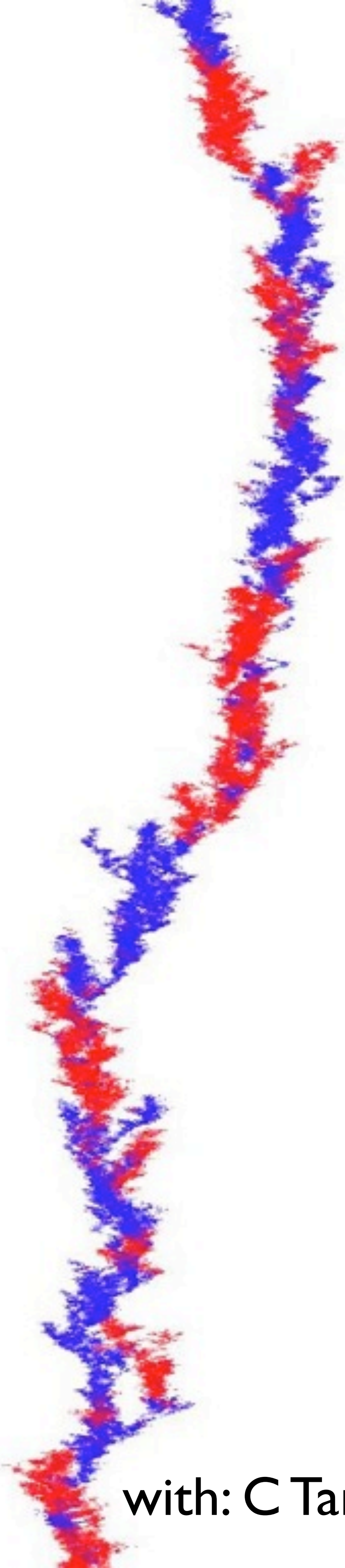


Who laughs last?
perturbation theory of games

Tibor Antal, Program for Evolutionary Dynamics, Harvard

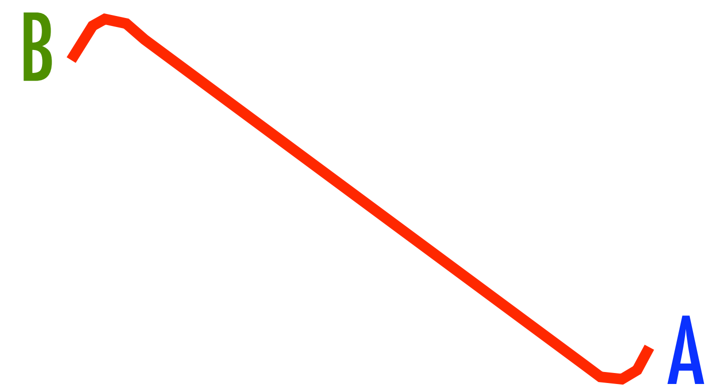
- games in phenotype space
- perturbation method: two key aspects
- further examples, general results

with: C Tarnita, H Ohtsuki, J Wakeley, P Taylor, A Traulsen, F Fu, N Wage, M Nowak

What is the question?

Two strategies: A and B: Which one is better?

John Forbes Nash,
John Maynard Smith
fixation probabilities ...
C Taylor, Nowak



Or: Which outnumbers the other in the long run?

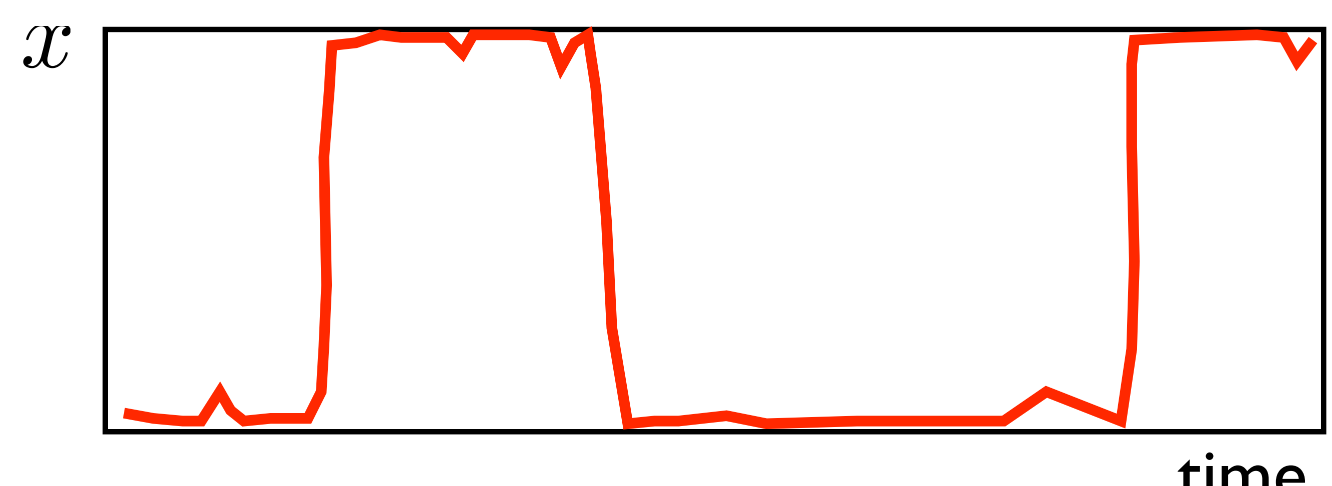
with two way mutation u
(who laughs last?)

$$\langle x \rangle > 1/2$$

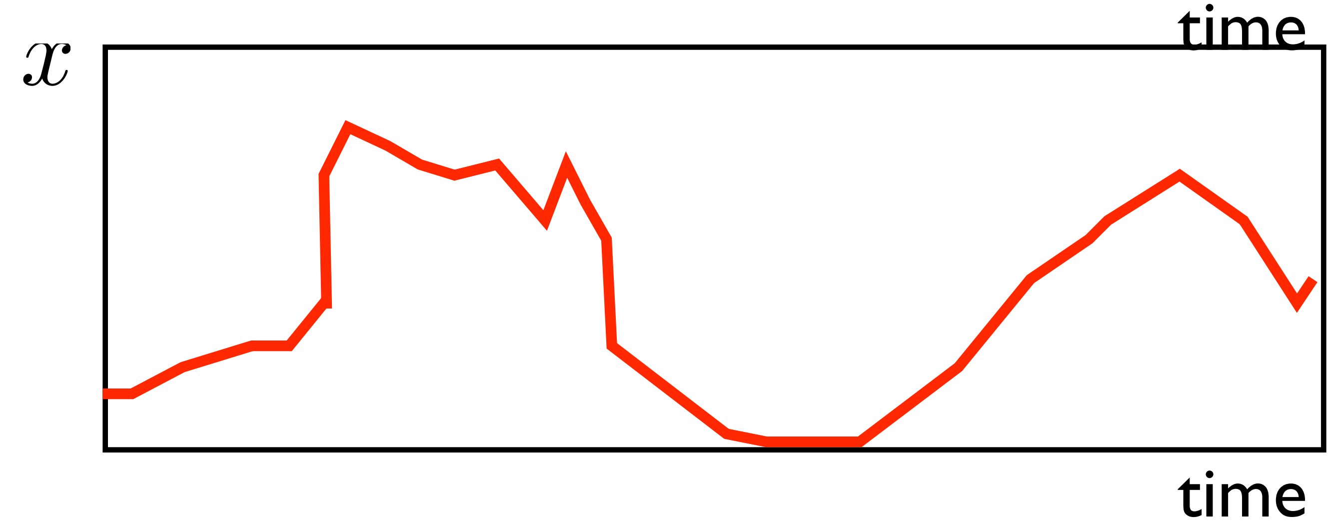
Kandori '93

$u \to 0$

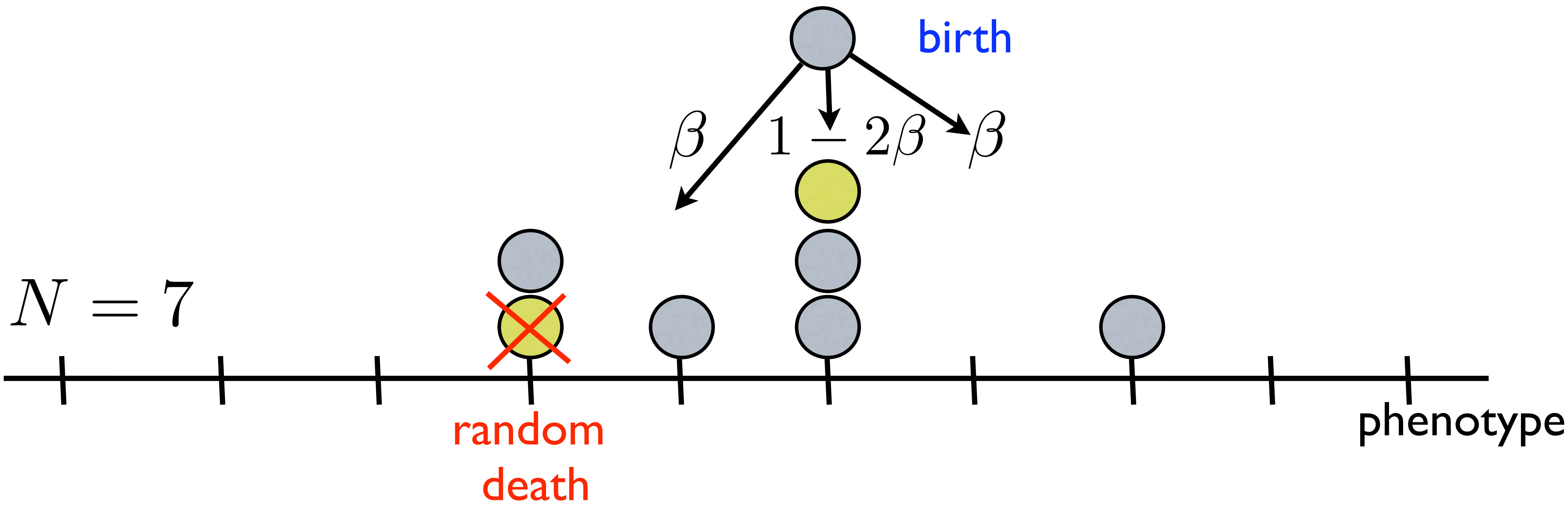
fixation probabilities
$\rho_A > \rho_B$



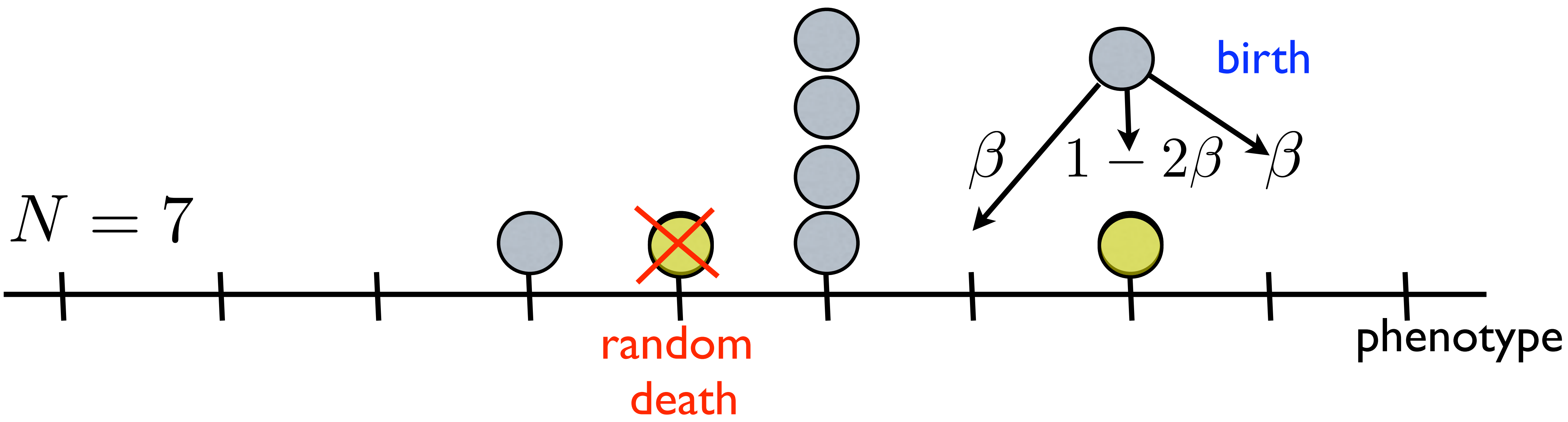
general u



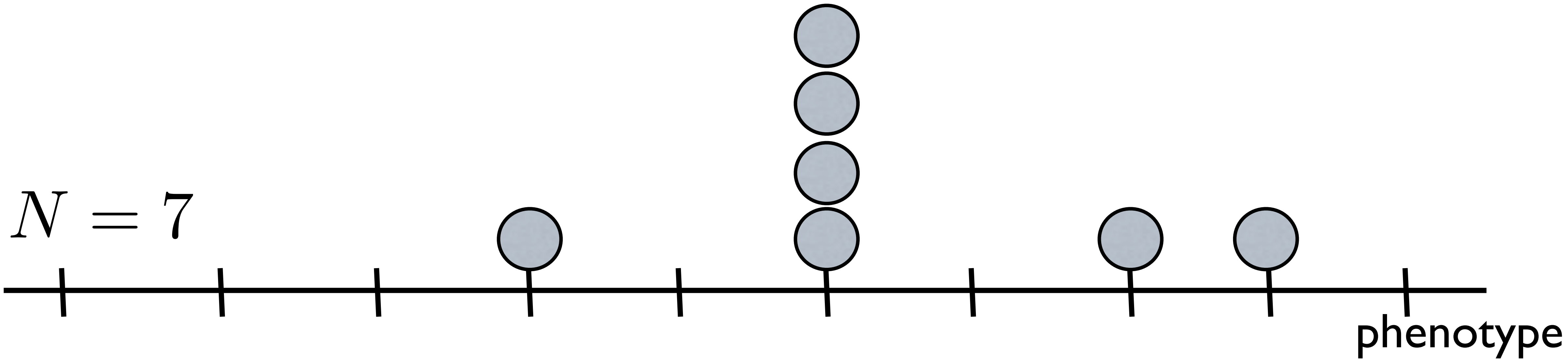
Evolution in phenotype space



Evolution in phenotype space

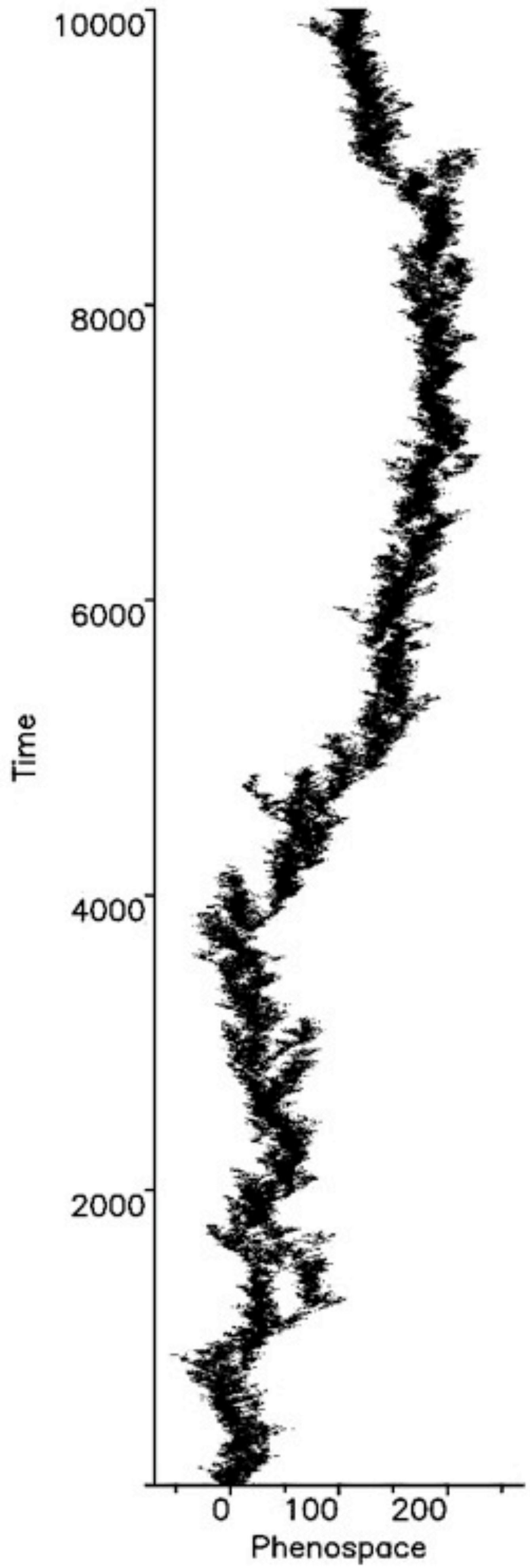


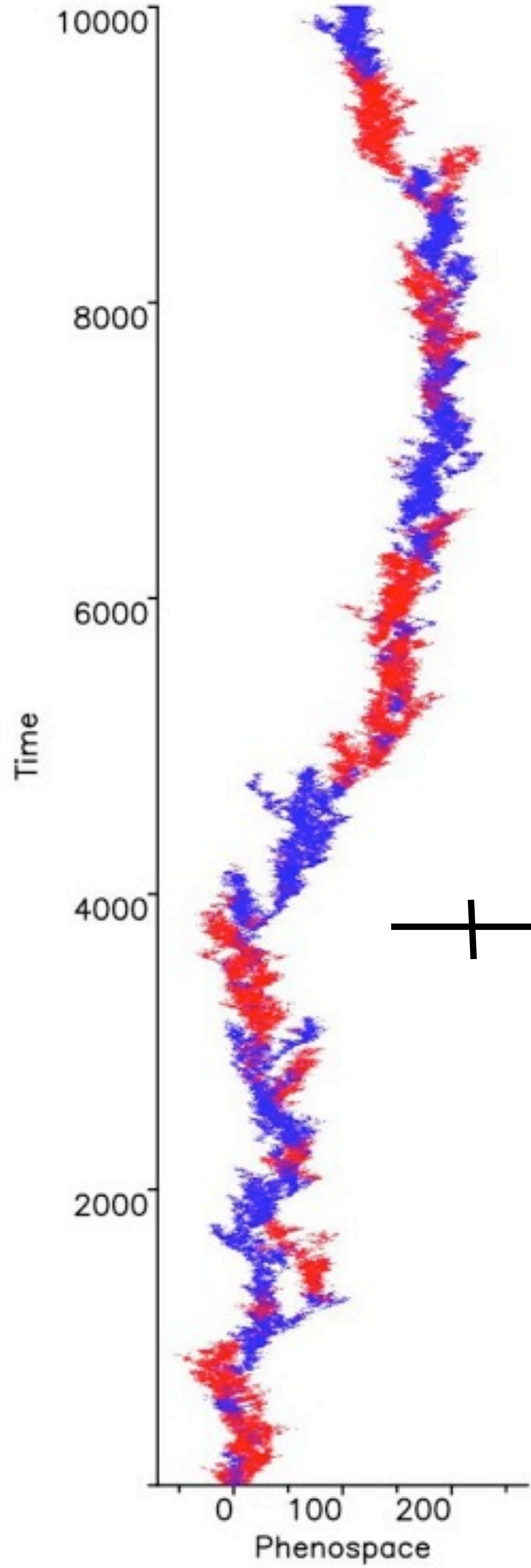
Evolution in phenotype space



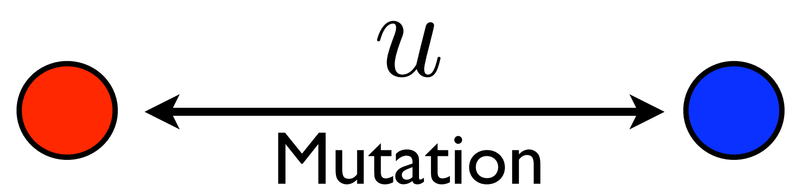
disperse or condense ?

Group of size $\sqrt{N\beta}$ diffuses as $D = N\beta/2$



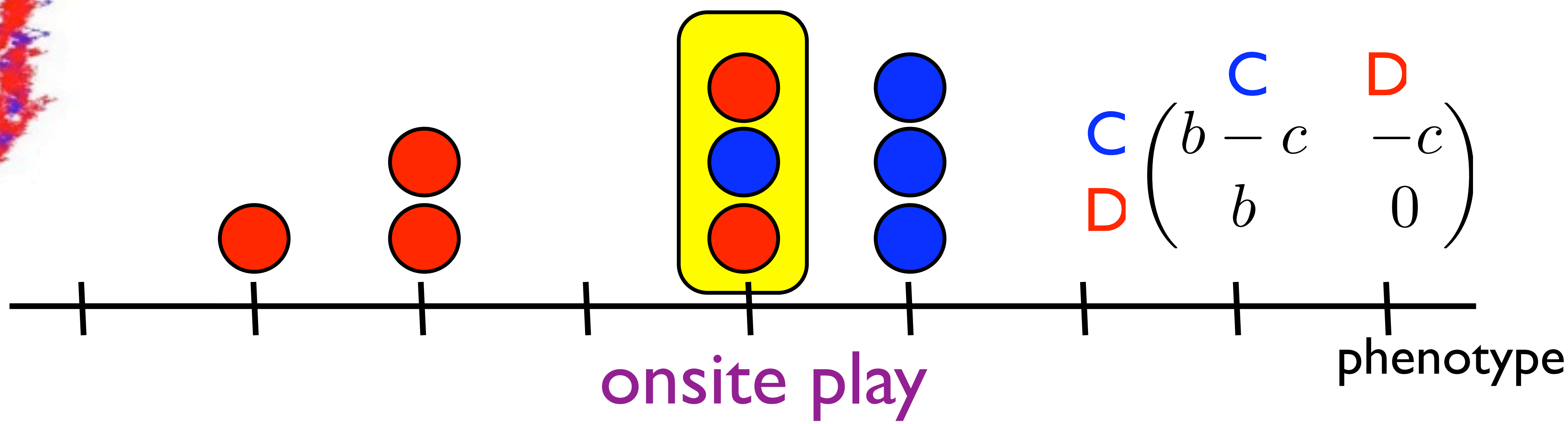


Colors



$$y = \Pr(S_k = S_q)$$
$$z = \Pr(X_k = X_q)$$
$$g = \Pr(S_k = S_q,\ X_k = X_q)$$
$$h = \Pr(S_l = S_k,\ X_k = X_q)$$

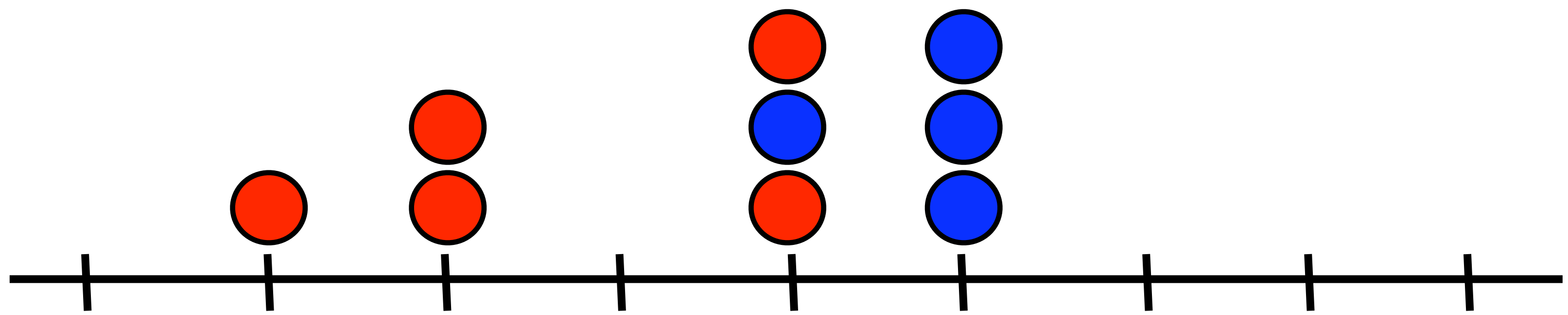
S_k strategy
X_k position

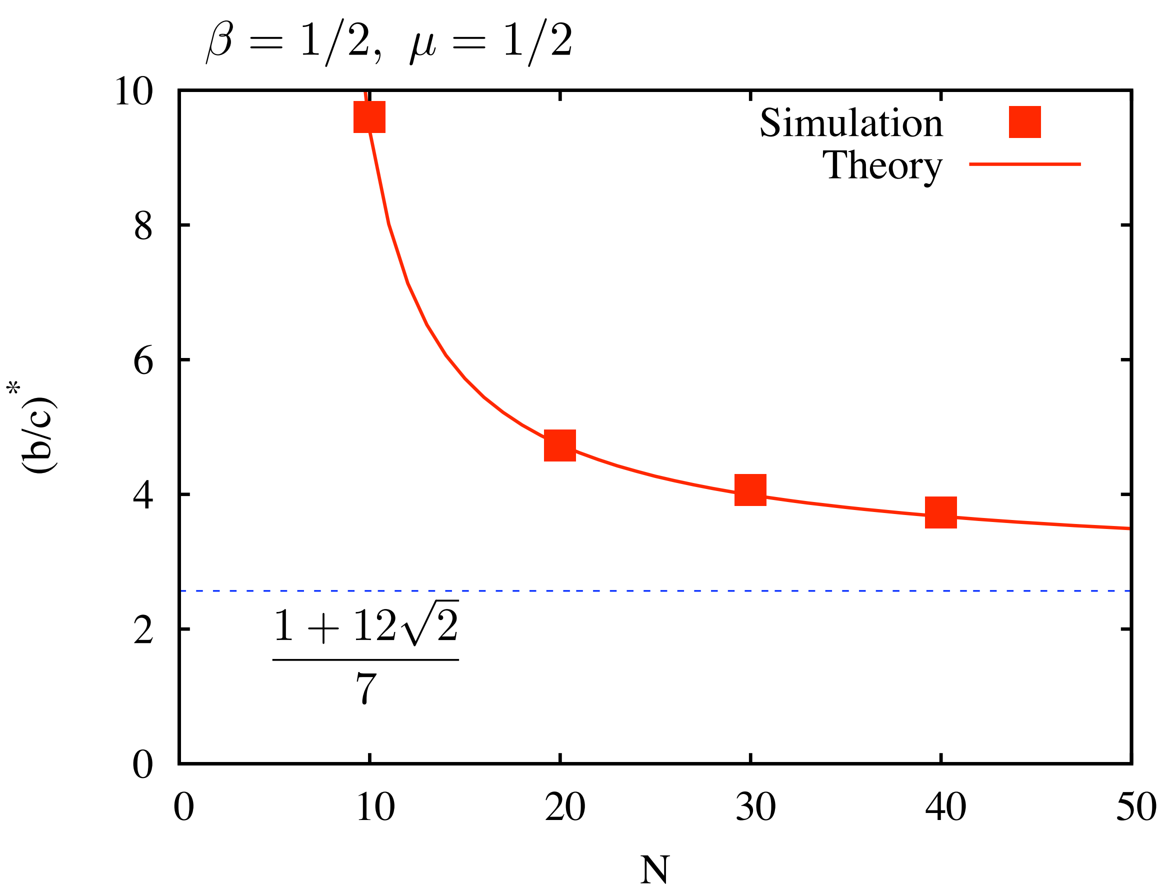


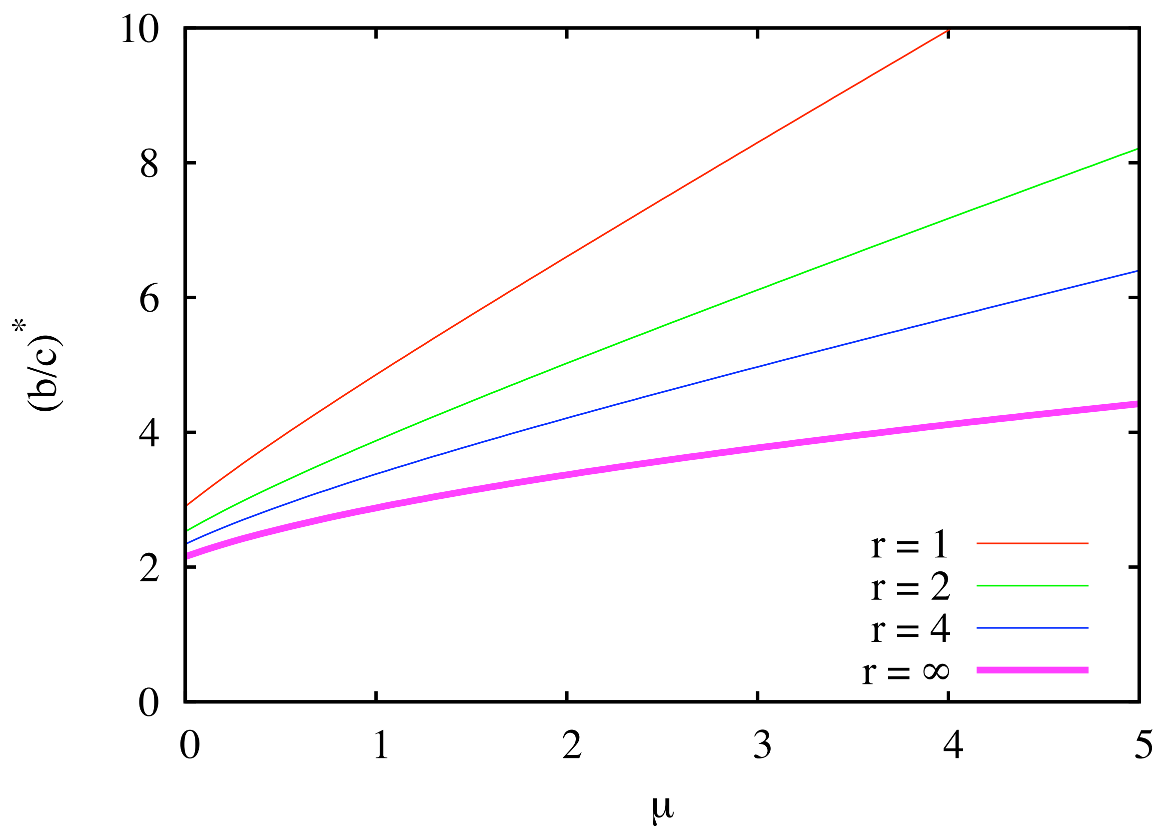
	C	D
C	$b - c$	$-c$
D	b	0

$$\left(\frac{b}{c}\right)^* = \frac{z-h}{g-h}$$

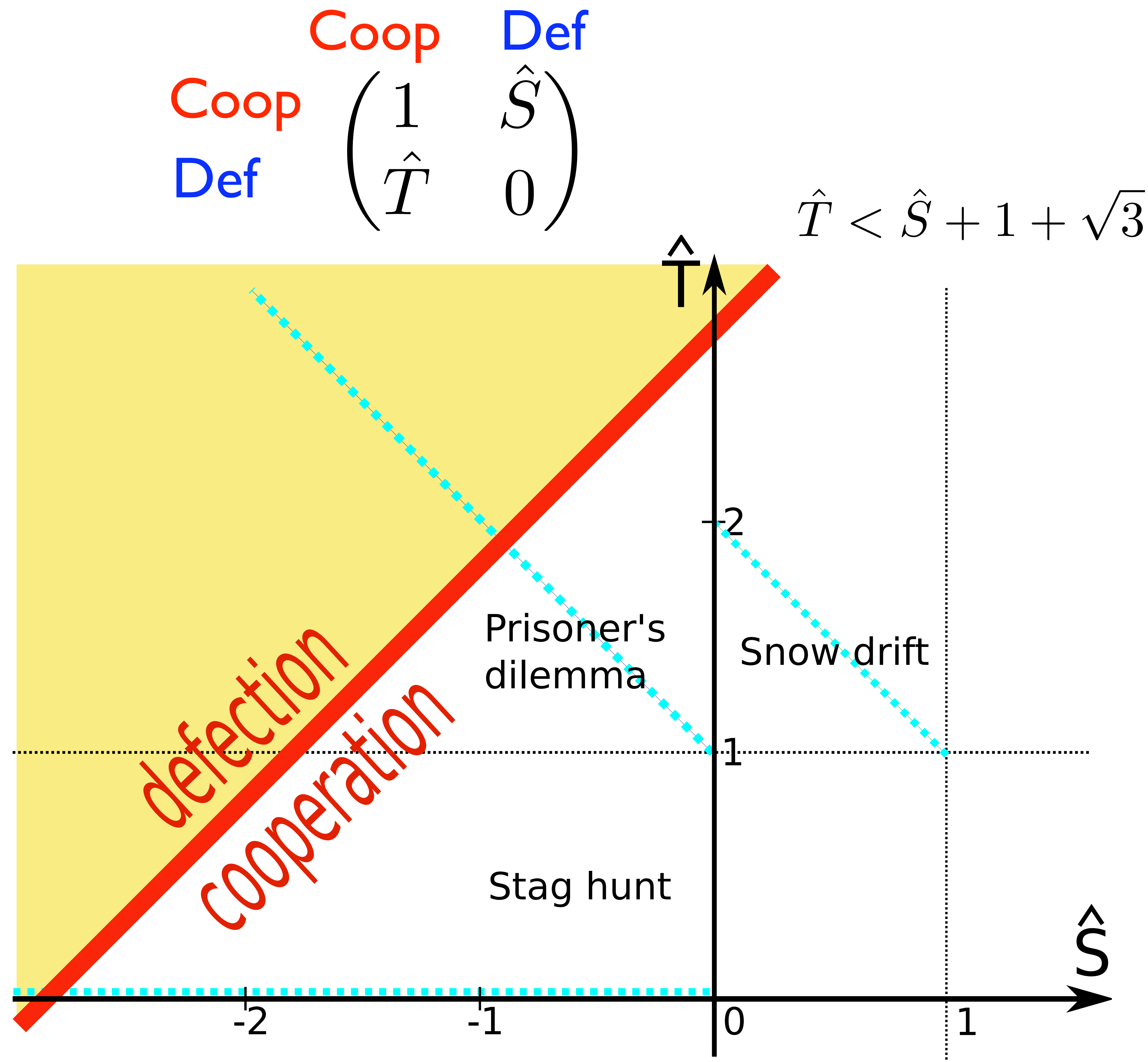
TA '09







$$\mu = 2Nu$$
$$r = 2N\beta$$



Coop
Def
Coop
Def
$$\begin{pmatrix} 1 & \hat{S} \\ \hat{T} & 0 \end{pmatrix}$$
$\hat{T} < \hat{S} + 1 + \sqrt{3}$
T̂
Ŝ
2
1
0
-2
-1
1
Prisoner's dilemma
Snow drift
Stag hunt
defection
cooperation

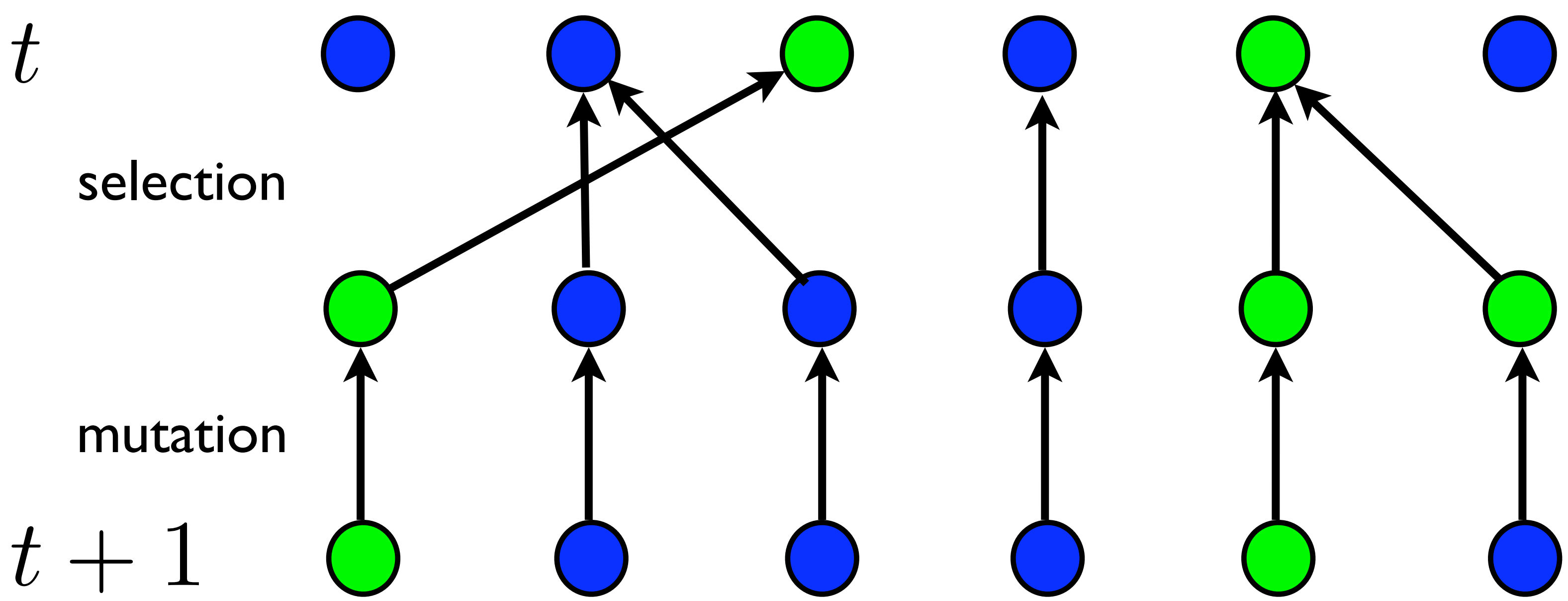
Perturbation method: 2 key points

Payoff = $1 + \delta \times$ payoff of

	when playing against A	B
A	a_{11}	a_{12}
B	a_{21}	a_{22}

x frequency of A
δ selection strength
u mutation probability

Wright-Fisher



Δx^{sel}

Δx^{mut}

$$\Delta x^{\mathrm{tot}} = \Delta x^{\mathrm{sel}} - \frac{u}{2}(x + \Delta x^{\mathrm{sel}}) + \frac{u}{2}(1 - x - \Delta x^{\mathrm{sel}})$$

$$\langle x \rangle = \frac{1}{2} + \frac{1-u}{u}\langle \Delta x^{\mathrm{sel}} \rangle$$

$$\langle x \rangle > \frac{1}{2} \iff \langle \Delta x^{\mathrm{sel}} \rangle > 0$$

Perturbation method: 2 key points

Payoff = $1 + \delta \times$ payoff of

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x frequency of A

δ selection strength

u mutation probability

$$\langle x \rangle > \frac{1}{2} \Longleftrightarrow \langle \Delta x^{\mathrm{sel}} \rangle > 0$$

Easy perturbation method for small δ

$$\langle \Delta x \rangle = \sum \Delta x_i \, \pi_i$$

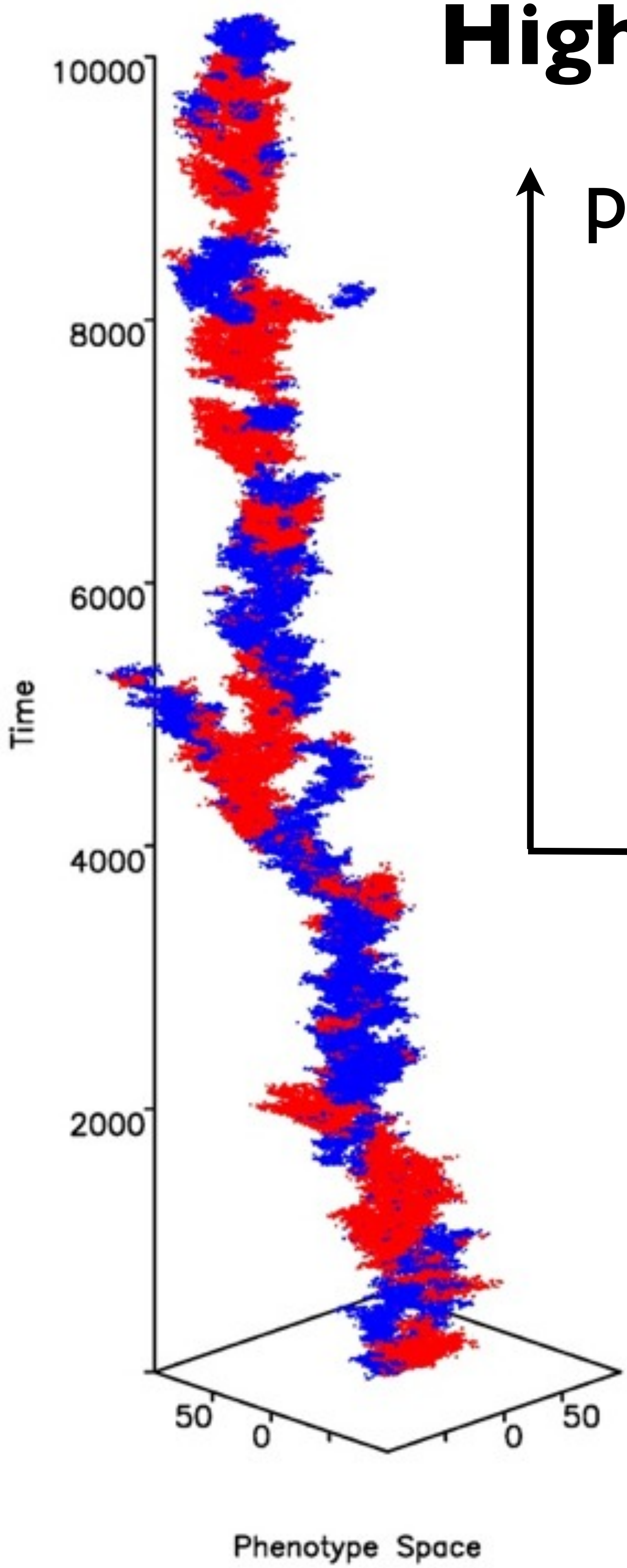
$$\Delta x_i = 0 + \delta \Delta x_i^{(1)}$$

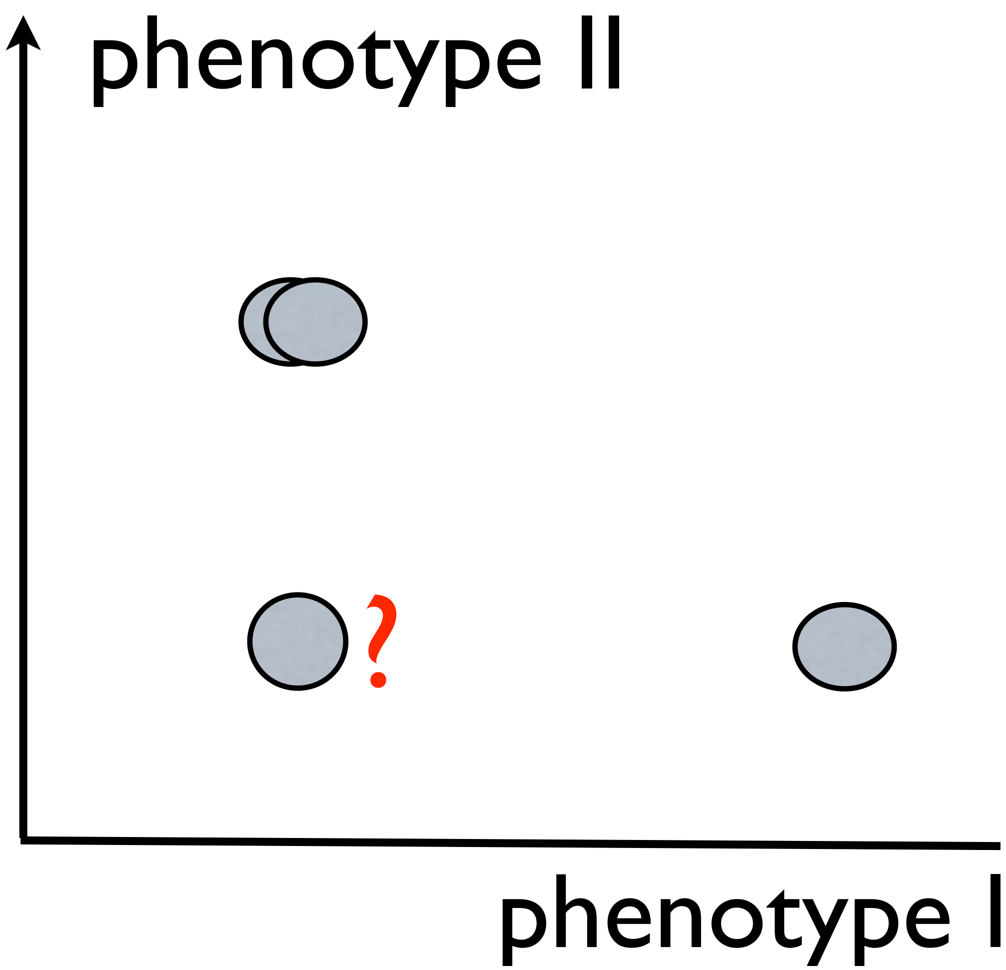
$$\pi_i = \pi_i^{(0)} + \delta \pi_i^{(1)}$$

$$\langle \Delta x \rangle = \delta \sum \Delta x_i^{(1)} \, \pi_i^{(0)} + \mathcal{O}(\delta^2)$$

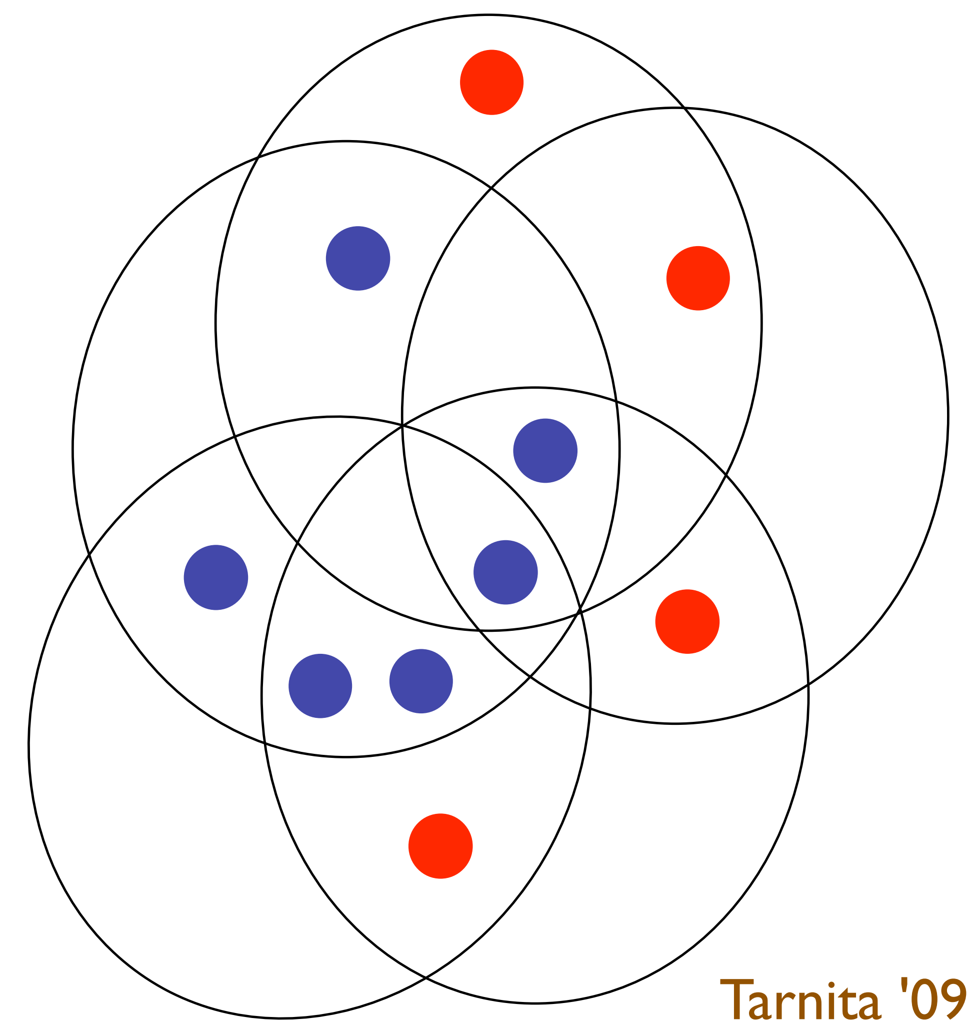
neutral probabilities only !

Higher dimensions

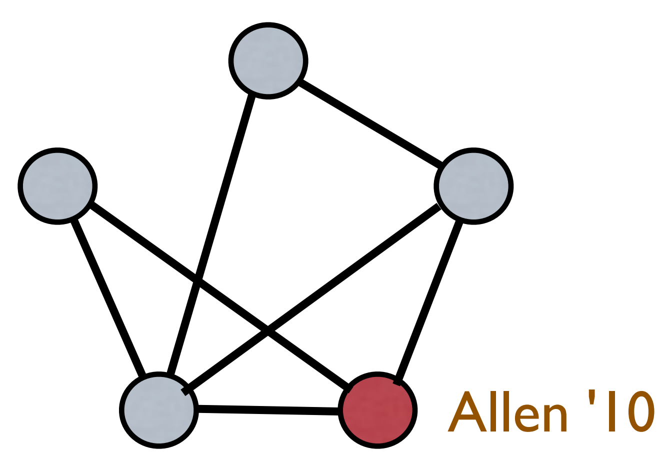




Sets



Graphs



One parameter to rule them all

Tarnita '09

A wins iff $\sigma a + b > c + \sigma d$

when playing against

$$\text{payoff of} \quad \begin{matrix} & A & B \\ A & a & b \\ B & c & d \end{matrix}$$

single parameter for all structures

classical well mixed $\sigma = 1$

$a + b > c + d$ (risk dominance) or $\sigma = 1 - 2/N$

phenotype game $\sigma = 1 + \sqrt{3}$

more strategies on structure?

Wage, Tarnita '10

# strategies	# parameters
2	1
≥ 3	2

Relations to relatedness

A wins iff $\frac{b}{c} > \frac{1}{R}$ (Hamilton's rule)

same size islands

$$R = \frac{\Pr(S_k = S_q | X_k = X_q) - \Pr(S_k = S_q)}{1 - \Pr(S_k = S_q)}$$

fluctuating size islands,
phenotype walk

$$\left(\frac{b}{c}\right)^* = \frac{z-h}{g-h} \qquad R = \frac{\Pr(S_k = S_q | \, X_k = X_q) - \Pr(S_l = S_k | \, X_k = X_q)}{1 - \Pr(S_l = S_k | \, X_k = X_q)}$$

TA '09, Taylor '10

more general structures

is there always a relatedness interpretation
of the general formulas?

Final slide

general method to study weak selection

TA, Ohtsuki, Wakeley, Taylor, Nowak, PNAS '09

some papers can be found on my website

thanks