Interference.

Let $\alpha = \frac{\log INR}{\log SNR}$. The integer-forcing linear receiver achieves the generalized degrees of freedom:

$$d_{\text{INT}} = M - K\alpha$$

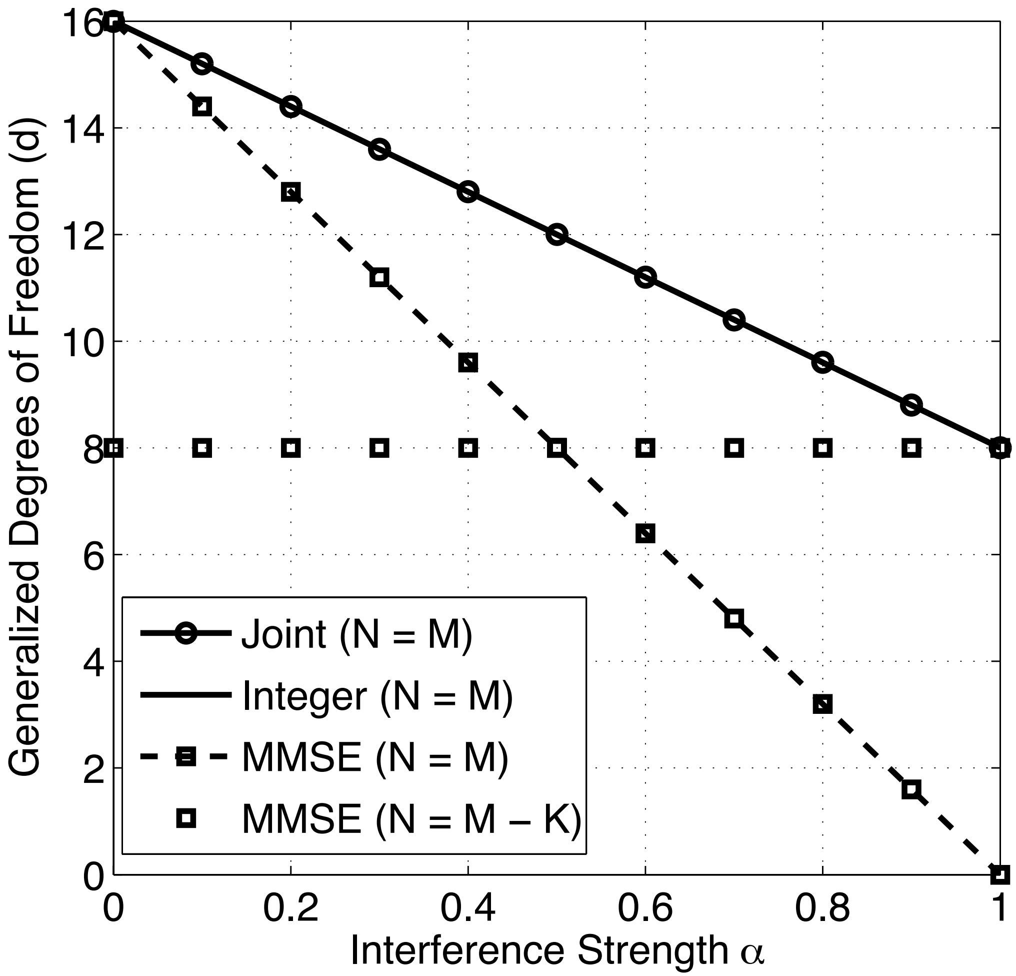
If $\mathbf{H} \in \mathbb{R}^{M \times M}$ and $\mathbf{J} \in \mathbb{R}^{M \times K}$ are such that the rows of $\mathbf{H}^{-1}\mathbf{J}$ are rationally independent.

$$d_{\text{JOINT}} = M - K\alpha$$

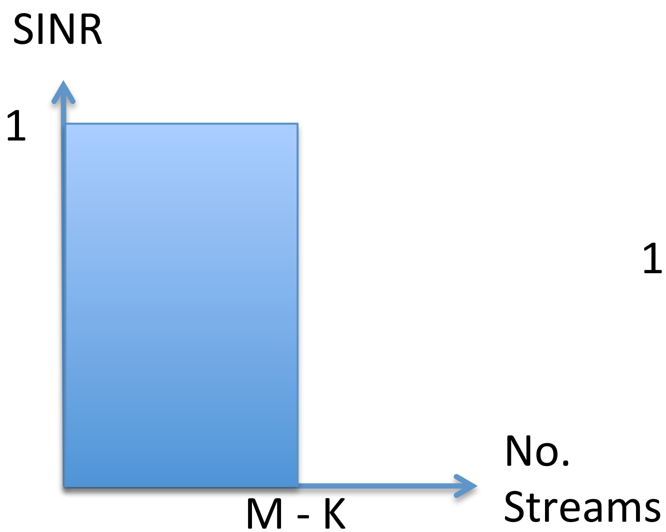
$$d_{\text{MMSE,M-K}} = M - K$$

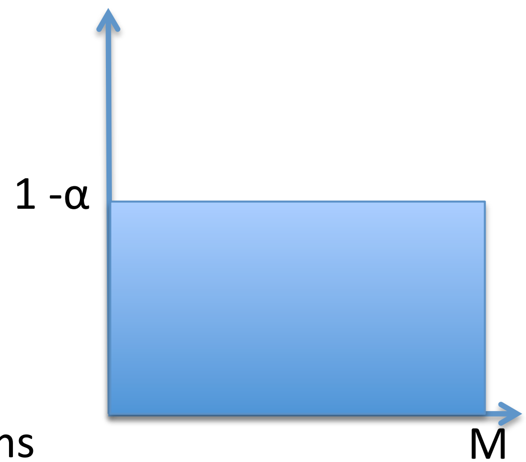
$$d_{\text{MMSE,M}} = M - M\alpha$$

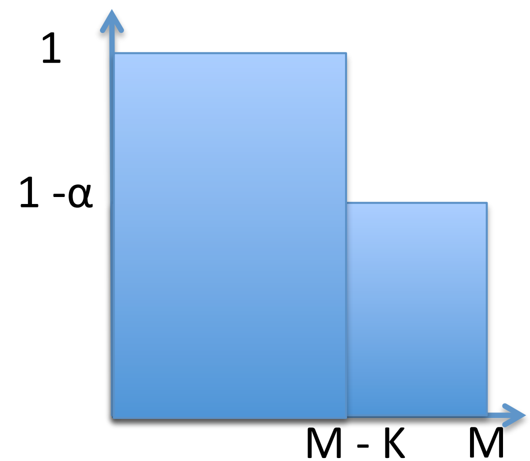
# Generalized Degrees of Freedom



# Generalized Degrees of Freedom




Transmit M – K Data Streams
Null Interference

Transmit M Data Streams
Treat Interference as Noise

Integer-Forcing

$d = M - K$

$d = M(1 - \alpha)$

$d = M - K\alpha$

# Proof Ideas

- Choose $\mathbf{a}_1 \cdots \mathbf{a}_M$ to minimize:

$$\max_m \lambda^2_{\text{MAX}}(\mathbf{H}^{-1})\|\mathbf{a}_m\|^2 + \|\tilde{\mathbf{J}}^T \mathbf{a}_m\|^2 INR$$

Find M linearly independent integer vectors $(\mathbf{q}_1, \mathbf{p}) \cdots (\mathbf{q}_M, \mathbf{p}_M)$ $\in \mathbb{Z}^{M-K} \times \mathbb{Z}^K$ that minimizes:

$$\|\mathbf{q}_m\|^2 + \|\mathbf{X}\mathbf{q}_m - \mathbf{p}_m\|^2 INR$$

where $\mathbf{X} \in \mathbb{R}^{K \times (M-K)}$

# Dirichlet's Approximation Theorem

- In the scalar case, the tradeoff is:

$$|q| \leq Q \qquad \|qx - p\| \leq \frac{1}{Q}$$

- Proof is by Pigeon hole principle

**Lemma (Dirichlets)**

For any $\mathbf{X} \in \mathbb{R}^{K \times (M-K)}$ and any $Q \in \mathbb{Z}_+$, there exists a $(\mathbf{q}, \mathbf{p}) \in \mathbb{Z}^{M-K} \times \mathbb{Z}^K \backslash \mathbf{0}$ with $\|\mathbf{q}\|_\infty \leq Q$ such that

$$\|\mathbf{X}\mathbf{q} - \mathbf{p}\| \leq \frac{1}{Q^{\frac{M-K}{K}}}$$

- What happens with multiple linearly independent approximations?

# Diophantine Approximations

- In the scalar case, the tradeoff becomes:

$$|q| \leq \Theta\left((\log Q)^2 Q\right) \qquad \|qx - p\| \leq \Theta\left(\frac{(\log Q)^2}{Q}\right)$$

- Proof uses Khintchine, Minkowski and Lagrange Multipliers

**Lemma**

Let $\mathbf{X} \in \mathbb{R}^{K\times(M-K)}$ where $|x_{i,j}| \leq 1$ and $x_{i,j}$ are rationally independent for all $i, j$. There exists an $Q' \in \mathbb{N}$ such that all for $Q > Q'$, there exist $M$ linearly independent integer vectors $(\mathbf{q}_1, \mathbf{p}), \cdots (\mathbf{q}_M, \mathbf{p}_M) \in \mathbb{Z}^K \times \mathbb{Z}^{M-k}$ such that

$$\|\mathbf{X}\mathbf{q}_m - \mathbf{p}_m\| \leq \frac{C(\log Q)^2}{Q^{\frac{M-K}{K}}} \qquad \|\mathbf{q}_m\|_\infty \leq CQ(\log Q)^2$$

where $C$ is a constant that is independent of $Q$.

# Some Connections

- **Lattice Reduction**: Yao-Wornell '02, Taherzadeh-Mobasher-Khandani '07, Jalden-Elia '09
- **Lattices for AWGN Capacity:** Erez-Zamir '04
- **Lattices for DMT:** El Gamal-Caire-Damen '04
- **Practical compute-and-forward:** Feng-Silva-Kschischang '10, Hern and Narayanan '11, Ordentlich and Erez '10, Osmane and Belfiore '11
- **The Degrees of Freedom of Compute-and-Forward:** Niesen-Whiting '11