Faces of the Barvinok-Novik orbitope

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The odd trigonometric moment curve

$$SM_{2k}(\theta) = (\cos(\theta), \sin(\theta), \cos(3\theta), \sin(3\theta), \ldots, \cos((2k-1)\theta), \sin((2k-1)\theta)),$$

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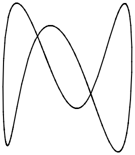

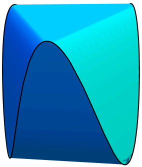

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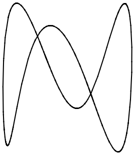

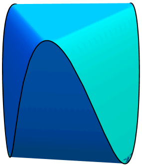

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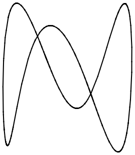

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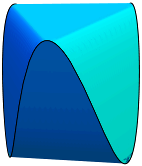

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  $\rightarrow$ good for making polytopes with many faces

Introduction

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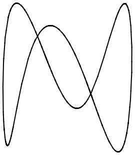
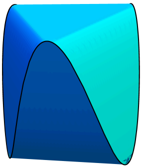


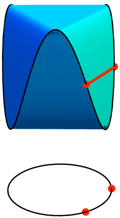


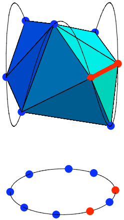
Why?

- B_{2k} centrally symmetric and has many faces
  $\longrightarrow$ good for making polytopes with many faces
- An interesting convex body in its own right
  (orbitope, projection of a spectrahedron)

Motvation: centrally symmetric polytopes with many faces

Idea: If $SM_{2k}(\theta_1), \ldots, SM_{2k}(\theta_j)$ form a face on B_{2k} then they form a face on $\text{conv}\{SM_{2k}(\theta_1), \ldots, SM_{2k}(\theta_j), SM_{2k}(\theta_{j+1}) \ldots, SM_{2k}(\theta_r)\}$.





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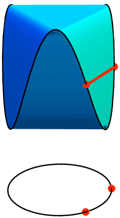

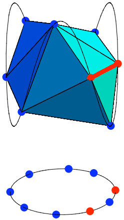

Theorem (Barvinok, Novik 2008)

For $d = 2k$ fixed, $j \leq k-1$ and $n \to \infty$, there is $c_j(d) \in \mathbb{R}_+$ with

$$c_j(d) + o(1) \leq \frac{fmax(d, n; j)}{\binom{n}{j+1}} \leq 1 - \frac{1}{2^d} + o(1),$$

where $fmax(d, n; j)$ is the maximum number of $j - faces$ on a centrally symmetric polytope with dimension d and n vertices.

Motivation: an interesting convex body

Sanyal, Sottile, & Sturmfels (2009) remark that convex hull of the full trigonometric moment curve,

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$$B_4 = \left\{ (x_1, y_1, x_3, y_3) \ : \ \exists\, x_2, y_2 \text{ with } \begin{bmatrix} 1 & z_1 & z_2 & z_3 \\ \overline{z_1} & 1 & z_1 & z_2 \\ \overline{z_2} & \overline{z_1} & 1 & z_1 \\ \overline{z_3} & \overline{z_2} & \overline{z_1} & 1 \end{bmatrix} \succeq 0 \right\}$$

where $z_j = x_j + iy_j$.

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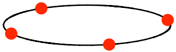

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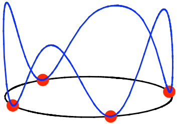

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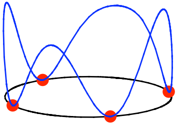

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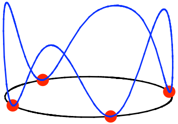

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- Warm up: B_4
- Main theorem: Edges of B_{2k}
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Main Theorem

For $\alpha, \beta \in [0, 2\pi]$, the line segment $[SM_{2k}(\alpha), SM_{2k}(\beta)]$ is

an exposed edge	if $\lvert\alpha - \beta\rvert < 2\pi(k-1)/(2k-1)$
a non-exposed edge	if $\lvert\alpha - \beta\rvert = 2\pi(k-1)/(2k-1)$
not an edge	if $\lvert\alpha - \beta\rvert > 2\pi(k-1)/(2k-1)$.

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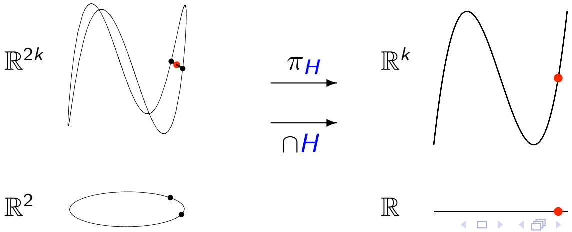
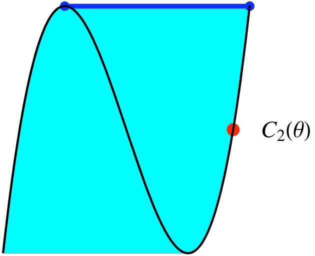
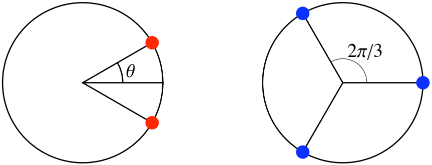


Warm up : C_2 and B_4



The faces of $\mathrm{conv}(C_2)$ tell us the *balanced* faces of B_4.



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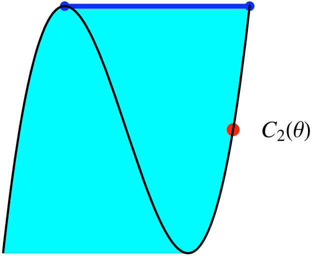

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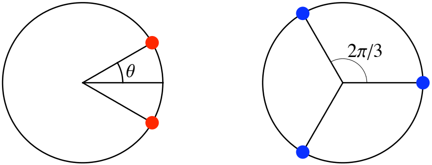

As shown by Barvinok & Novik (2008), the exposed faces of B_4 are

dim	conv($\cdot$)		
0	$SM_4(\alpha)$	:	$\alpha \in [0, 2\pi]$
1	$SM_4(\alpha),\ SM_4(\beta)$	:	$\lvert\alpha - \beta\rvert < 2\pi/3$
2	$SM_4(\alpha),\ SM_4(\alpha + 2\pi/3),\ SM_4(\alpha + 4\pi/3)$	:	$\alpha \in [0, 2\pi]$

Edges of B_{2k}

Theorem (Barvinok, Novik, 2008) There exists $\phi_k \geq \frac{2k-2}{2k-1}\pi$ so that for $\alpha, \beta \in [0, 2\pi]$, the line segment $[SM_{2k}(\alpha), SM_{2k}(\beta)]$ is

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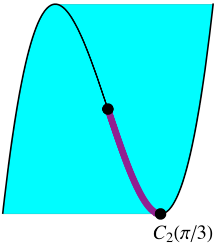

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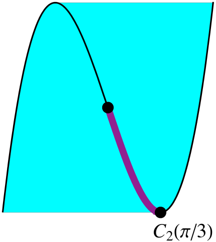
Corollary $\phi_k = \frac{2k-2}{2k-1}\pi$.



Curves dipping behind facets

$C(t) = (f_1(t), \ldots, f_d(t))$, a polynomial curve
$F = \text{conv}\{C(t_0), \ldots, C(t_r)\}$, a facet of $\text{conv}(C)$

Claim: If C is smooth at t_0 and $C(t_0) + \epsilon C'(t_0)$ is in the relative interior of F then $C(t_0 + \epsilon)$ is in the interior of $\text{conv}(C)$.

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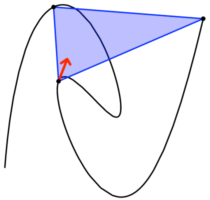

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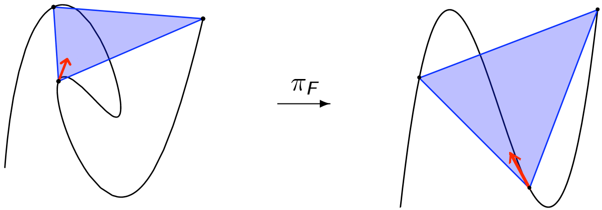

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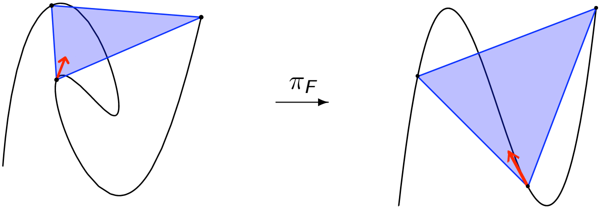

$\pi_F(C(t_0 + \epsilon)) \in$ interior$(\pi_F F)$

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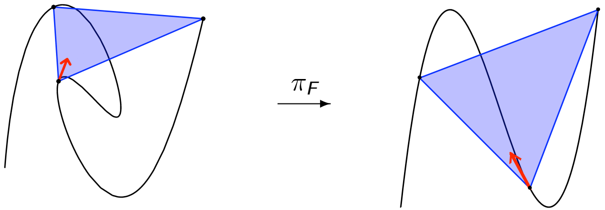
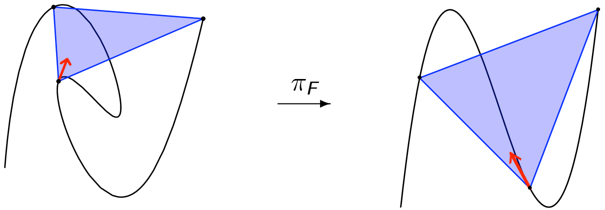


$$\pi_F(C(t_0 + \epsilon)) \in \text{ interior}(\pi_F F) \Rightarrow C(t_0) + \epsilon C'(t_0) \in \text{ rel.interior}(F)$$

Curves dipping behind facets

$C(t) = (f_1(t), \ldots, f_d(t))$, a polynomial curve
$F = \text{conv}\{C(t_0), \ldots, C(t_r)\}$, a facet of conv$(C)$

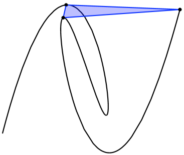
Claim: If C is smooth at t_0 and $C(t_0) + \epsilon C'(t_0)$ is in the relative interior of F then $C(t_0 + \epsilon)$ is in the interior of conv(C).



$$\pi_F(C(t_0+\epsilon)) \in \text{ interior}(\pi_F F) \Rightarrow C(t_0) + \epsilon C'(t_0) \in \text{ rel.interior}(F)$$
$$\Rightarrow C(t_0+\epsilon) \in \text{ interior}(\text{conv}(C))$$

Trigonometry is useful.

Now use $C = C_5$, and
$F = \text{conv}\{C_3(0\pi), C_3(\frac{2\pi}{5}), C_3(\frac{4\pi}{5})\}$.



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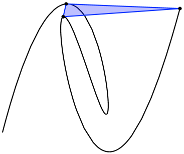
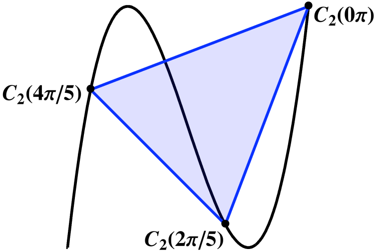
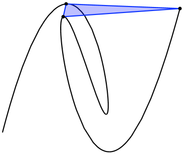




Need to show $C_2(\frac{2\pi}{5} + \epsilon) \in \pi_F(F)$.

Trigonometry is useful.



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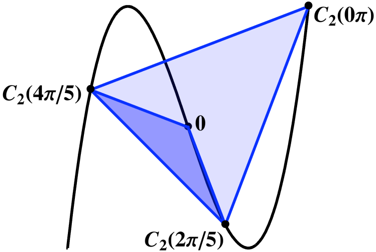

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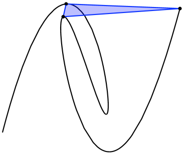

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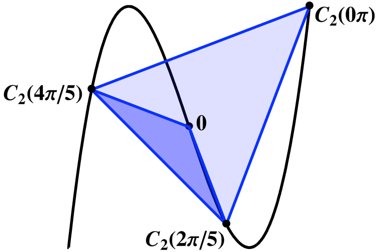
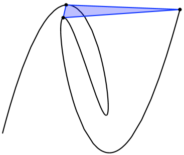


Need to show $C_2(\frac{2\pi}{5} + \epsilon) \in \pi_F(F)$.

Use trigonometry to explicitly find functions giving facets of $\pi_F(F)$

Trigonometry is useful.



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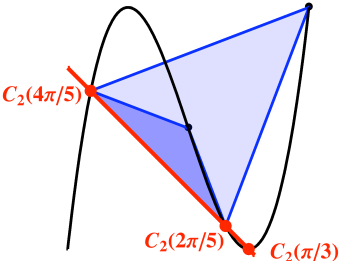

$x_1 + x_2 = -\frac{1}{2}$

Need to show $C_2(\frac{2\pi}{5} + \epsilon) \in \pi_F(F)$.

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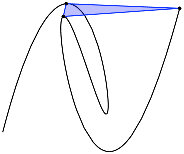
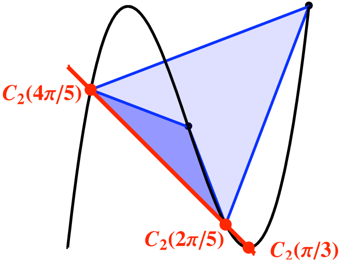




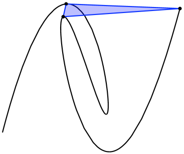
$x_1 + x_2 = -\frac{1}{2}$

Need to show $C_2(\frac{2\pi}{5} + \epsilon) \in \pi_F(F)$.

Use trigonometry to explicitly find functions giving facets of $\pi_F(F)$

and their roots and signs.

Trigonometry is useful.



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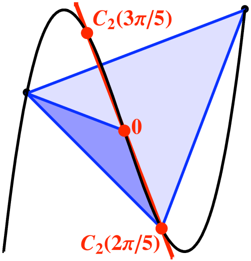

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Use trigonometry to explicitly find functions giving facets of $\pi_F(F)$

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$\cos(4\pi/5)x_1 + \cos(2\pi/5)x_2 = x_1 + x_2$

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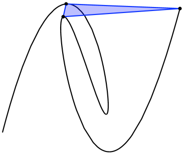

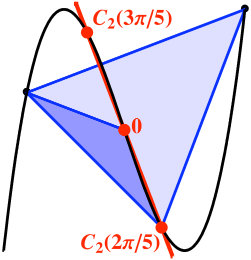

Need to show $C_2(\frac{2\pi}{5} + \epsilon) \in \pi_F(F)$.

Use trigonometry to explicitly find functions giving facets of $\pi_F(F)$

and their roots and signs.

$\rightarrow$ all positive on $C_2(\frac{2\pi}{5} + \epsilon)$.

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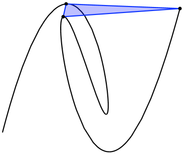

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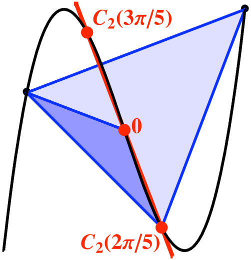

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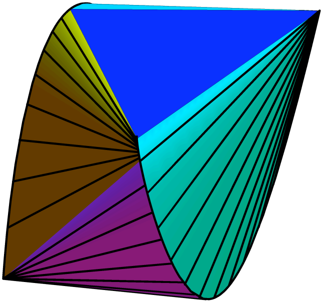
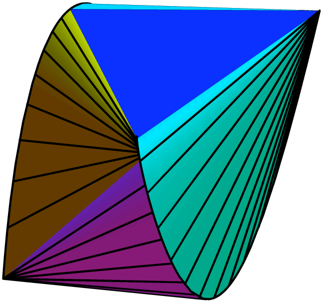
$\cos(4\pi/5)x_1 + \cos(2\pi/5)x_2 = x_1 + x_2$ □

Finale: C_3 and B_6



The Zariski-closure of $\partial\,\mathrm{conv}(C_3)$ has 5 components: 2 tritangent planes and 3 edge surface components of degrees 4,4, and 7.

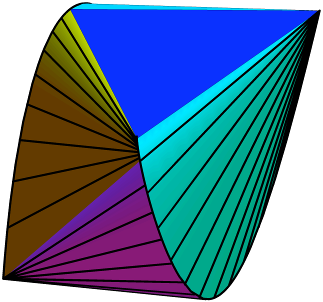
Finale: C_3 and B_6



The Zariski-closure of $\partial\,\mathrm{conv}(C_3)$ has 5 components: 2 tritangent planes and 3 edge surface components of degrees 4,4, and 7.

The balanced faces of B_6:

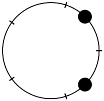
Finale: C_3 and B_6



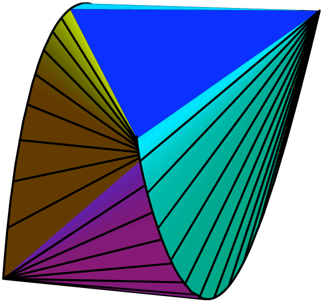
The Zariski-closure of $\partial\,\mathrm{conv}(C_3)$ has 5 components: 2 tritangent planes and 3 edge surface components of degrees 4,4, and 7.

The balanced faces of B_6:

exposed points of C_3 $\leftrightarrow$ edges of B_6



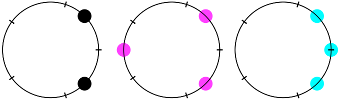
Finale: C_3 and B_6



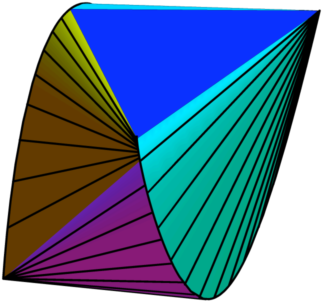
The Zariski-closure of $\partial\,\mathrm{conv}(C_3)$ has 5 components: 2 tritangent planes and 3 edge surface components of degrees 4,4, and 7.

The balanced faces of B_6:

exposed points of C_3 $\leftrightarrow$ edges of B_6
some edges $\leftrightarrow$ 2-dim. faces on B_6



Finale: C_3 and B_6



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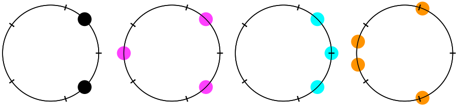
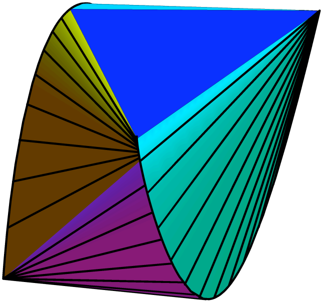
The balanced faces of B_6:

exposed points of C_3	$\leftrightarrow$	edges of B_6
some edges	$\leftrightarrow$	2-dim. faces on B_6
some edges	$\leftrightarrow$	3-dim. faces on B_6



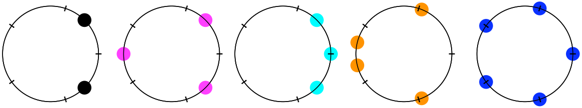
Finale: C_3 and B_6



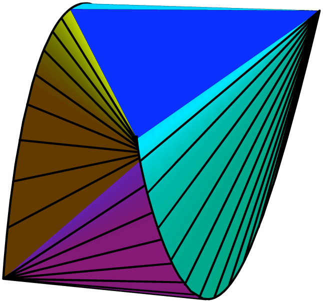
The Zariski-closure of $\partial\,\mathrm{conv}(C_3)$ has 5 components: 2 tritangent planes and 3 edge surface components of degrees 4,4, and 7.

The balanced faces of B_6:

exposed points of C_3 $\leftrightarrow$ edges of B_6
some edges $\leftrightarrow$ 2-dim. faces on B_6
some edges $\leftrightarrow$ 3-dim. faces on B_6
triangent planes $\leftrightarrow$ 4-dim. faces on B_6



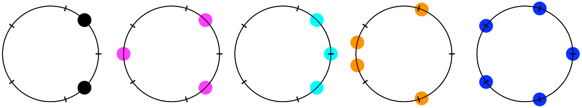
Finale: C_3 and B_6



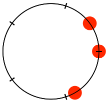
The Zariski-closure of $\partial\, \mathrm{conv}(C_3)$ has 5 components: 2 tritangent planes and 3 edge surface components of degrees 4,4, and 7.

The balanced faces of B_6:

exposed points of C_3 $\leftrightarrow$ edges of B_6
some edges $\leftrightarrow$ 2-dim. faces on B_6
some edges $\leftrightarrow$ 3-dim. faces on B_6
triangent planes $\leftrightarrow$ 4-dim. faces on B_6



no

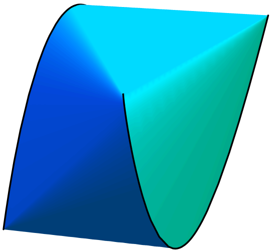


References

1. A. Barvinok, I. Novik, *A centrally symmetric version of the cyclic polytope*. Discrete Comput. Geom. 39 (2008), no. 1-3, 76–99.

2. R. Sanyal, F. Sottile, B. Sturmfels, *Orbitopes.* http://arxiv.org/pdf/0911.5436.pdf

3. C. Vinzant, *Edges of the Barvinok-Novik orbitope.* (in progress)



References

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