Prospects of Incoherent Control by Continuous Measurements

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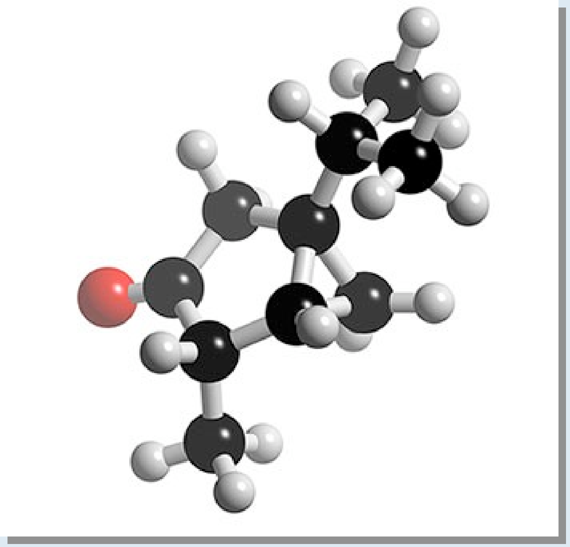
Motivation

- coherent control inefficient in systems with many dof (e.g. polyatomic molecules)



- incoherent dynamics: contractive evolution & steady states

  ➔ **robustness**

- ways to induce controlled incoherent dynamics:

  1. environment engineering (Prezhdo, PRL 85, 4413 (2000))
  2. optical pumping (Wang, Schirmer, PRA 81, 062306 (2010))
  3. measurements (Roa et al., PRA 73, 012322 (2006))

Generalized position measurement

- measurement operators: $M_i(x)$, with $\sum_i \int M_i^\dagger(x)\, M_i(x) dx = 1$
- detection probability: $P_i(\psi) = \int |M_i(x)\psi(x)|^2 \, dx$
- post measurement state: $\psi(x) \xrightarrow{\text{outcome } i} M_i(x)\psi(x)/\sqrt{P_i(\psi)}$
- left-right measurement: $M_l(x) = \Theta(-x)$ $M_r(x) = \Theta(x)$

dynamics under continuous non-selective measurement

$$\mathcal{L}\rho = \dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H,\rho] + \gamma\Big(\sum_{i\in\{\mathrm{l},\mathrm{r}\}} M_i\rho M_i^\dagger - \tfrac{1}{2}\{M_i^\dagger M_i,\rho\}\Big)$$

$$= \underbrace{-\frac{\mathrm{i}}{\hbar}\big(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger\big)}_{\text{deterministic evolution}} + \underbrace{\gamma \sum_i M_i \rho M_i^\dagger}_{\text{jumps}} = (\mathcal{L}_0 + \mathcal{J})\rho$$

where $H_{\mathrm{eff}} = H - \frac{\mathrm{i}\hbar\gamma}{2} \sum_i M_i^\dagger M_i$

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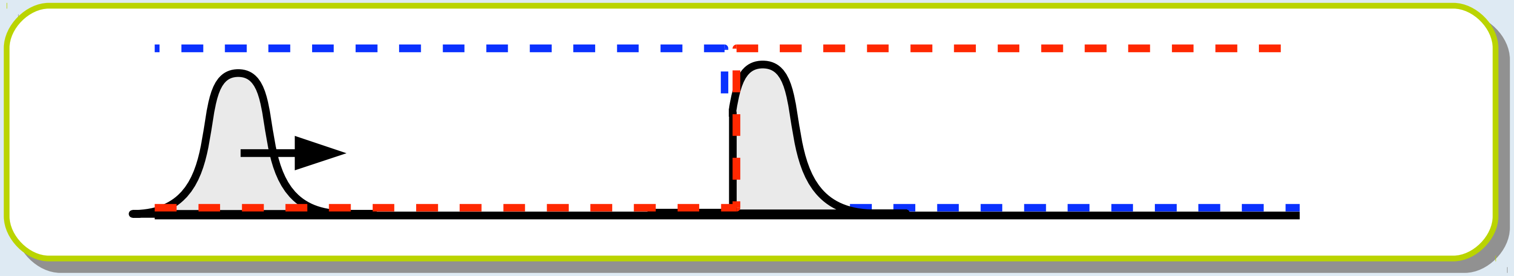
dynamics under continuous non-selective measurement

$$\begin{aligned}
\mathcal{L}\rho = \dot{\rho} &= -\frac{\mathrm{i}}{\hbar}[H,\rho] + \gamma\Big(\sum_{i\in\{\mathrm{l,r}\}} M_i\rho M_i^\dagger - \tfrac{1}{2}\{M_i^\dagger M_i,\rho\}\Big) \\
&= -\frac{\mathrm{i}}{\hbar}\big(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger\big) + \gamma \sum_i M_i\rho M_i^\dagger = (\mathcal{L}_0 + \mathcal{J})\rho \\
&= -\tfrac{\mathrm{i}}{\hbar}[H,\rho] + 2\gamma\big(M_r\rho M_r^\dagger - \tfrac{1}{2}\{M_r^\dagger M_r,\rho\}\big) = (\mathcal{L}_r + \mathcal{J}_r)\rho \\
&= -\tfrac{\mathrm{i}}{\hbar}[H,\rho] + 2\gamma\big(M_l\rho M_l^\dagger - \tfrac{1}{2}\{M_l^\dagger M_l,\rho\}\big) = (\mathcal{L}_l + \mathcal{J}_l)\rho
\end{aligned}$$

Analytical treatment in terms of jump expansion and reordering

- expand time evolution in Dyson series

$$e^{(\mathcal{L}_0+\mathcal{J})t} = e^{\mathcal{L}_0 t} + \sum_i \int dt_i..dt_1 e^{\mathcal{L}_0(t_i-t_{i-1})}\mathcal{J} e^{\mathcal{L}_0(t_{i-1}-t_{i-2})}\mathcal{J}\ldots\mathcal{J} e^{\mathcal{L}_0 t_1}$$



- reordering of jump expansion:

$$e^{\mathcal{L}t} = e^{\mathcal{L}_r t} + \int dt_1 e^{\mathcal{L}_l(t-t_1)}\mathcal{J}_r e^{\mathcal{L}_r t_1}$$
$$+ \int dt_1 dt_2 e^{\mathcal{L}_r(t-t_2)}\mathcal{J}_l e^{\mathcal{L}_l(t_2-t_1)}\mathcal{J}_r e^{\mathcal{L}_r t_1} + \ \ldots$$

➔ few jumps
➔ most relevant terms in front

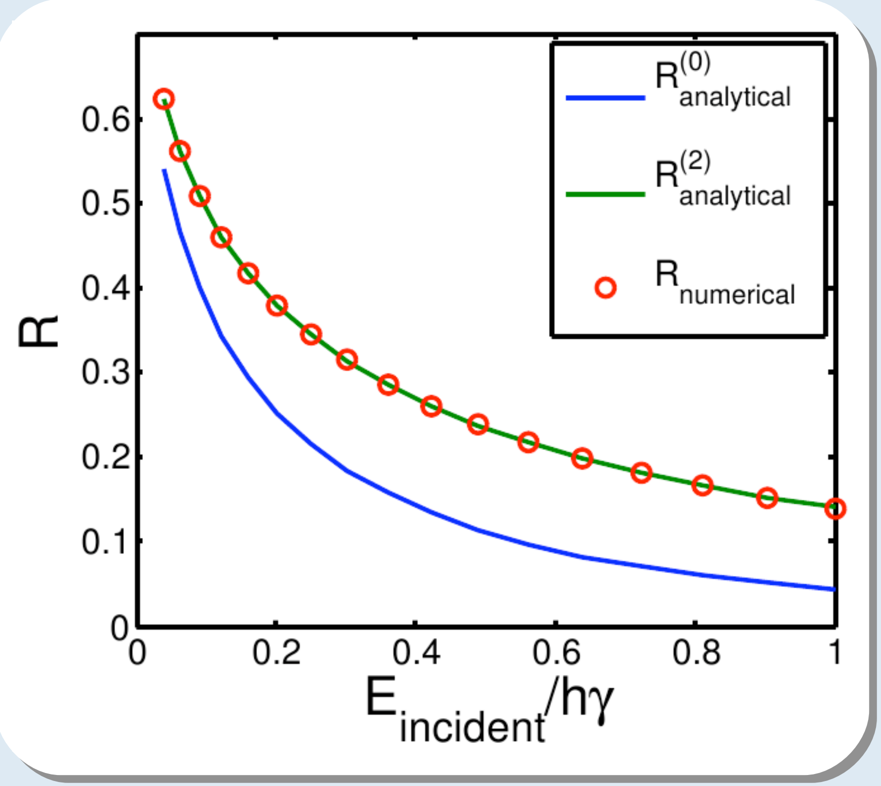
Steering molecular wave packets: reflection by measurement & Zeno effect

- reflection coeffcient of imaginary potential step (0^{th} order)

$$R^{(0)}(k) = \left| \frac{1-\sqrt{1+i\gamma/k^2}}{1-\sqrt{1+i\gamma/k^2}} \right|^2 \xrightarrow{\gamma\to\infty} 1 \quad \text{(Zeno effect)}$$

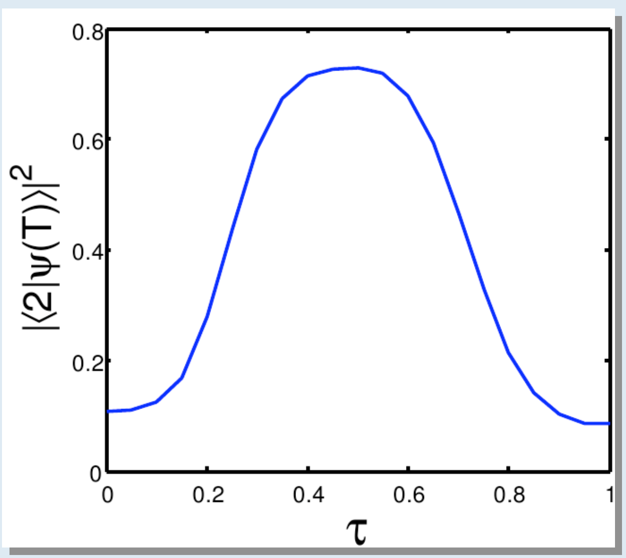
- next contribution (2^{nd} order): approximate $\psi_{t_1}(k)$ after first jump & integrate negative velocity components

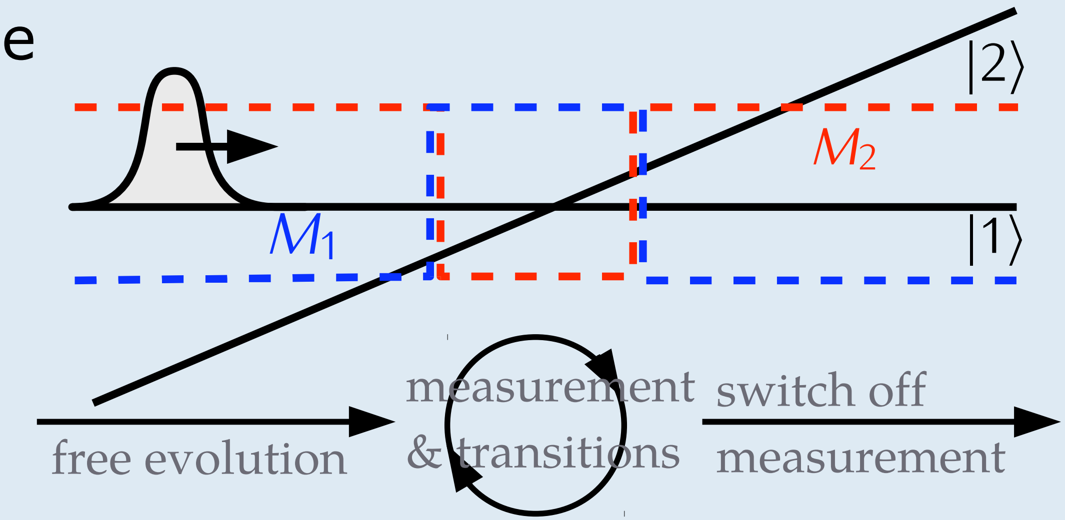
$$R^{(2)}(k) \approx \int_{-\infty}^{0} (1-R^{(0)}(k))\,|\psi_{t_1}(k)|^2$$



Control of branching ratio between coupled Born-Oppenheimer surfaces

- Zeno control ($\gamma \to \infty$): trap wave packet inside strong coupling region





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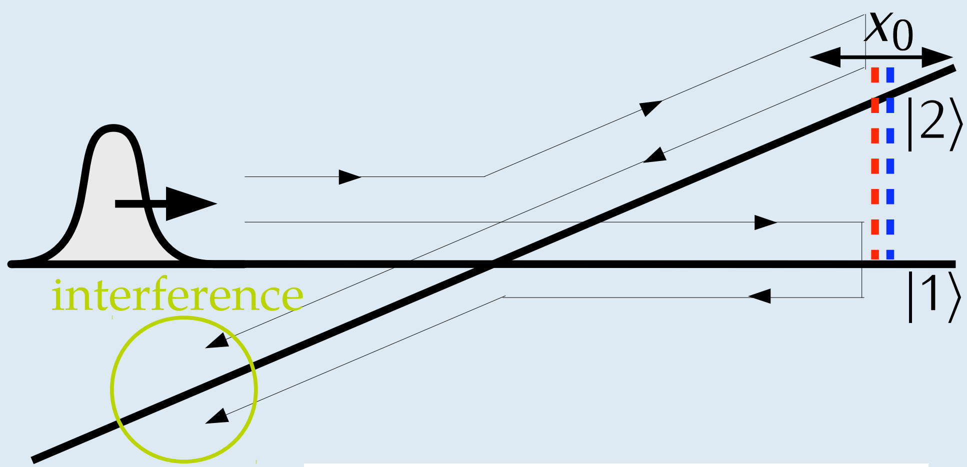
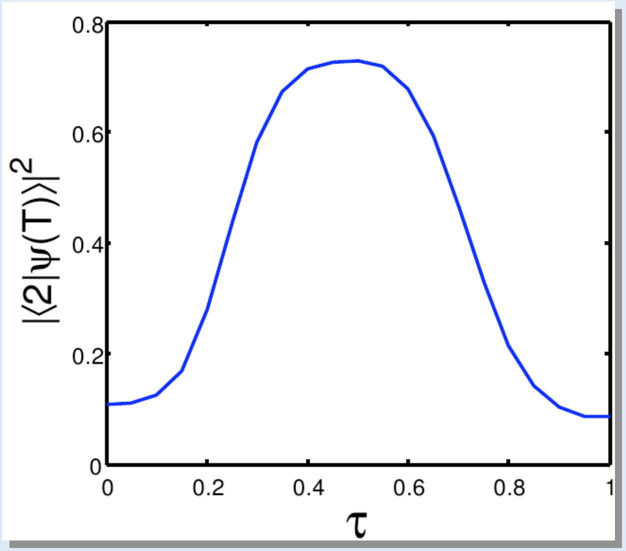
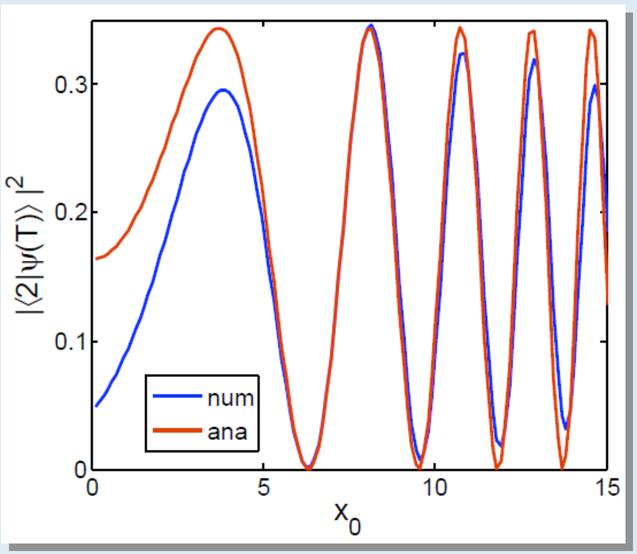
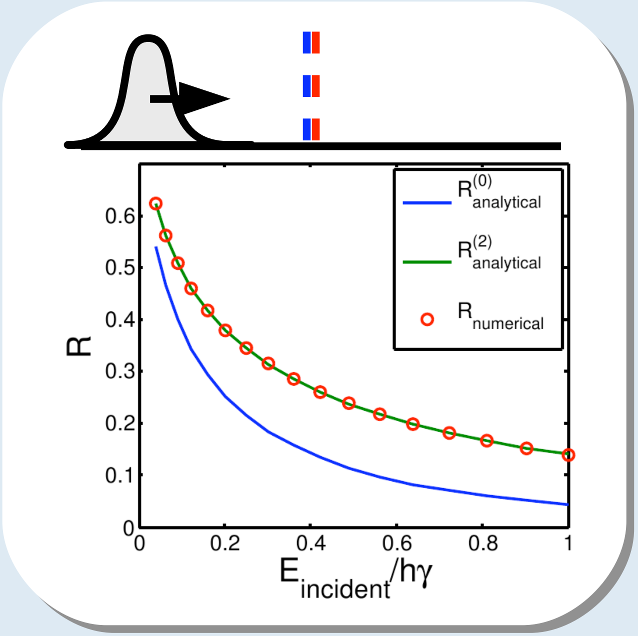


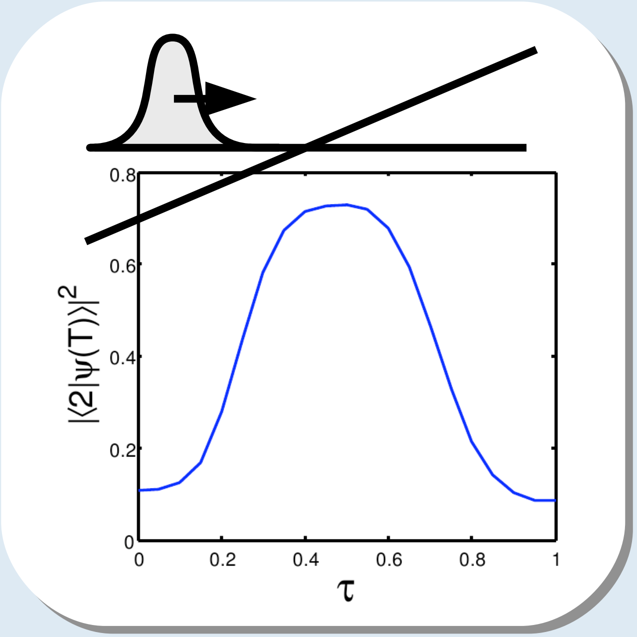




- for finite measurement rate γ, use Stückelberg interference

Summary and outlook





- incoherent scattering formalism
- optimal incoherent control