Further Comments on Lattices for Gaussian Relay Networks

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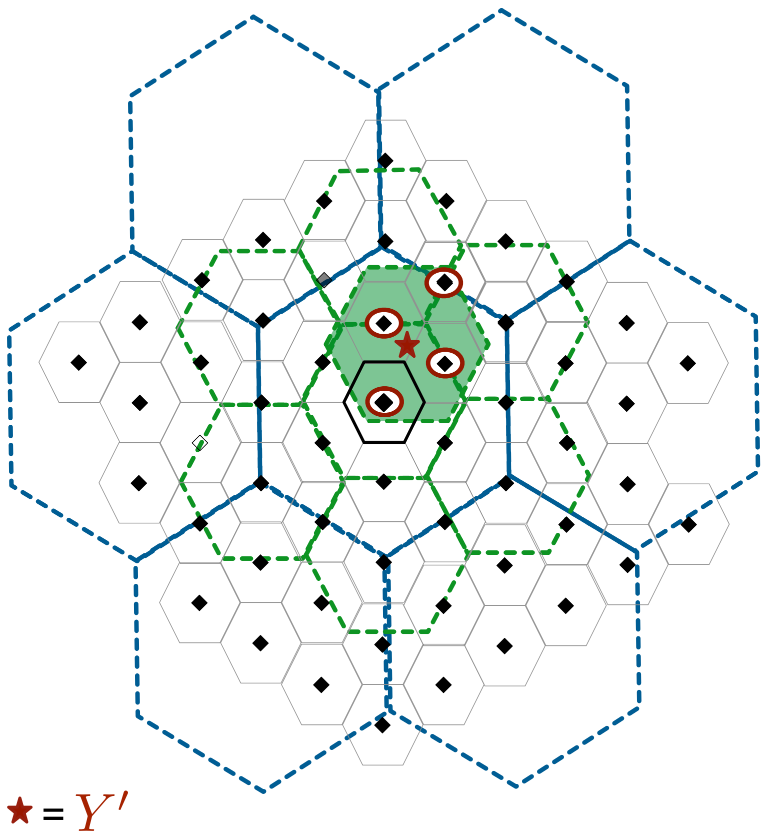
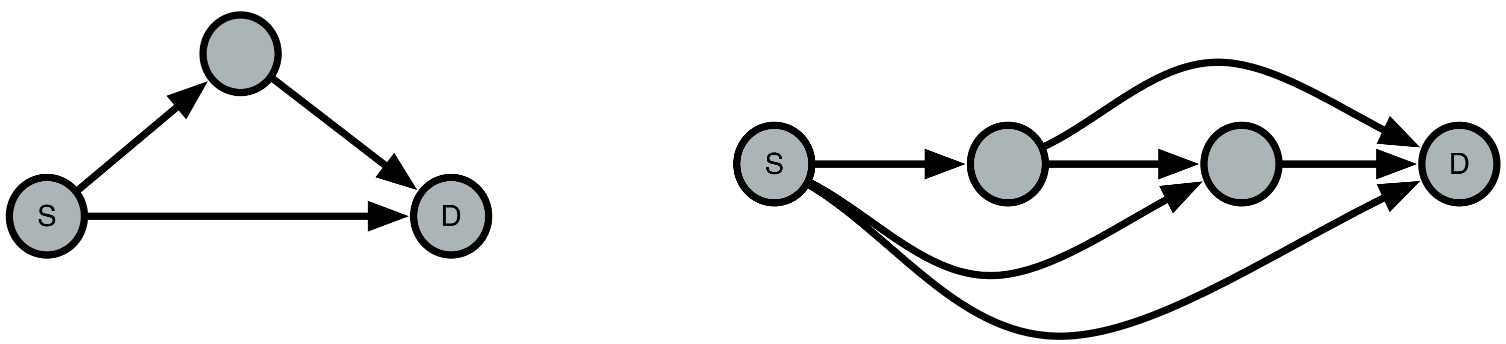
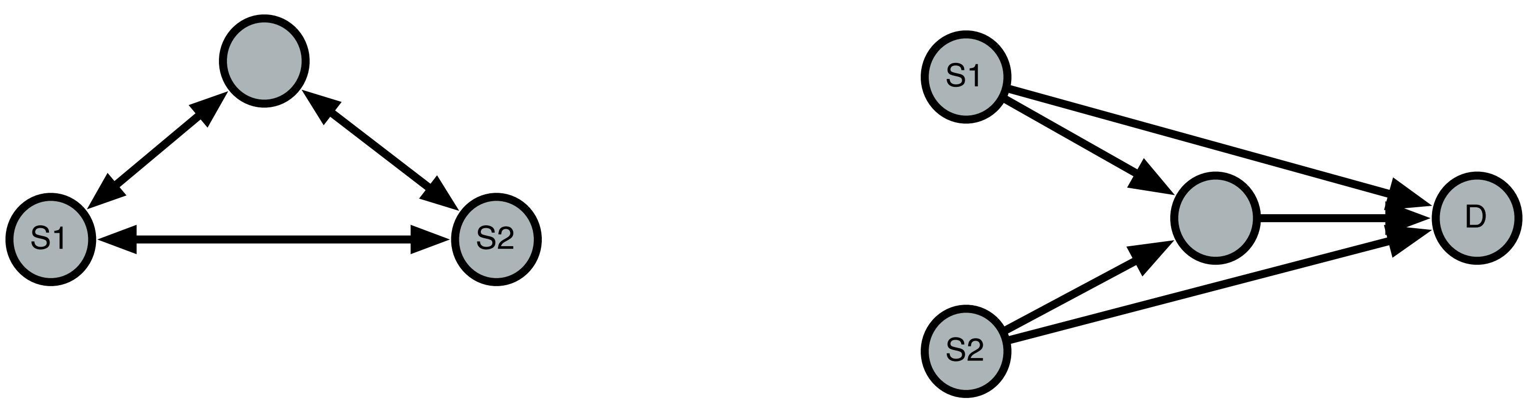
Review on “Lattices for Gaussian Relay Networks”

Lattice list decoding





- Lattice codes as good as Random codes



- Lattice codes outperform Random codes

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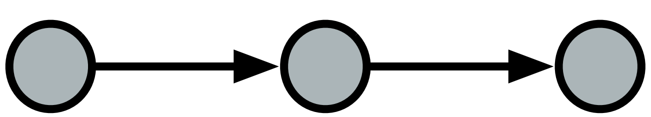
List Decoding

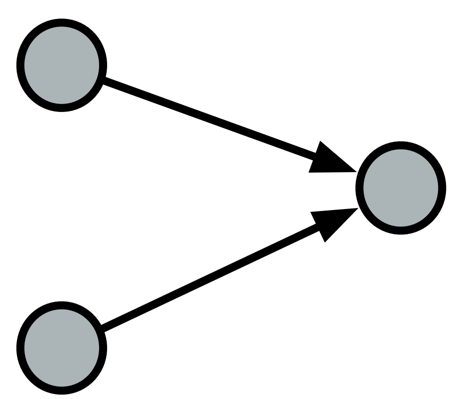
- When coding rate is above capacity, decoder can decode a list of possible codewords/messages rather than the unique one.

What is a lower bound on the size of this list?

 $: 2^{n(R-C)}$

How about:





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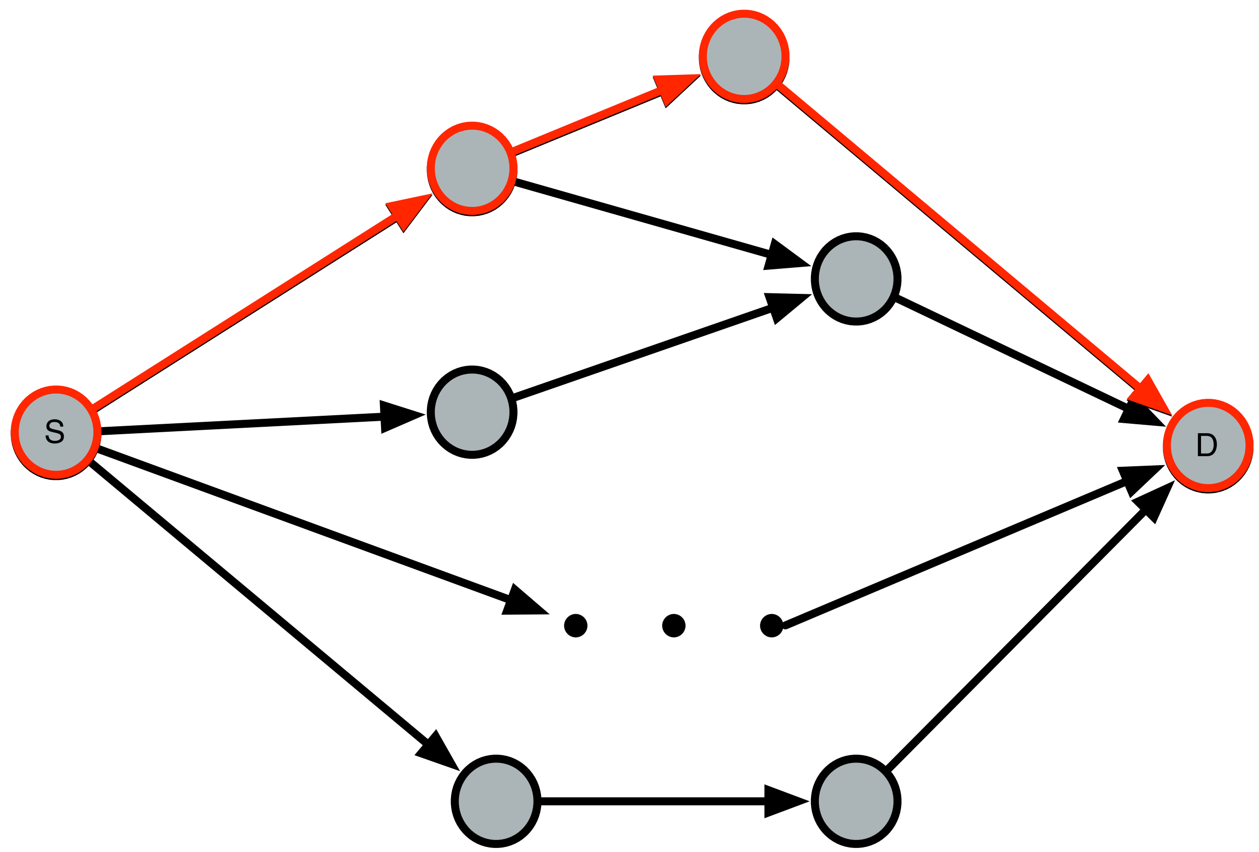
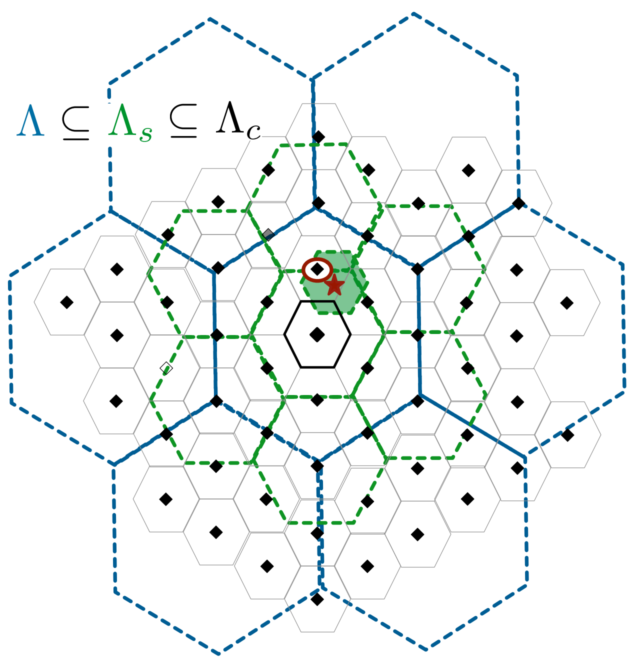
List Decoding May Be Important

- Multiple links between source and destination.

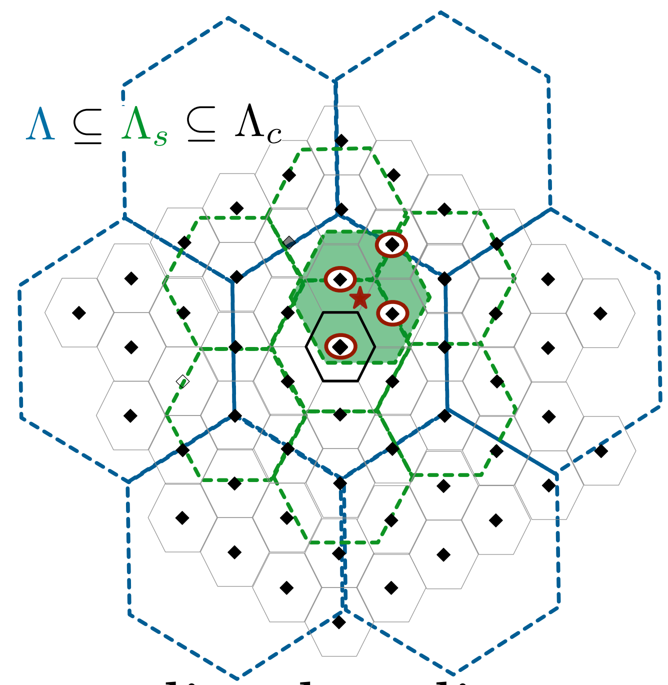


$C \geq C_{red}$: list decoding for each link? for each node?

List Decoding Is Interesting

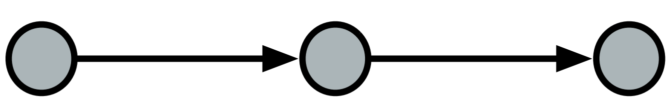


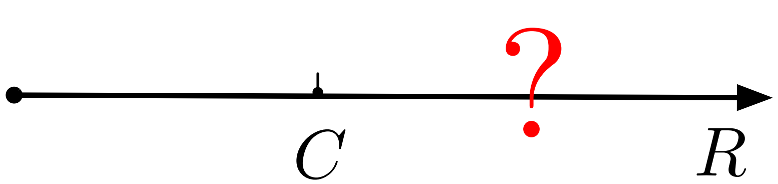
unique decoding

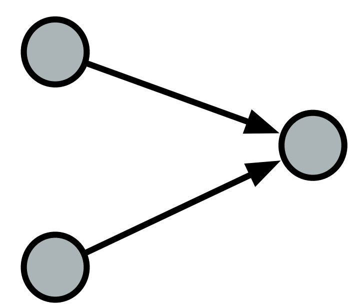


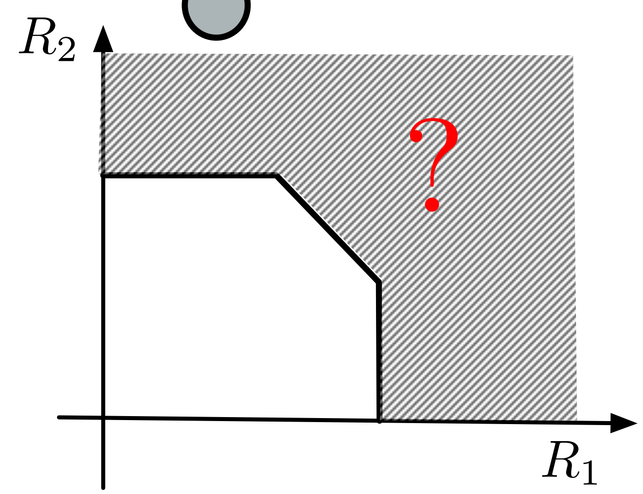
list decoding

- What is outside capacity?









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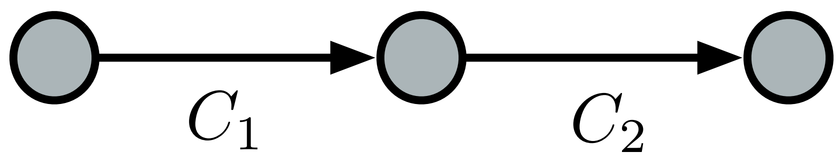
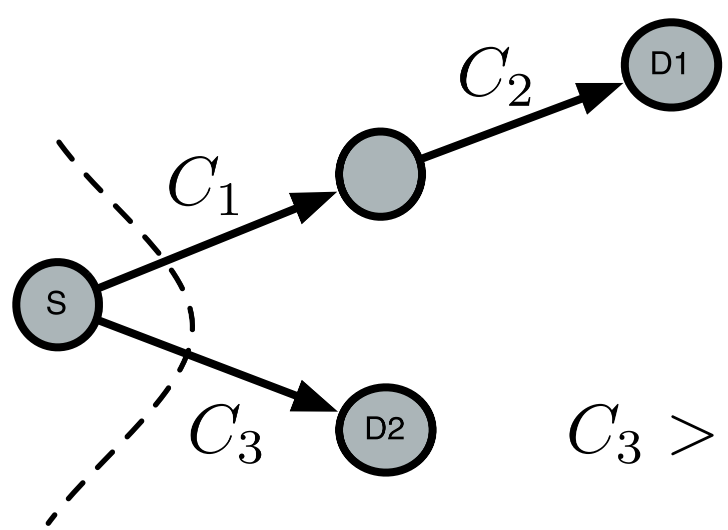
List Decoding for Gaussian Multi-hop Channel



- When $R > \min(C_1, C_2)$, destination can decode a list of possible codewords of size ?

- A possible guess can be $2^{n(R-\min(C_1,C_2))}$ since we can always bin/compress the source to be of rate $\min(C_1, C_2)$

- Why not binning?

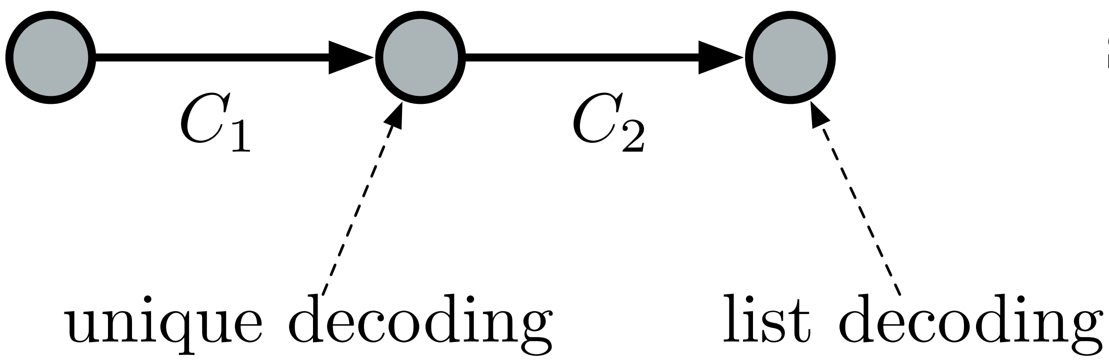


$C_3 > \min(C_1, C_2)$

Broadcast nature of wireless

List Decoding for Gaussian Multi-hop Channel

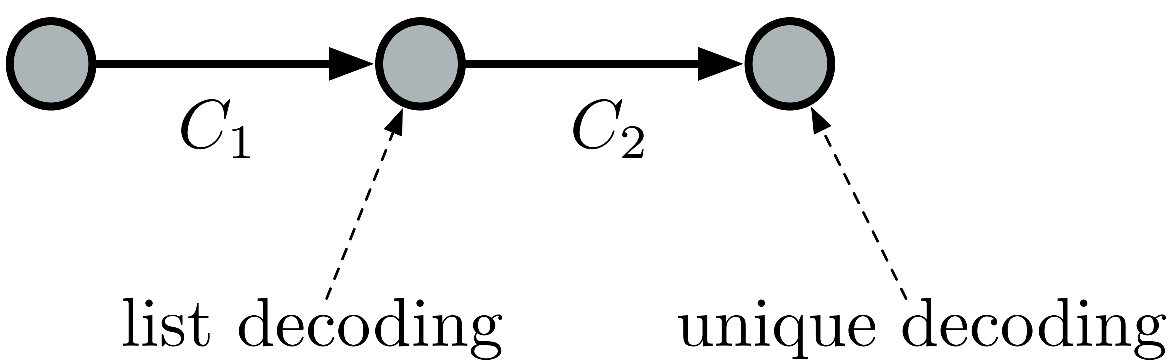
- Case 1: $C_1 > R > C_2$



size : $2^{n(R-C_2)}$

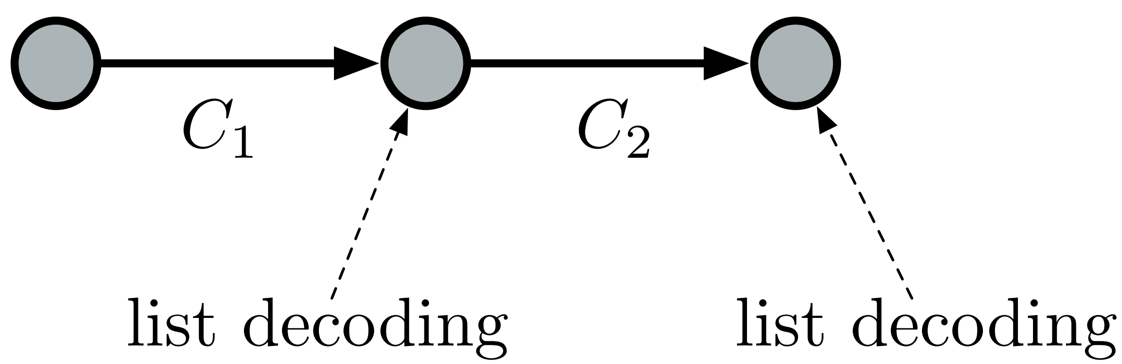
- Case 2: $C_1 < R < C_2$

size: ??



List Decoding for Gaussian Multi-hop Channel

- What about Case 3: $R > \max(C_1, C_2)$?



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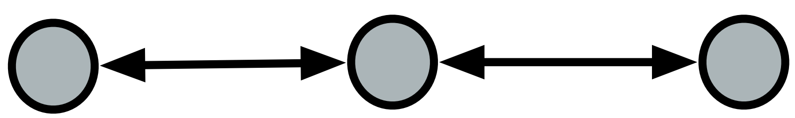
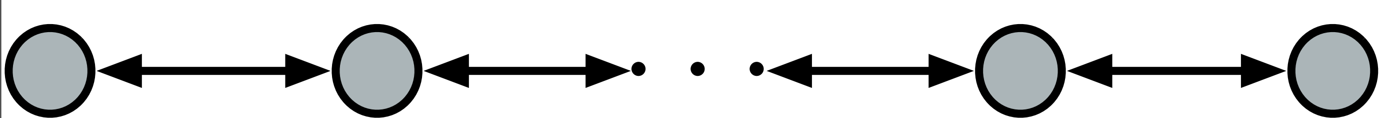
Gaussian Two-Way Multi-Relay Channel

- In AWGN two-way communications scenarios, can two streams flow at the same time without interfering with each other?

- *[Han, Trans. IT, 1984]*



- *[Nam, Chung, Lee, 2010], [Wilson, Narayanan, Pfister, Sprintson, 2010]*



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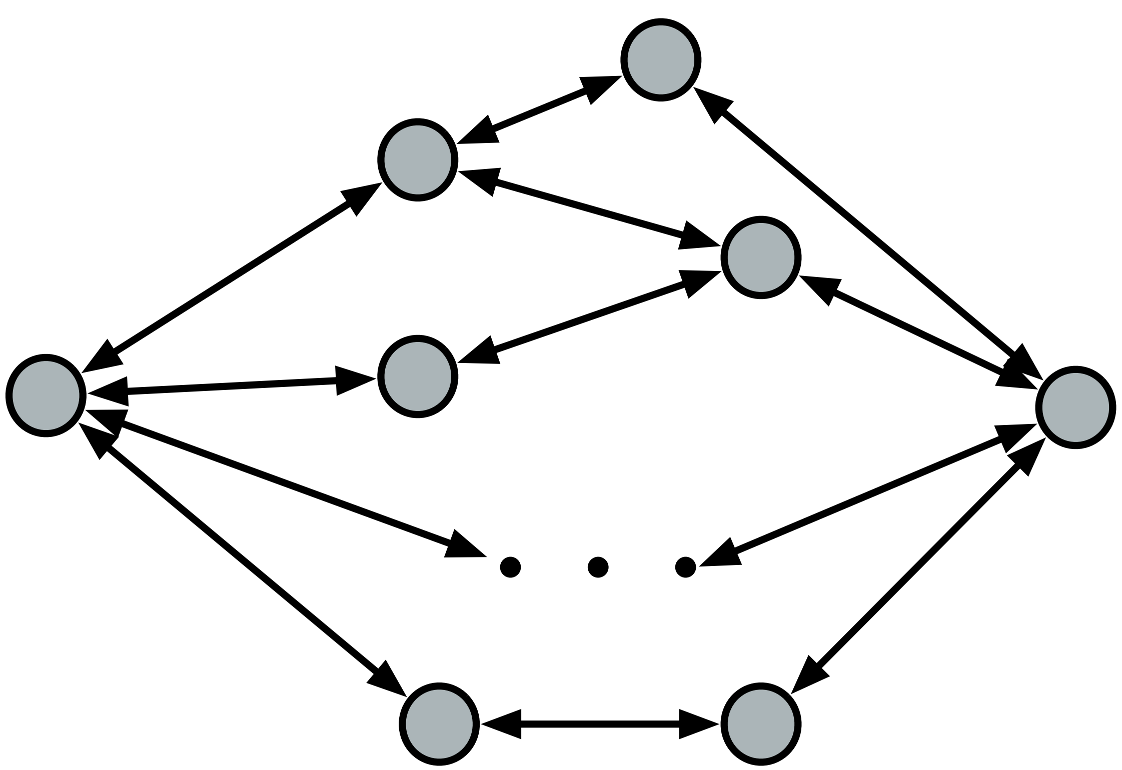

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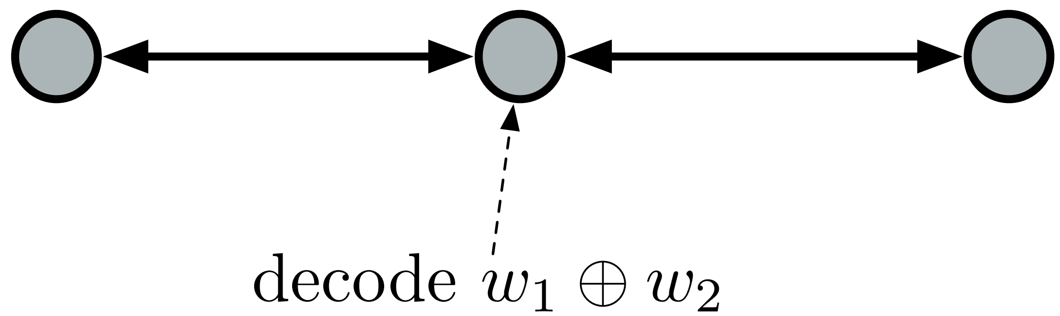
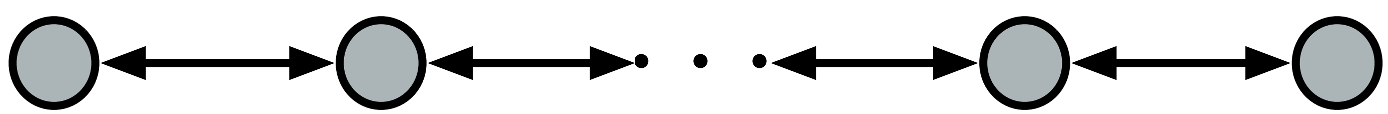
Applying Lattices in TW-MR Channel: The Possibility



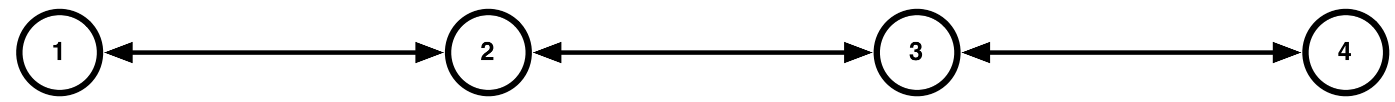
- With side information w_1/w_2 , $w_1 \oplus w_2$ is enough to determine w_2/w_1 .

MAC phase: lattice codes. BC phase: random codes.



- Every relay needs to exploit its own side information.

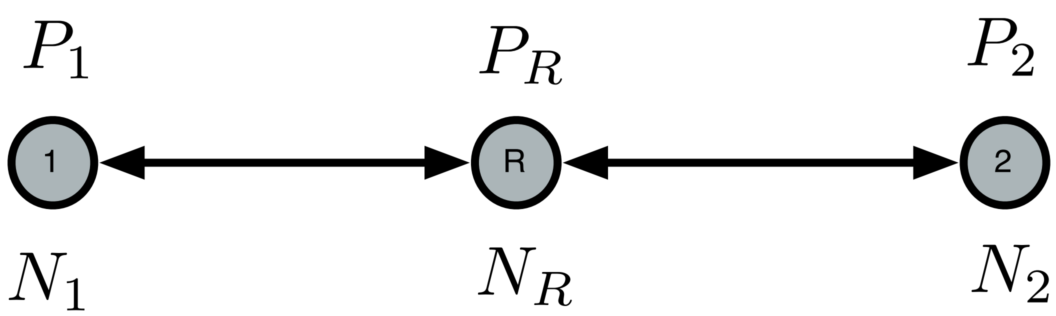
MAC phase: lattice codes. BC phase: lattice codes.



		1	2		3		4
beginning of stage (1)	send	t_{a1}					t_{b1}
end of stage (1)	decode		t_{a1}		t_{b1}		
	send	t_{a2}	t_{a1}		t_{b1}		t_{b2}
end of stage (2)	decode		$(t_{a2}+t_{b1})$	mod	$(t_{a1}+t_{b2})$	mod	
	send	t_{a3}	$(t_{a2}+t_{b1})$	mod	$(t_{a1}+t_{b2})$	mod	t_{b3}
				$-t_{a1}$	$-t_{b1}$		
end of stage (3)	decode	t_{b1}	$(t_{a3}+t_{b2})$	mod	$(t_{a2}+t_{b3})$	mod	t_{a1}
	send	t_{a4}	$(t_{a3}+t_{b2})$	mod	$(t_{a2}+t_{b3})$	mod	t_{b4}
				$-(t_{a2}+t_{b1})$	$-(t_{a1}+t_{b2})$		
end of stage (4)	decode	t_{b2}	$(t_{a4}+t_{b3}+t_{b1})$	mod	$(t_{b4}+t_{a3}-t_{a1})$	mod	t_{a2}
	send	t_{a5}	$(t_{a4}+t_{b3}+t_{b1})$	mod	$(t_{b4}+t_{a3}-t_{a1})$	mod	t_{b5}
				t_{a1} $-(t_{a3}+t_{b2})$	t_{b1} $-(t_{b3}+t_{a2})$		
end of stage (5)	decode	t_{b3}	$(t_{a5}+t_{b4}-t_{b2})$	mod	$(t_{b5}+t_{a4}-t_{a2})$	mod	t_{a3}
	send	t_{a6}	$(t_{a5}+t_{b4}-t_{b2})$	mod	$(t_{b5}+t_{a4}-t_{a2})$	mod	t_{b6}
				$(t_{a2}+t_{b1})$ $-(t_{a4}+t_{b3}+t_{b1})$	$(t_{b2}+t_{a1})$ $-(t_{b4}+t_{a3}+t_{a1})$		
end of stage (6)	decode	t_{b4}	$(t_{a6}+t_{b5}-t_{b3}+t_{b1})$	mod	$(t_{b6}+t_{a5}-t_{a3}+t_{a1})$	mod	t_{a4}

Applying Lattices in TW-MR Channel: The Difficulty

- To apply lattice codes in BC phase, the difficulty is power asymmetry.



assume $P_1 \geq P_2$

- Relay decodes $T = (t_1 + t_2 - Q_2(t_2 + U_2)) \mod \Lambda_1$, re-encodes it and broadcasts it. Is there an encoding/permutation function f(T) such that , for any given t_1 , f(T) is a good lattice which occupies the whole space of $\mathcal{V}(\Lambda_1)$

[Nam, Chung, Lee, 2010]

- Notice for any given t_1 , T is a good lattice which is spread over $\mathcal{V}(\Lambda_2)$

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