Multi-Scale Kernel Bundles for LDDMM

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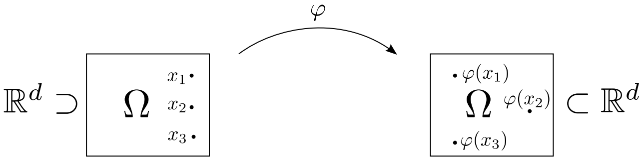
- Outline
  - Short Overview of LDDMM
  - Regularization and Kernels
  - LDDKBM, Kernel Bundles, and Scale
  - Some Experimental Results

LDDMM: Large Deformation Diffeomorphic Metric Mapping

- diffeomorphisms act on landmarks, images, etc.

Landmarks:

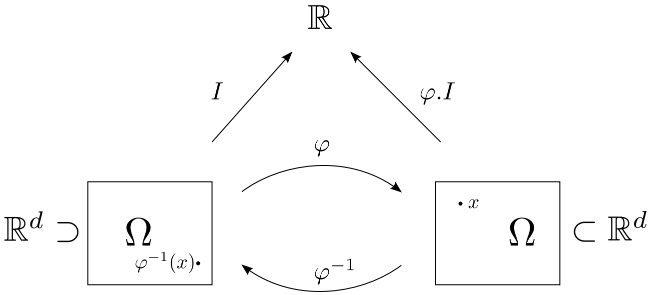


action: $\varphi.(x_1, x_2, x_3) = \big(\varphi(x_1), \varphi(x_2), \varphi(x_3)\big)$

LDDMM: Large Deformation Diffeomorphic Metric Mapping

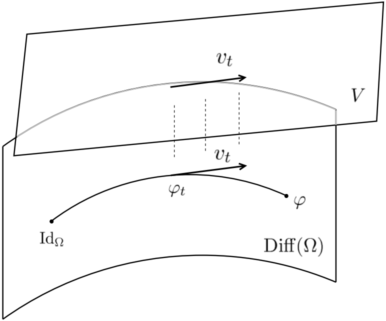
- diffeomorphisms act on landmarks, images, etc.

Images:



action: $\varphi.I = I \circ \varphi^{-1}(x) = I\big(\varphi^{-1}(x)\big)$

- manifold of diffeomorphisms $G_V \subset \mathrm{Diff}(\Omega)$ with tangent space V (subset of vector fields on domain Ω)
- G_V parametrized by time-dependent vector fields $v_t : [0, T] \to V$
- associated flow $\partial_t \varphi_t^v = v(t) \circ \varphi_t^v$



- norm on V gives metric on G_V

Registration with LDDMM

- search for $\varphi \in G_V$ minimizing

$$E(\varphi) = E_1(\varphi) + \lambda U(\varphi) \ ,$$

$$E_1(\varphi) = \min_{v_t \in V, \varphi_{01}^v = \varphi} \int_0^1 \|v_s\|_V^2 \, ds$$

- optimal paths satisfy the *EPDiff* evolution equations
- the search can be phrased in terms of the initial velocity/momentum:

$$E(v_0) = E_1(v_0) + \lambda U(v_0) \ ,$$

$$E_1(v_0) = \min_{v_t \text{ satisfy } \mathit{EPDiff}} \int_0^1 \|v_s\|_V^2 \, ds$$

- necessitates a choice of norm $\| \cdot \|_V$

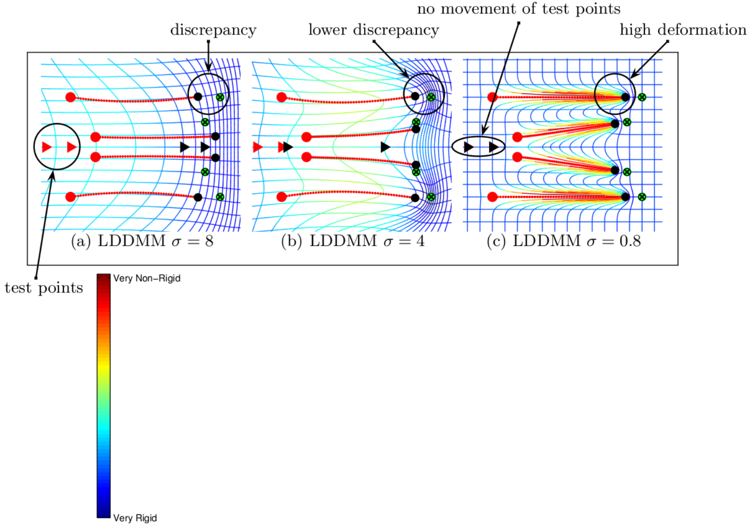
Regularization

- common norms $\|\cdot\|_V$: Sobolev, induced from Gaussians
- under certain conditions, *RHKS* structure gives correspondence between kernels $K(\cdot,\cdot) \in \mathbb{R}^{d\times d}$ and norm
- the scale of the kernel affects the regularization
- the norm/kernel duality makes the choice of norm concrete, e.g. the EPDiff equations asserts that

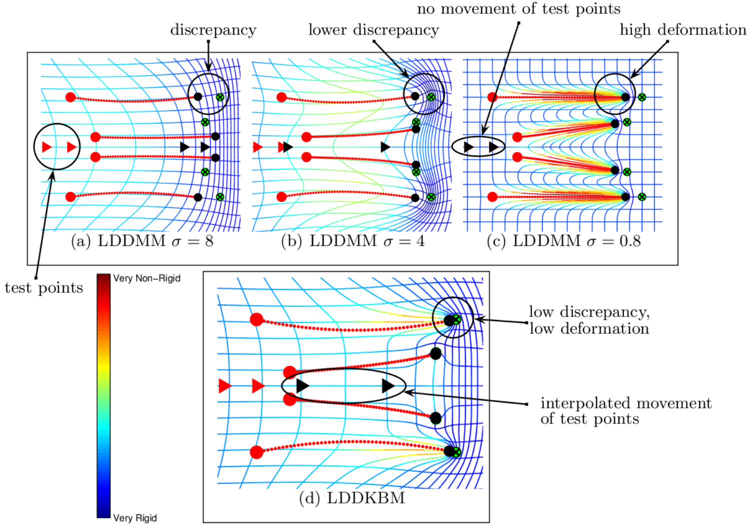
$$v_t(\cdot) = \sum_{l=1}^{N} K(\cdot, x_{t,l}) a_{t,l}$$

for landmarks

- a simple example: registration of four points with Gaussian kernels of different scales:



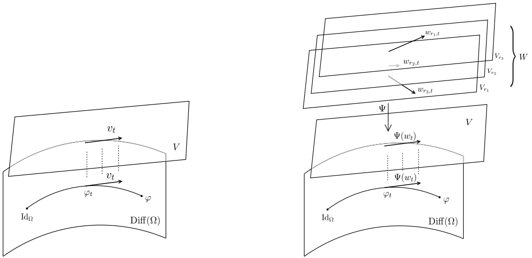
- a simple example: registration of four points with Gaussian kernels of different scales:



- sparse data provide good examples but the same phenomena arise for image data (consider e.g. areas of constant intensity)
- Modelling questions:
  - what is the right regularization? (if there is one)
  - does deformation occur at different scales?
- Performance improvements:
  - sparse descriptions of large deformations
  - can faster representations be compatible with norm? (sparse initial data must imply sparsity throughout evolution)

LDDKBM: Large Deformation Diffeomorphic Kernel Bundle Mapping

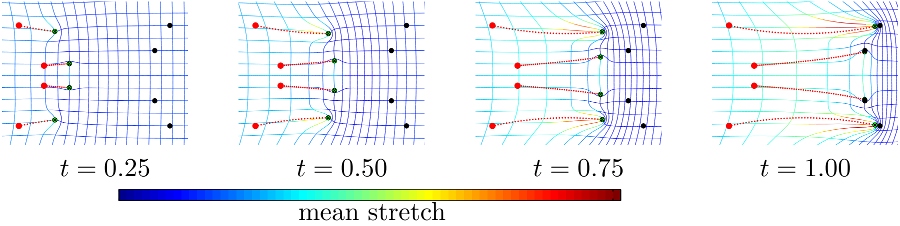
- the tangent space V is extended to a bundle W allowing different kernels/scales



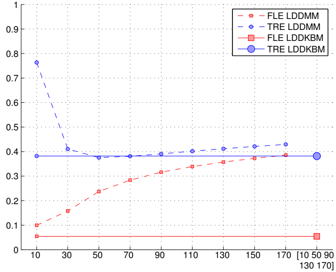
- we regularize using the norm $\|w\|_W^2 = \int_{I_W} \|w_r\|_r^2 \, dr$ on W (the contribution of each scale is split) and

$$E_1^W(\varphi) = \min_{w_t \in W, \varphi_{01}^{\Psi(w)} = \varphi} \int_0^1 \|w_s\|_W^2 \, ds$$

- Evolution of optimal 3-scale diffeomorphism path:



- in essence: we decouple the contribution of each scale in the registration
- advantages of kernel bundle extension:
  - models deformation at multiple scales/kernel shapes
  - many of the mathematical properties of LDDMM are preserved: momentum conservation, EPDiff evolution equations, well-posedness, etc.
  - statistics using scale information
  - possibility for faster registration: sparsity and smoothness at different scales
  - removes the need for scale selection:



KB-EPDiff equations

- the EPDiff equations

$$v_t = \int_\Omega K(\cdot, x) a_t(x) dx \ ,$$
$$\partial_t a_t = -Da_t v_t - a_t \nabla \cdot v_t - (Dv_t)^T a_t$$

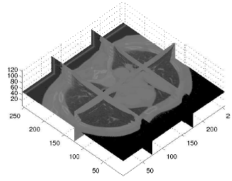
extend to the KB-EPDiff equations (scale: r)

$$w_{r,t} = \int_\Omega K_r(\cdot, x) a_{r,t}(x) dx \ ,$$
$$\partial_t a_{r,t} = \int_{I_W} -Da_{r,t} w_{s,t} - a_{r,t} \nabla \cdot w_{s,t} - (Dw_{s,t})^T a_{r,t} \, ds$$

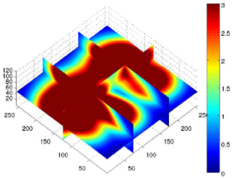
- in a more general form:

$$w_{t,r} = \mathrm{Ad}^{T,r}_{\varphi^{\Psi(w)}_{t0}} w_{0,r} \qquad \text{and} \qquad \partial_t a_{t,r} = -\mathrm{ad}^*_{\Psi(w_t)} a_{t,r}$$

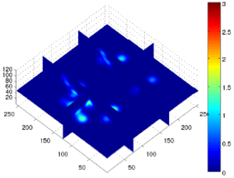
- towards sparsity: initial vector fields of lung registration, LDDMM and LDDKBM



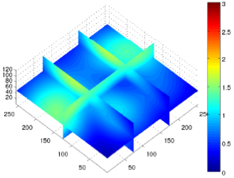
(a) Slices of 3D lung image



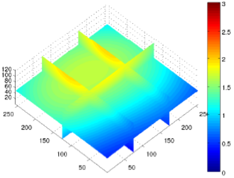
(b) LDDMM $\sigma = 50$



(c) LDDKBM $\sigma = 10$



(d) LDDKBM $\sigma = 90$



(e) LDDKBM $\sigma = 170$

- References:

  - *A Multi-Scale Kernel Bundle for LDDMM: Towards Sparse Deformation Description Across Space and Scales*
    S. Sommer, M. Nielsen, F. Lauze, X. Pennec, IPMI 2011
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    S. Sommer, F. Lauze, M. Nielsen, X. Pennec, SSVM 2011
  - *Accelerating Multi-Scale Flows for LDDKBM Diffeomorphic Registration*
    S. Sommer, GPUCV workshop at ICCV 2011