A Stochastic Multicloud Model for Tropical Convection

Boualem Khouider

University of Victoria

Joint work with:

Joseph Biello (UC Davis) and Andrew J. Majda (Courant Institute)

Multiscale Processes in the Tropics, BIRS, 2009

Background papers

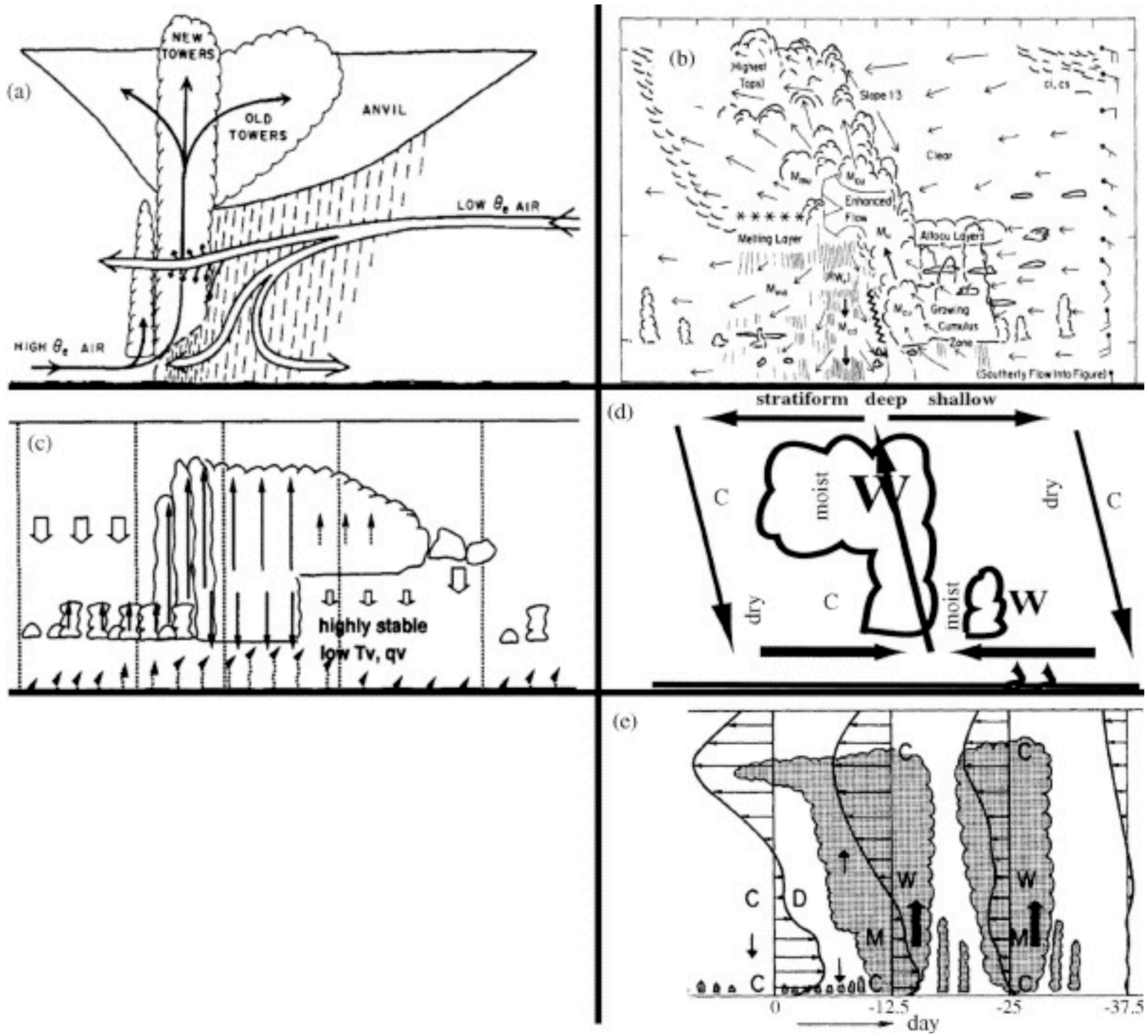
- Stochastic models for CIN:
  - ★Majda, Franzke, & Khouider, (2008), Phil. Trans. Roy. Soc.
  - ★Khouider, Majda, & Katsoulakis (2003), PNAS:
  - ★Majda & Khouider (2002), PNAS:
- Deterministic Multicloud Models (Khouider and Majda J. Atmos. Sci., 2006, 2007, 2008a,2008b)
- Stochastic Multicloud models (with Biello and Majda): Submitted to Comm. Math. Sci.

Self-similar structure of organized convection based on three cloud types: congestus, deep, and stratiform

Squall lines

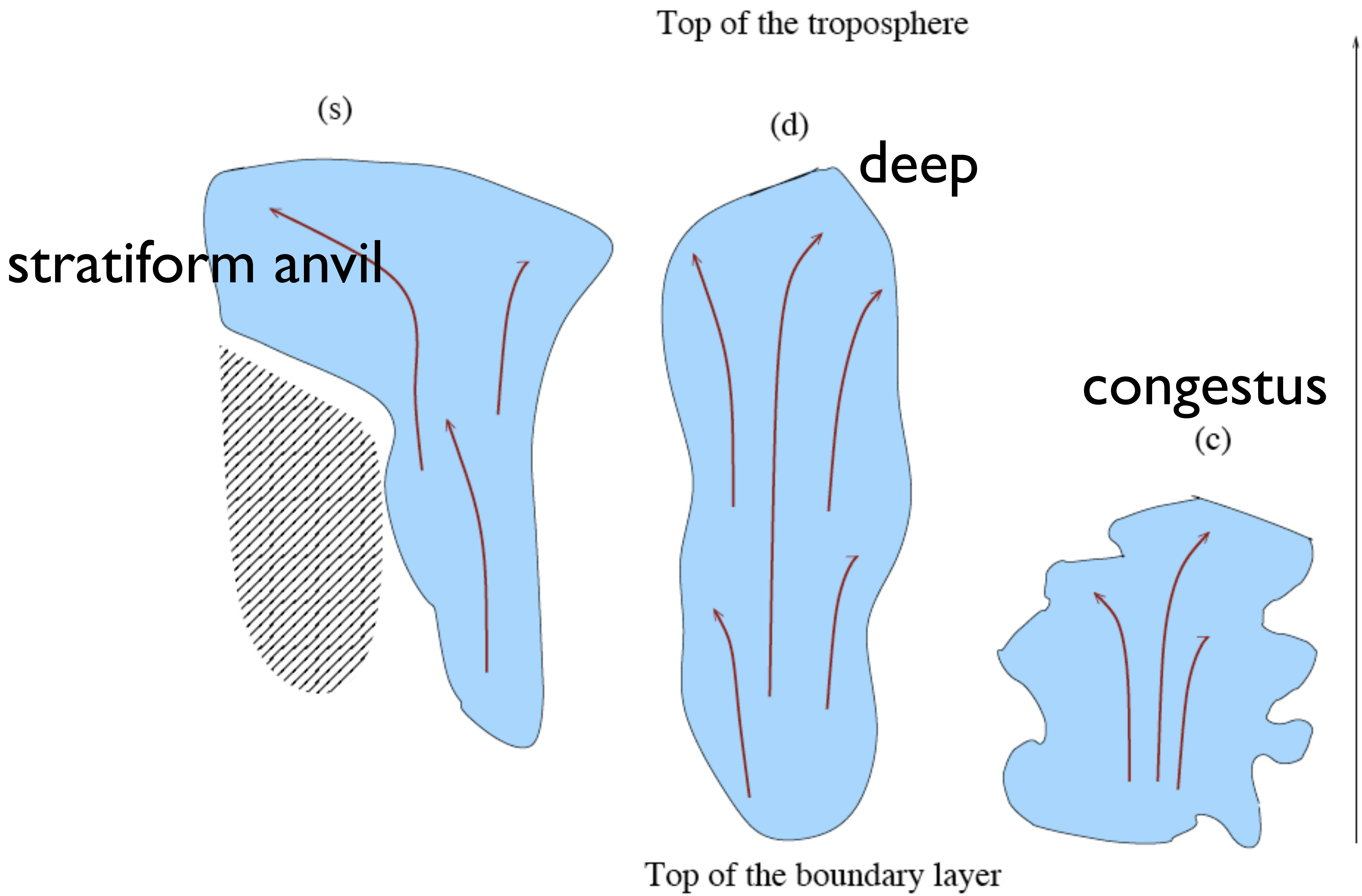
C.C.W.

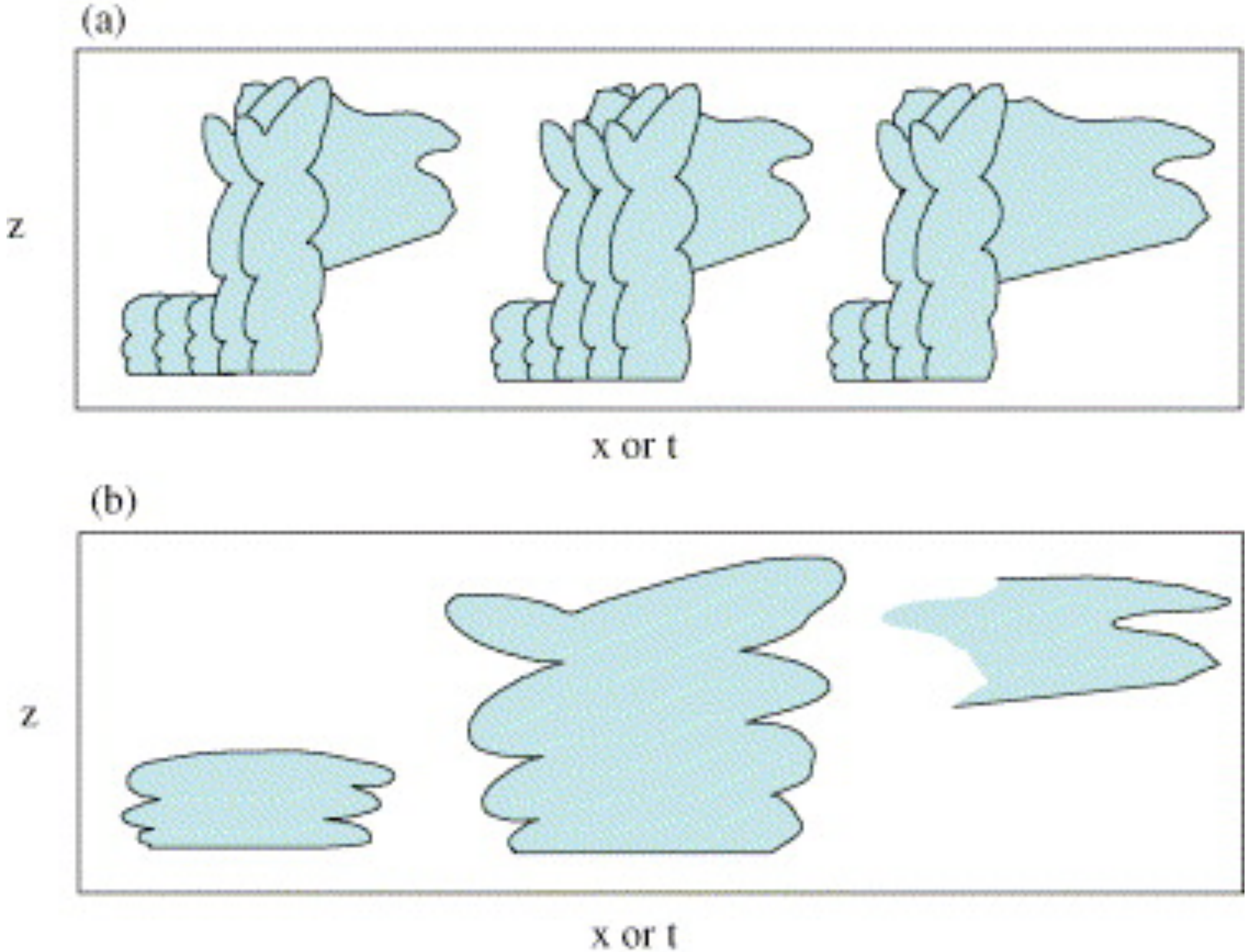
M.J.O.



Compiled by Mapes et al. DAO. 2006

Schematic of three cloud types

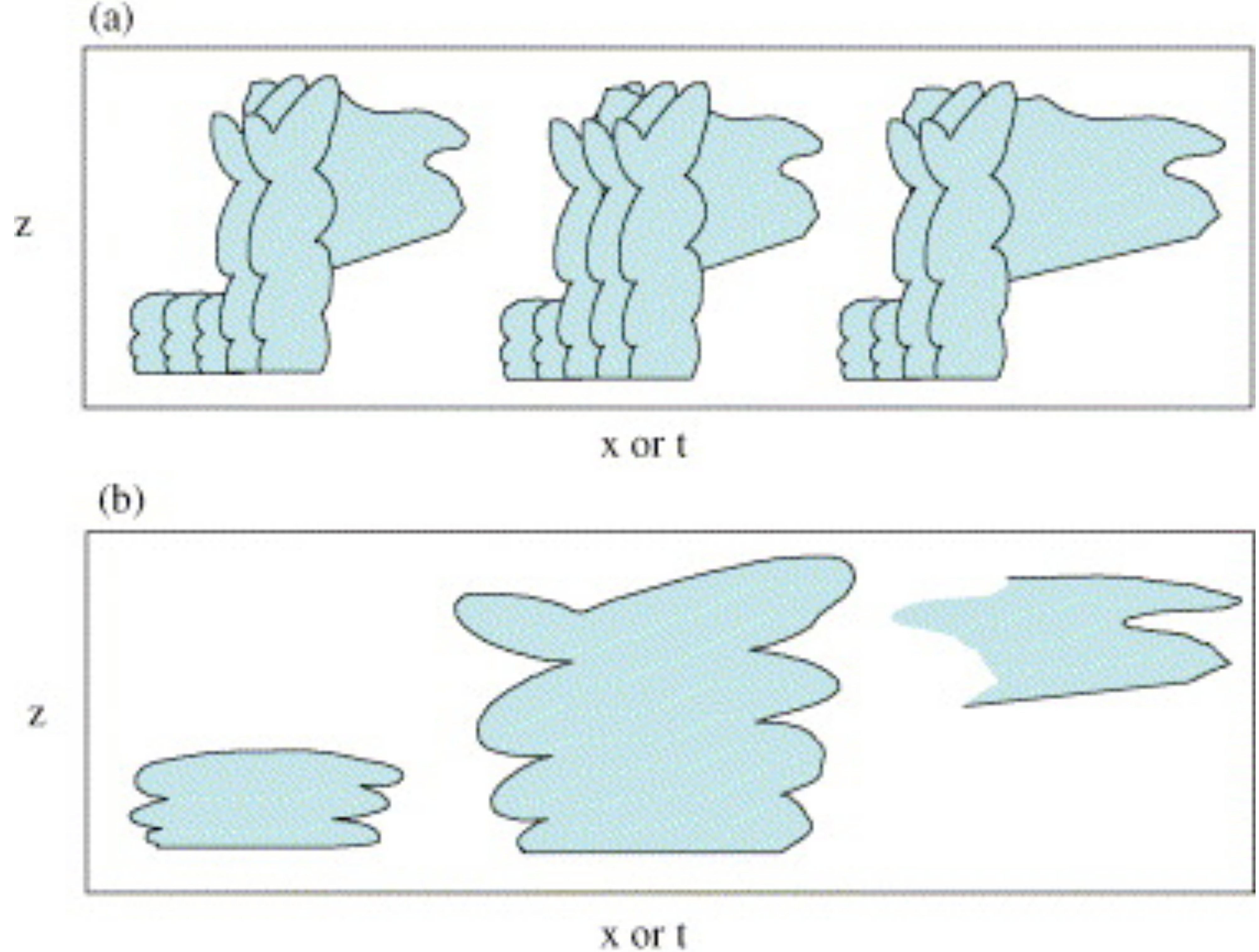




(b) = statistical average of (a)

Mapes et al. DAO 2006

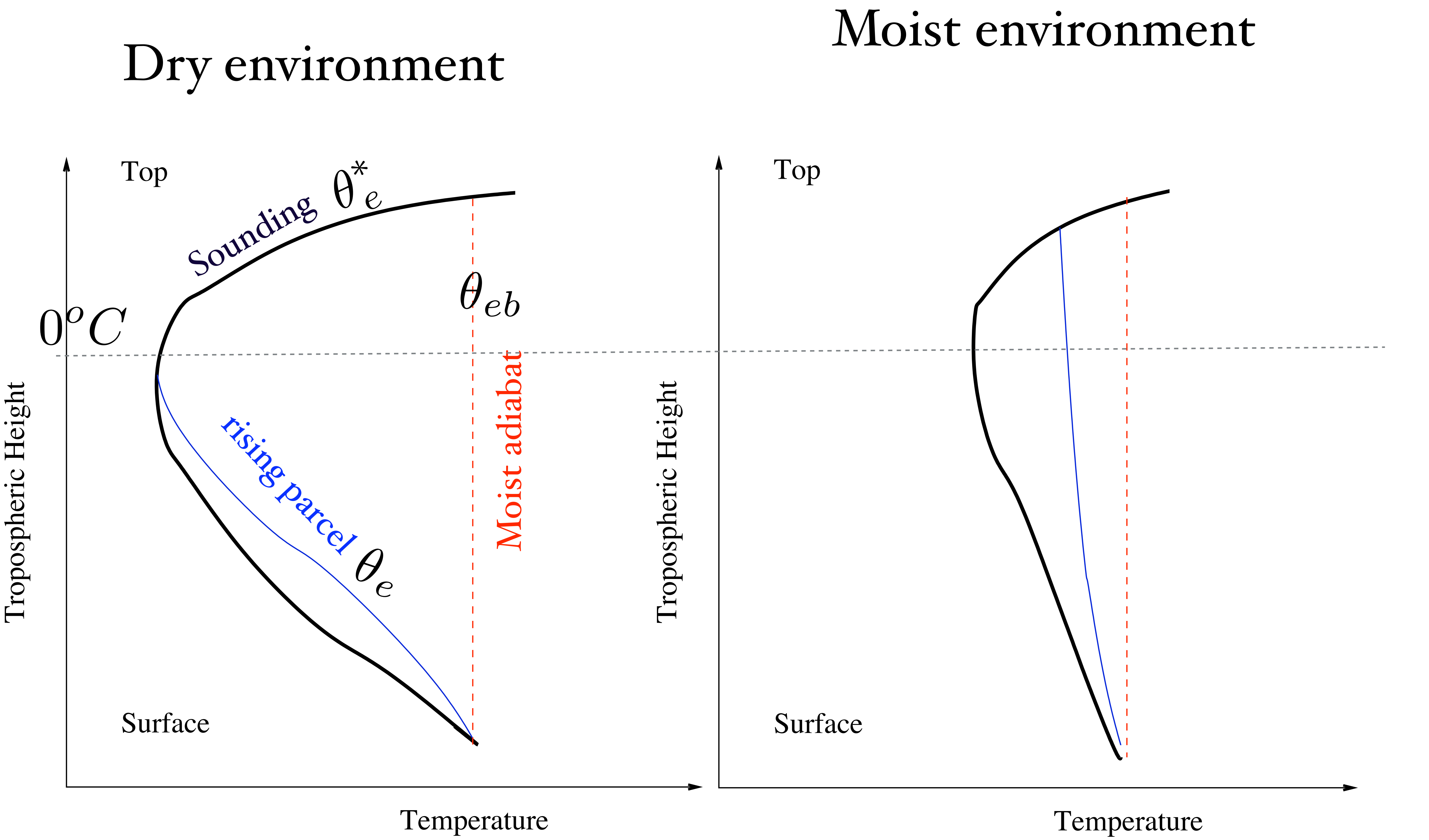
(Stretched) Building Block Hypothesis



(b) = statistical average of (a)

Mapes et al. DAO 2006

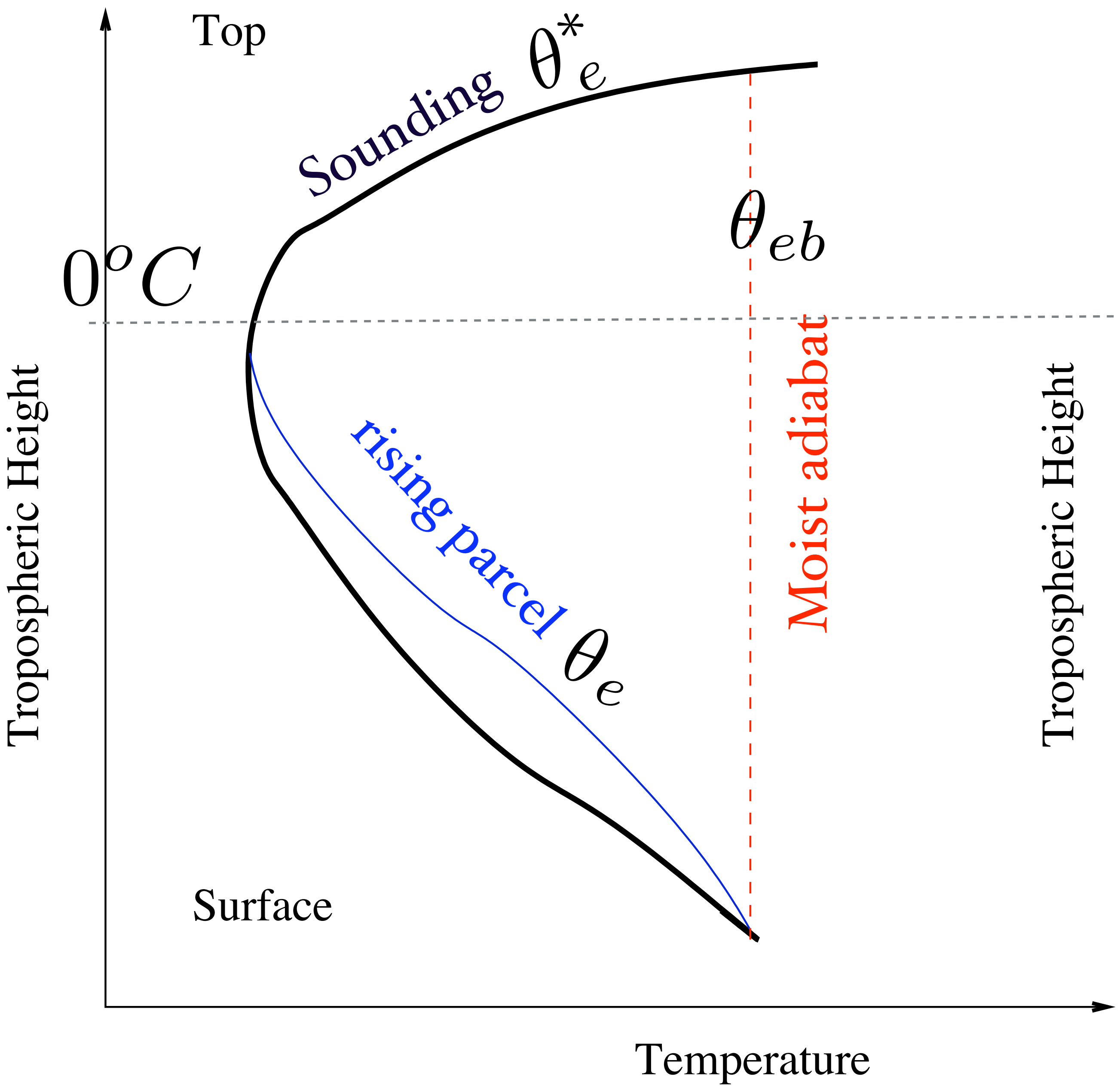
Dilute parcel lifting

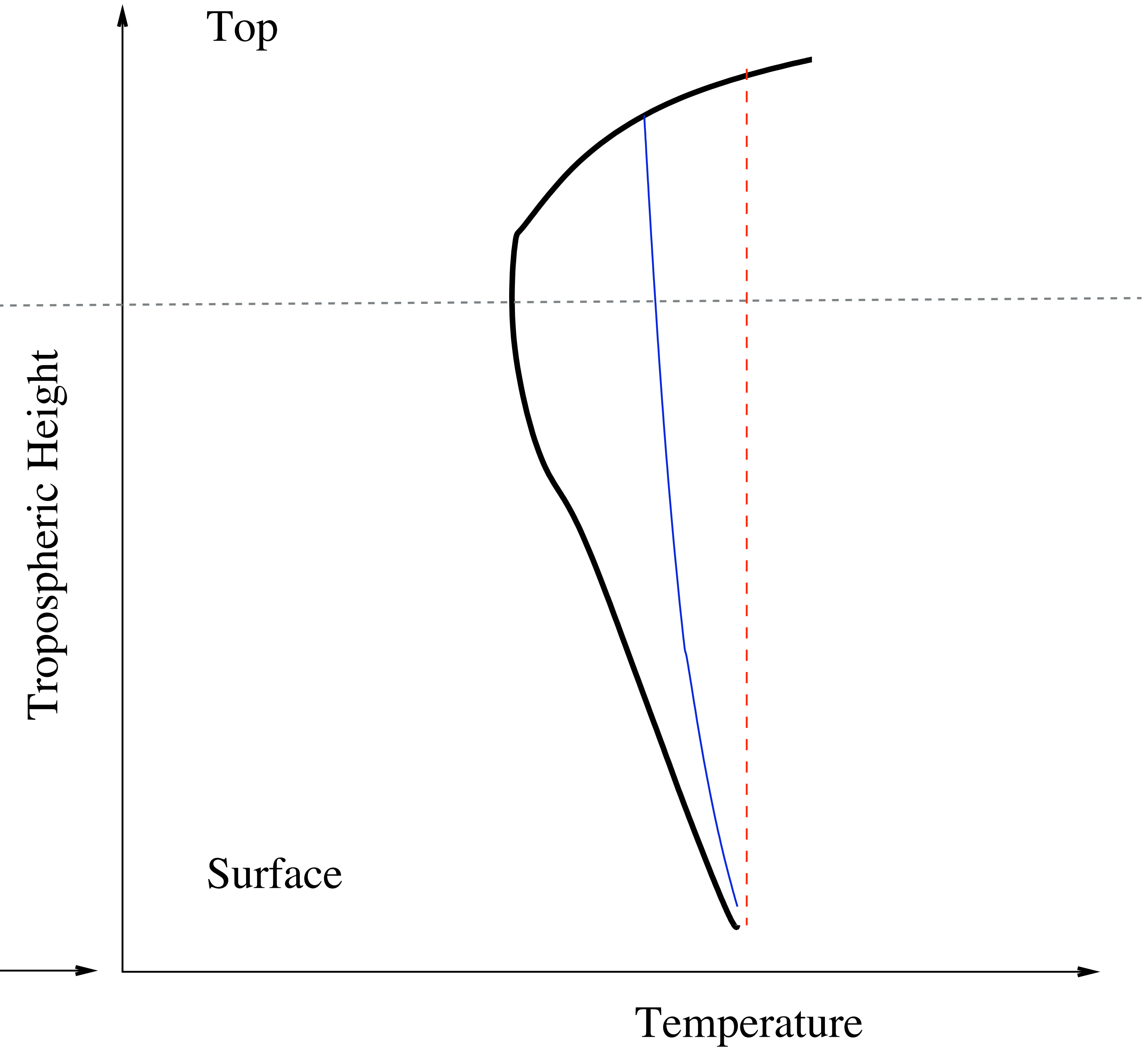


Dilute parcel lifting

Dry troposphere with positive CAPE favors congestus clouds

Moist environment

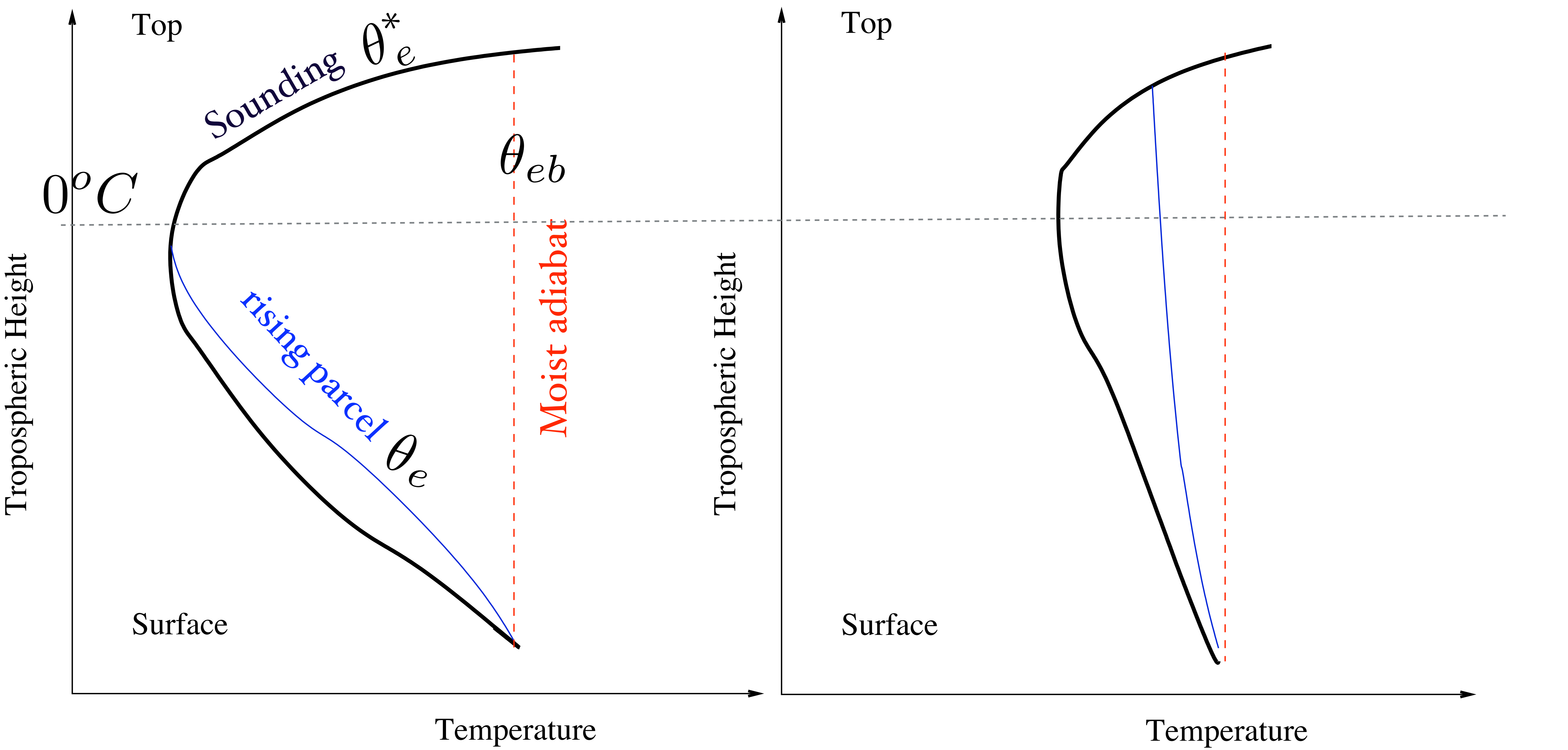




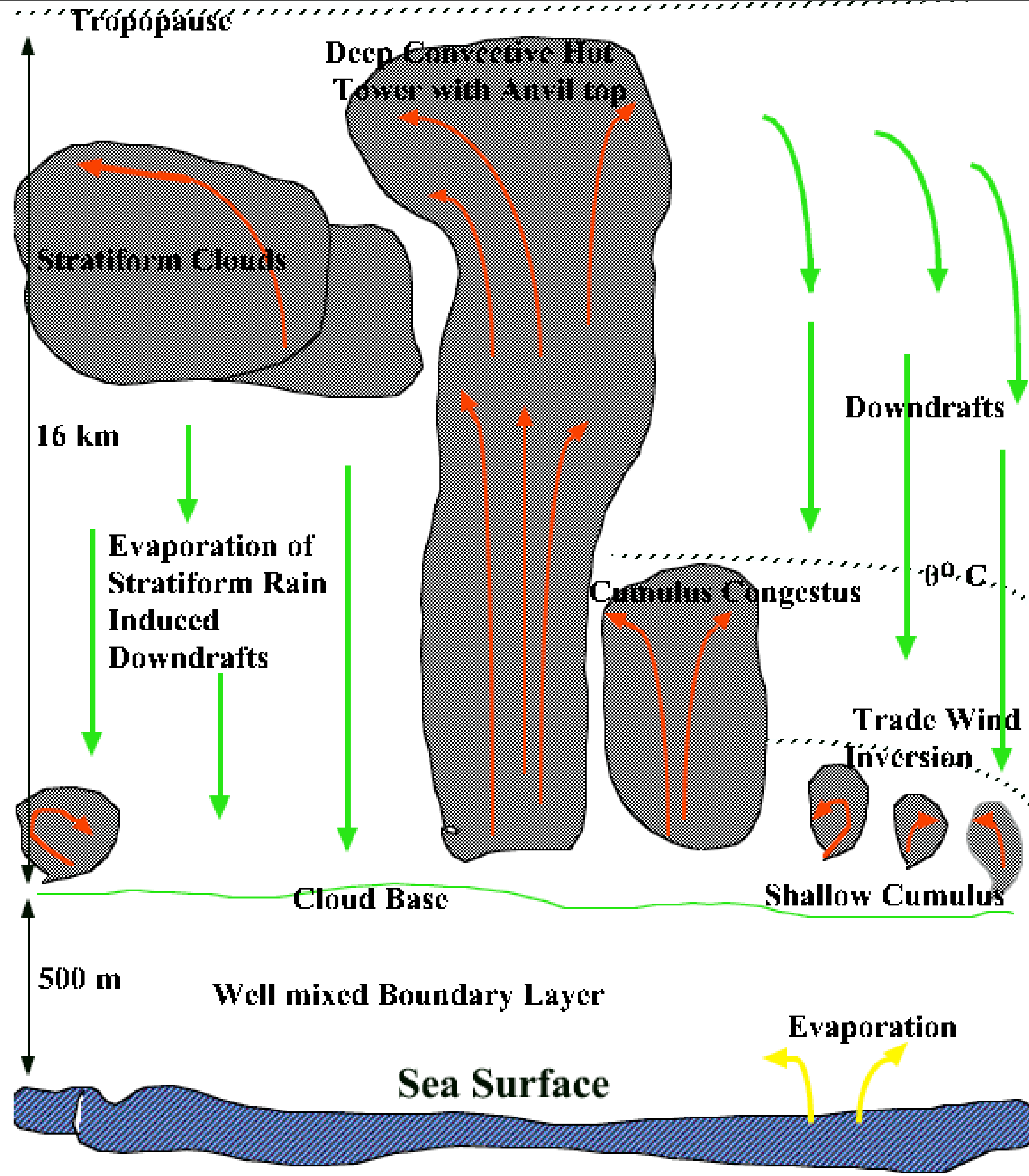
Dilute parcel lifting

Dry troposphere with positive CAPE favors congestus clouds

Deep convection is allowed (beyond freezing level) when troposphere is moist



Conceptual elementary convective cell and its life cycle



THE THREE CLOUD MODEL: AN IDEALIZED PICTURE

Stochastic Multicloud Model

- Lattice points take values 0,1,2, or 3 ==> a Four state Markov chain
- Three order parameters c,d,s taking values 1 or 0, at a given lattice point according to whether there is a congestus, a deep, a stratiform cloud or none: ===> multistate state multivariable stochastic process

C		S
	D	
C		
	C	
S		D

Intuitive transition rules

- A clear sky site turns into a congestus site with high probability if CAPE>0 and middle troposphere is dry.
- A congestus or clear sky site turns into a deep site with high probability if CAPE>0 and middle troposphere is moist.
- A deep site turns into a stratiform site with high probability.
- All three cloud types decay naturally according to prescribed decay rates.

Transition probabilities

- Four state Markov chain at given site

$$X_t = \begin{cases} 0 & \text{at clear sky site} \\ 1 & \text{at congestus site} \\ 2 & \text{at deep site} \\ 3 & \text{at statiform site} \end{cases}$$

- Prob{ $X_{t+\Delta t} = k / X_t = l$ } = $R_{lk}\Delta t + O(\Delta t^2), l \neq k$
- Transition probability matrix, $\text{with } P_{lk} = R_{lk}\Delta t$

$$M = \begin{bmatrix} 1 - P_{01} - P_{02} & P_{01} & P_{02} & 0 \\ P_{10} & 1 - P_{10} - P_{12} & P_{12} & 0 \\ P_{20} & 0 & 1 - P_{20} - P_{23} & P_{23} \\ P_{30} & 0 & 0 & 1 - P_{30} \end{bmatrix}$$

Equilibrium distribution:

depends only on the transition rates not on time stepping

$$\pi = \frac{1}{Z}\begin{pmatrix} 1 \\ \frac{R_{01}}{R_{10}+R_{12}} \\ \frac{1}{R_{20}+R_{23}}\left(R_{02}+\frac{R_{12}R_{01}}{R_{10}+R_{12}}\right) \\ \frac{R_{23}}{R_{30}}\frac{1}{R_{20}+R_{23}}\left(R_{02}+\frac{R_{12}R_{01}}{R_{10}+R_{12}}\right) \end{pmatrix}$$

Transition rates

- C = normalized CAPE, D = mid-trop. dryness
- tau_kl transition time scales
- Local interactions ignored for simplicity

$$R_{01} = \frac{1}{\tau_{01}}\Gamma(C)\Gamma(D) \qquad R_{02} = \frac{1}{\tau_{02}}\Gamma(C)(1-\Gamma(D))$$

$$R_{10} = \frac{1}{\tau_{10}}\Gamma(D) \qquad R_{12} = \frac{1}{\tau_{12}}\Gamma(C)(1-\Gamma(D))$$

$$R_{20} = \frac{1}{\tau_{20}}[1-\Gamma(C)] \qquad R_{23} = \frac{1}{\tau_{23}} \text{ or } \frac{\Gamma(C)}{\tau_{23}}$$

$$\Gamma(x) = 1 - e^{-x} \text{ if } x > 0$$

$$R_{30} = 1/\tau_{30} \qquad \Gamma(x) = 0 \text{ if } x \leq 0$$

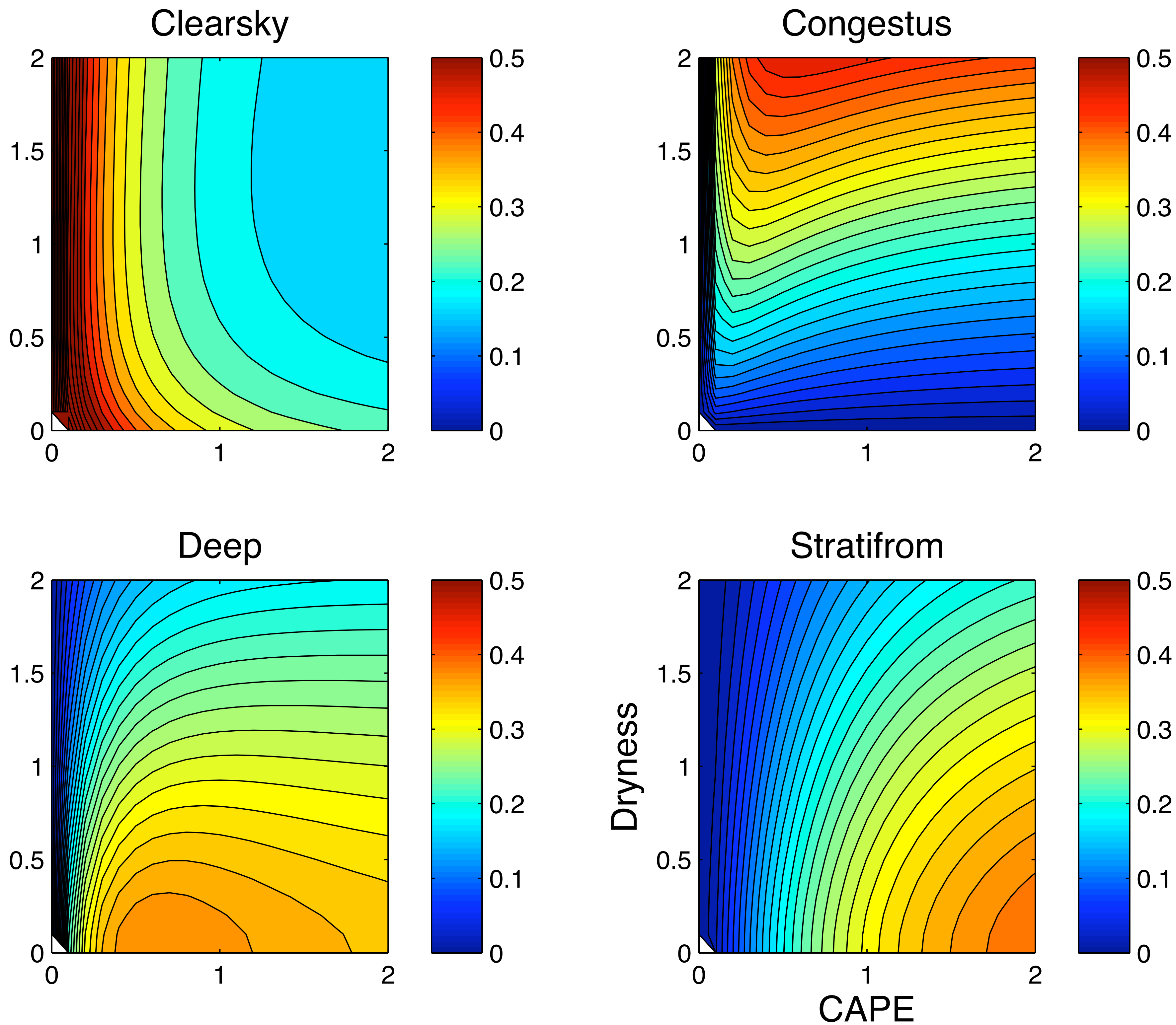
Design Principles for Transition Rates

- Ideally they should be inferred from Obs or CRM data
- Cloud formation time scales << decay time scale
- An effective stratiform time scale of about 3 hours, to agree with previous work.
- Equilibrium/stationary distribution will favor either, congestus, deep, or stratiform clouds according to large scale state in away that is consistent with Obs.

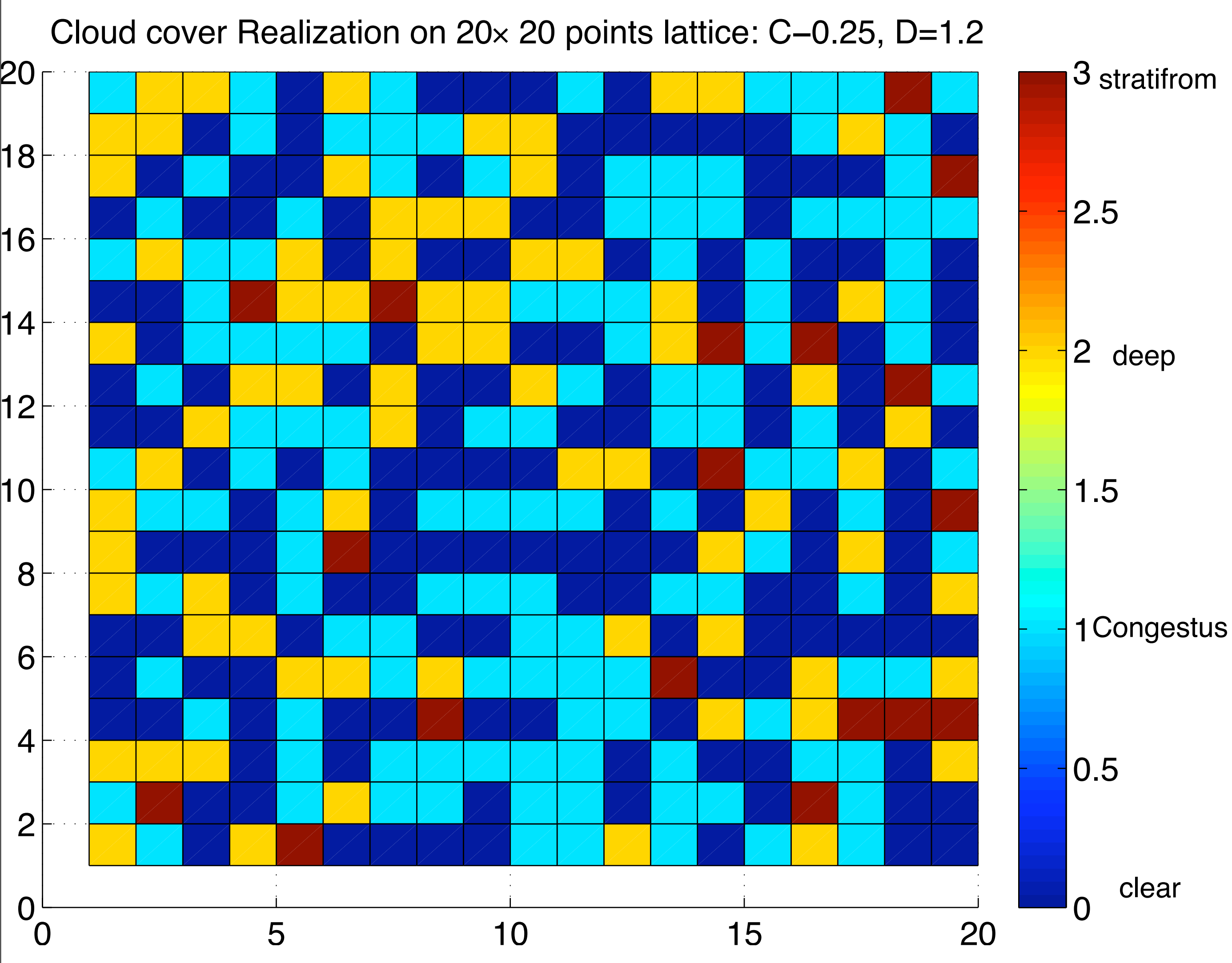
Two typical choices

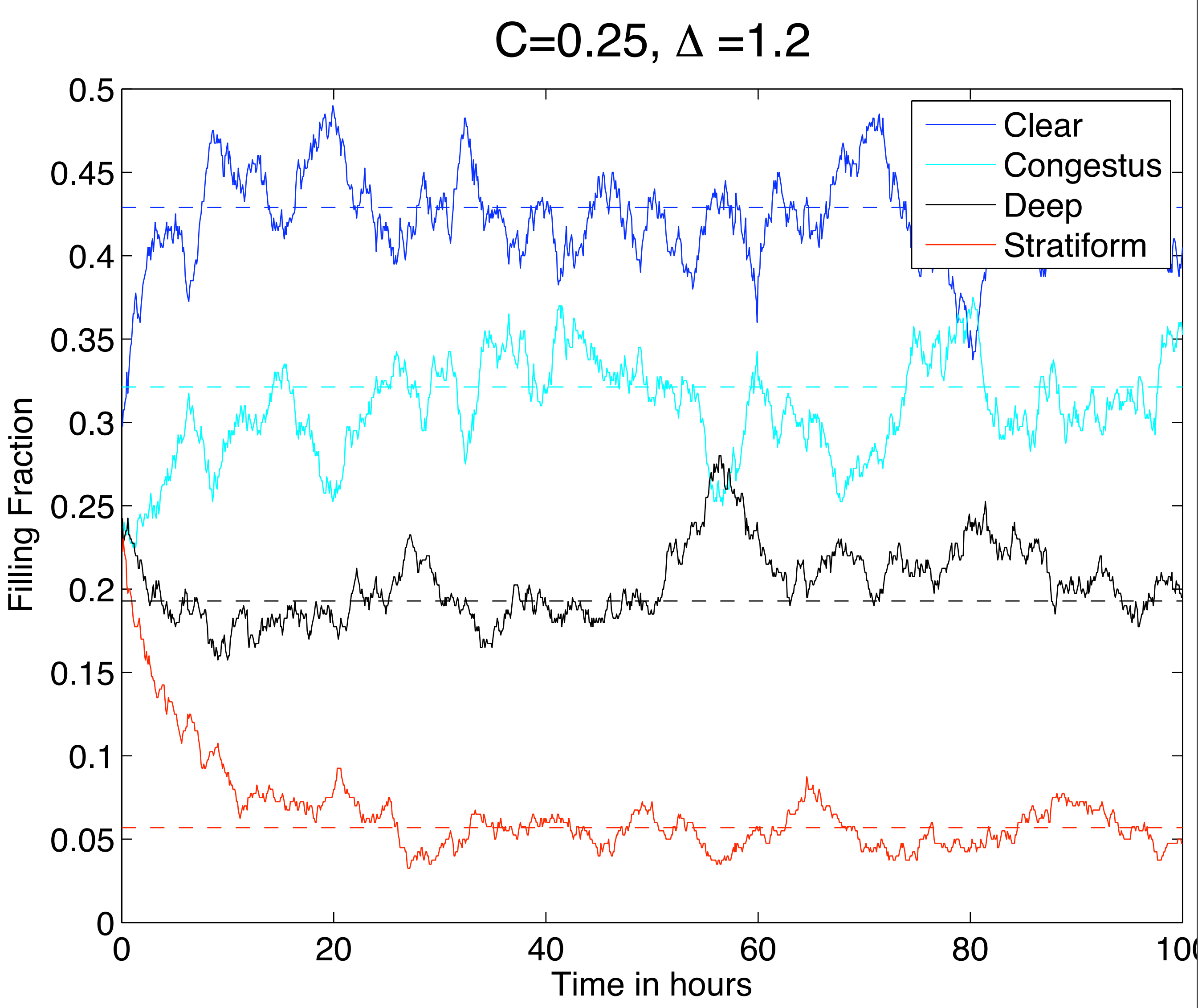
Time	description	Case 1	Case 2
τ_{01}	formation of congestus	1 hour	3 hours
τ_{10}	decay of congestus	5 hours	2 hours
τ_{12}	conversion of congestus to deep	1 hour	2 hours
τ_{02}	formation of deep	2 hours	5 hours
τ_{23}	conversion of deep to stratiform	3 hours	0.5 hour
τ_{20}	decay of deep	5 hours	5 hours
τ_{30}	decay of stratiform	5 hours	24 hours

Equilibrium distribution (filling fractions)



Time evolution and statistics of filling fraction

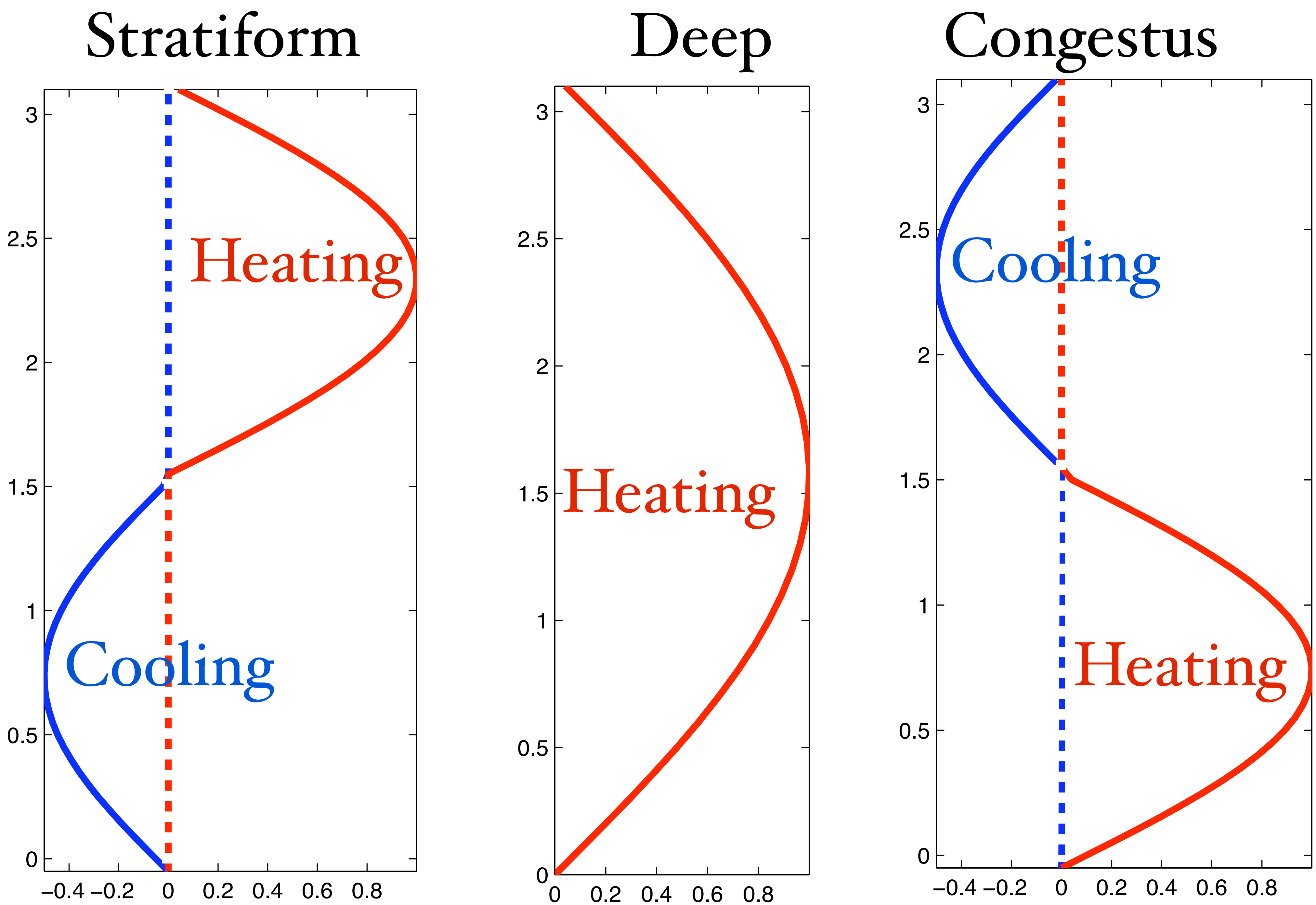




Coupling to deterministic Multicloud Equations

- Couple stochastic multicloud model to deterministic model convective parametrization discretized on coarse mesh
- A lattice of convective sites is imbedded within each coarse grid cell
- Filling fractions determine heating strength associated with each cloud type

Heating profiles in large-scale multicloud model (KM)



Large scale multicloud deterministic equations: Simple case of a one-column model

Governing equations:

$$\frac{d\theta_1}{dt} = H_d - Q_{R,1}^0 - \theta_1/\tau_R$$

$$\frac{d\theta_2}{dt} = -H_s + H_d - Q_{R,2}^0 - \theta_2/\tau_R$$

$$\frac{dq}{dt} = -P + \frac{D}{H}$$

$$\frac{d\theta_{eb}}{dt} = \frac{1}{\tau_e}(\theta_{eb}^* - \theta_{eb}) - \frac{D}{h}$$

Closure assumptions and coupling to stochastic model:

$$D = m_0 \left(1 + \mu \frac{H_s - H_c}{Q^0_{R,1}}\right)^+ (\theta_{eb} - \theta_{em}),$$

$$H_d = \langle d_{ij} \rangle \left(\sqrt{\overline{CAPE}/H_m} + \frac{1}{\langle \bar{d} \rangle \tau_c} (a_1 \theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2 \theta_2)) \right)^+$$

$$H_c = \alpha_c \langle c_{ij} \rangle \sqrt{\left(\overline{CAPE} + R * (\theta_{eb} - \gamma(\theta_1 + 2\theta_2))\right)^+ / H_m}$$

$$H_s = \langle s_{ij} \rangle \sqrt{\left(\overline{CAPE} + R * (\theta_{eb} - \gamma(\theta_1 + 0.1\theta_2))\right)^+ / H_m / 4}$$

$$P = \frac{2\sqrt{2}}{\pi} H_d$$

Radiative convective equilibrium of coupled system

- Given external parameters: radiative cooling, $Q^0_{R,1}$, and discrepancies $\theta^*_{eb} - \bar{\theta}_{eb}$ and $\bar{\theta}_{eb} - \bar{\theta}_{em}$, the RCE equations together with the equilibrium distribution relating the area fractions of convection $\langle \bar{c} \rangle, \langle \bar{d} \rangle, \langle \bar{s} \rangle,$ to CAPE and dryness through

$$\Delta = \frac{\theta_{eb} - \theta_{em}}{\bar{\theta}_0}, \bar{\theta}_0 = 10 \text{ Kelvin}$$

$$C = CAPE/CAPE_0, CAPE_0 = 2000J/kg$$

- RCE solution with $\theta_1 = \theta_2 = q = 0$ is uniquely determined

Coarse graining and mean-field eqns

- Nx = number of sites filled with cloud type x
- N = Nc + Nd + Ns + Ncs
- cloud area fractions: Nx/N
- System of Birth-death processes with exact stochastic dynamics because no coupling between lattice sites

$$Prob\{N_c^{t+\Delta t} = k + 1 / N_c^t = k\} = N_{cs} R_{01} \Delta t + o(\Delta t)$$

$$Prob\{N_c^{t+\Delta t} = k - 1 / N_c^t = k\} = N_c (R_{10} + R_{12}) \Delta t + o(\Delta t)$$

$$Prob\{N_d^{t+\Delta t} = k + 1 / N_d^t = k\} = (N_{cs} R_{01} + N_c R_{12}) \Delta t + o(\Delta t)$$

$\ldots$

Mean field equations:

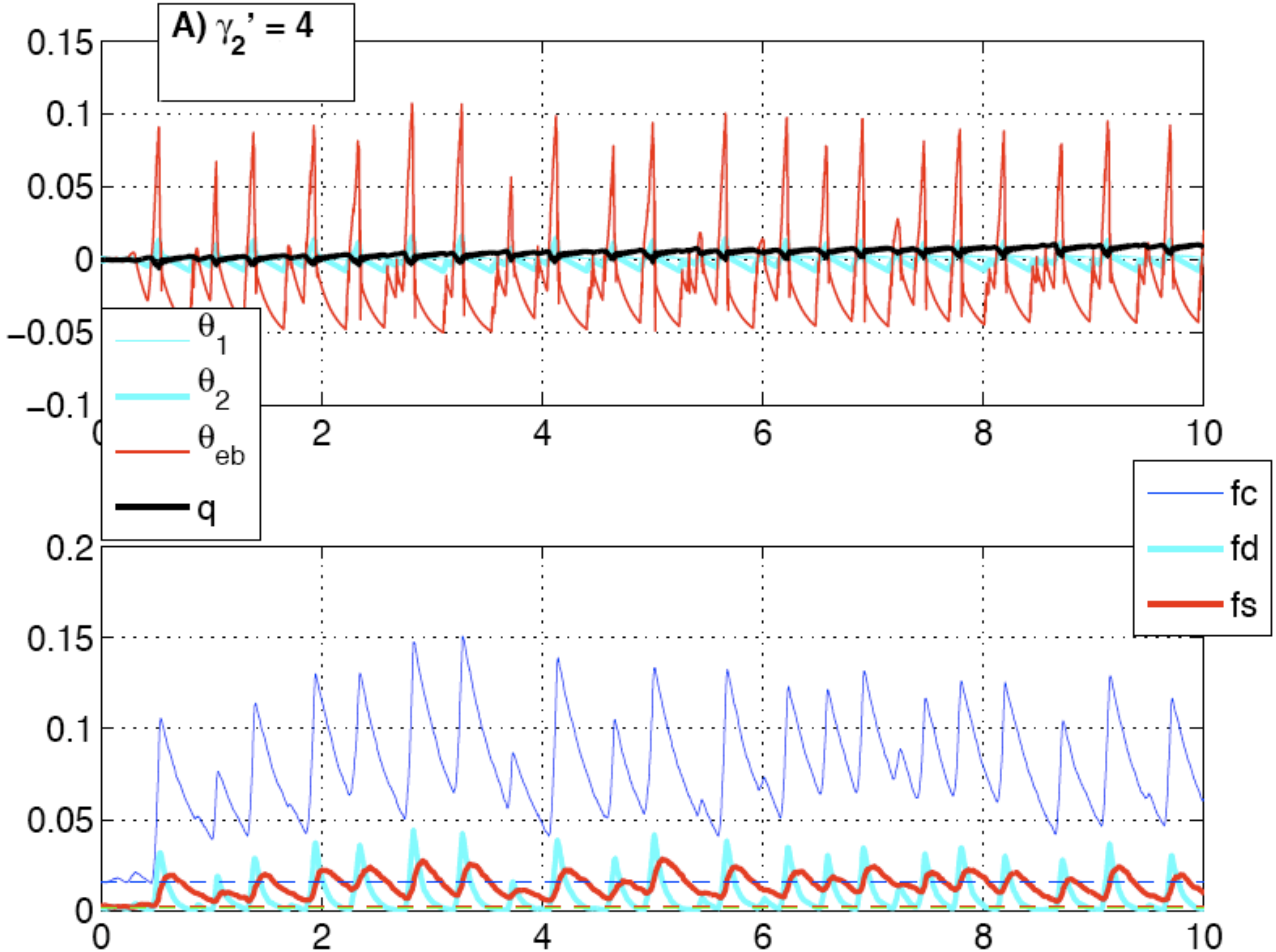
$$\dot{\sigma}_c = (1 - \sigma_c - \sigma_d - \sigma_s)R_{01} - \sigma_c(R_{10} + R_{12})$$

$$\dot{\sigma}_d = (1 - \sigma_c - \sigma_d - \sigma_s)R_{02} + \sigma_c R_{12} - \sigma_d(R_{20} + R_{23})$$

$$\dot{\sigma}_s = \sigma_d R_{23} - \sigma_s R_{30}.$$

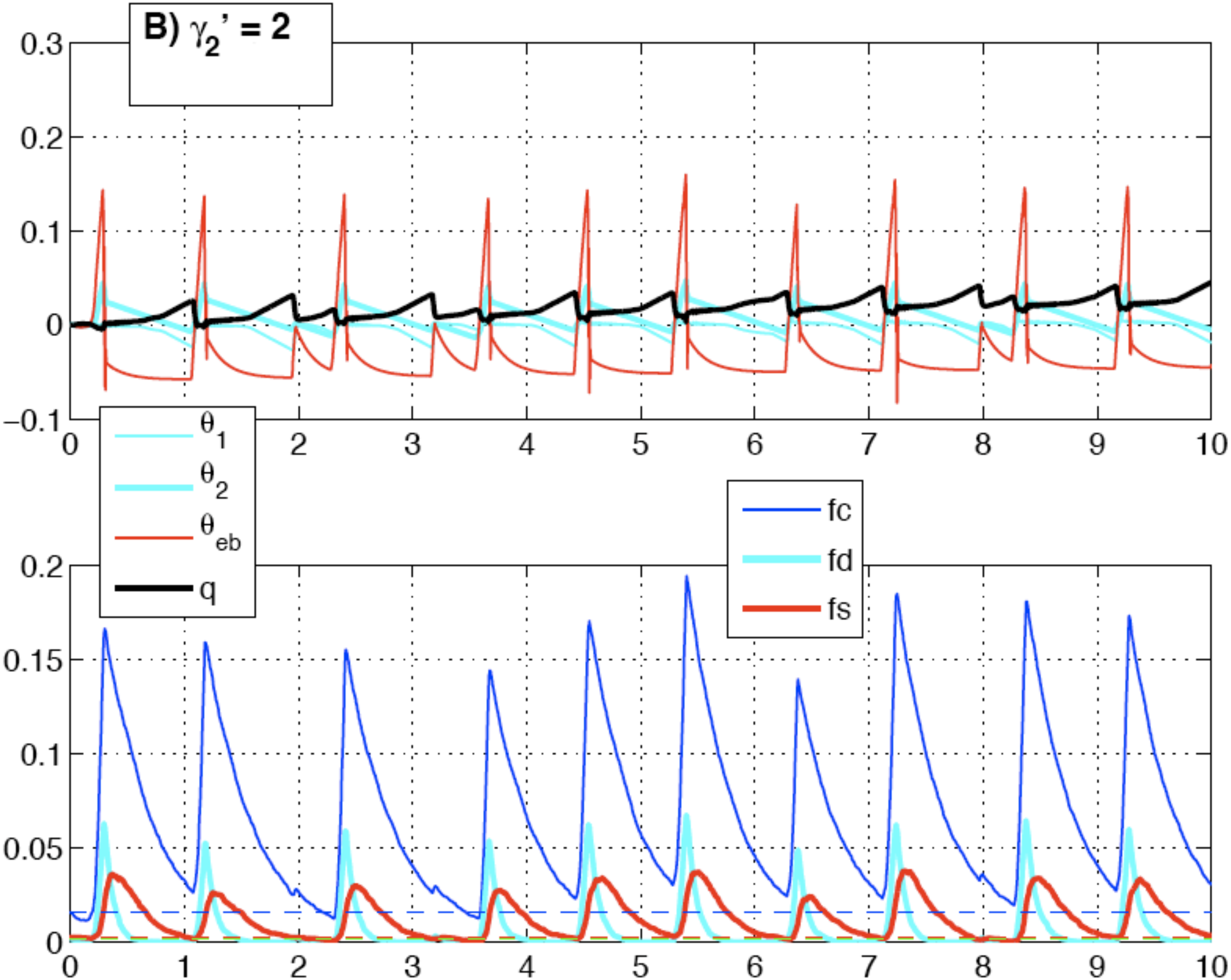
Steady State is the Equilibrium distribution

Coupled stochastic-deterministic

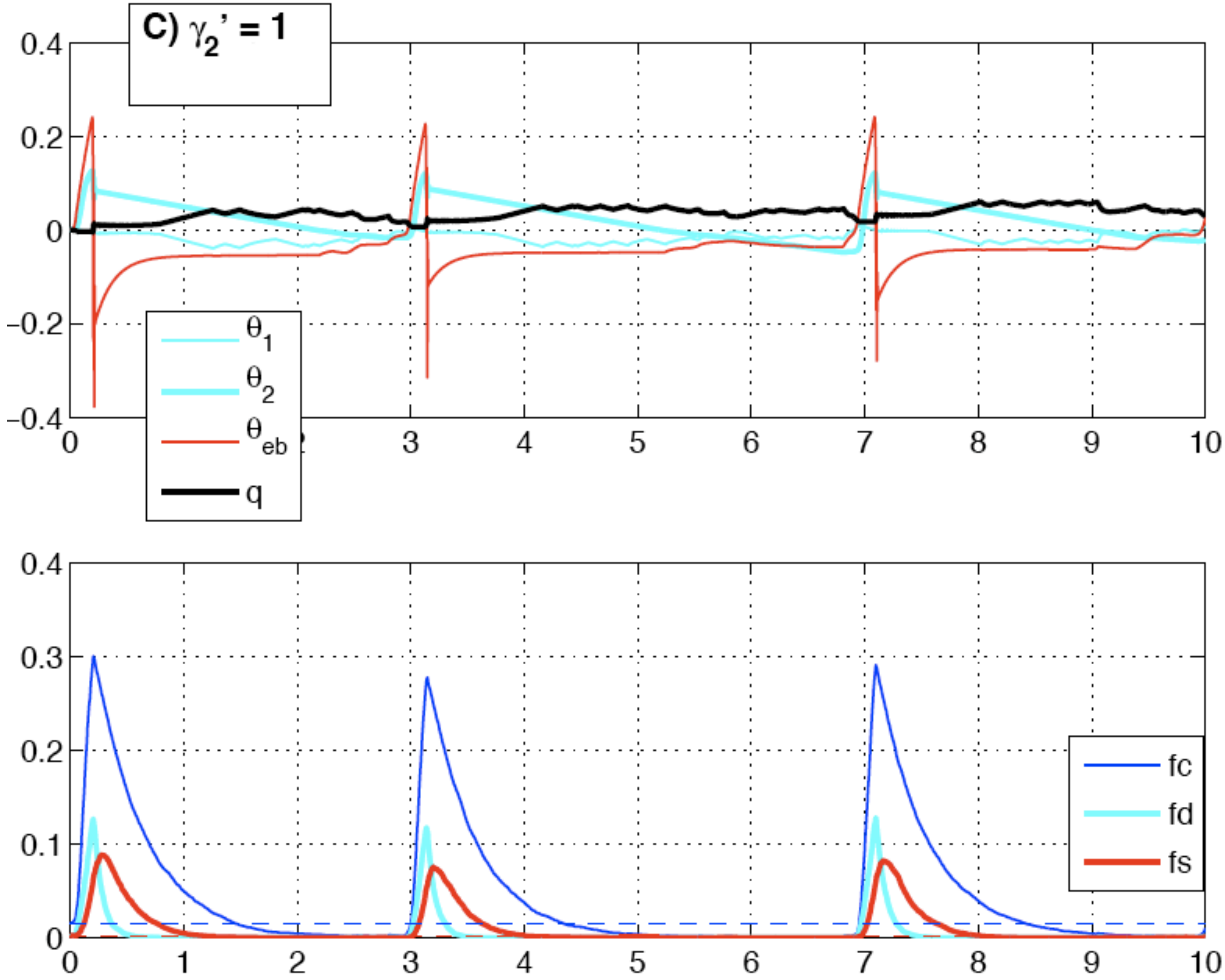


theta_eb(CAPE) >>> congestus >>> deep >>>stratiform

Slower low-level buoyancy time scale



...and much slower



Summary

- Stochastic model coupled to deterministic multicloud model of KM provides an alternative for evolving from congestus, to deep, to stratiform heating modes
- Stochastic model responds directly to CAPE variations and excites heating modes in a feed-back loop consistent with stochastic resonance theory.
- Low-level buoyancy time scale plays crucial role in stochastic response time scale.
- Statistical accumulation of congestus, deep, and stratiform sites in a fashion similar to stretched building block of Mapes et al. 2006

Remarks

- Local interactions ignored...might be important for better organization
- One major role of congestus heating is low level convergence of moisture. Cannot be captured here by the one column model.
- Also congestus moistening by detrainment is not included in multicloud model for simplicity.
- This kind of ideas can be easily adapted to realistic convective parametrizations ... work in progress

END

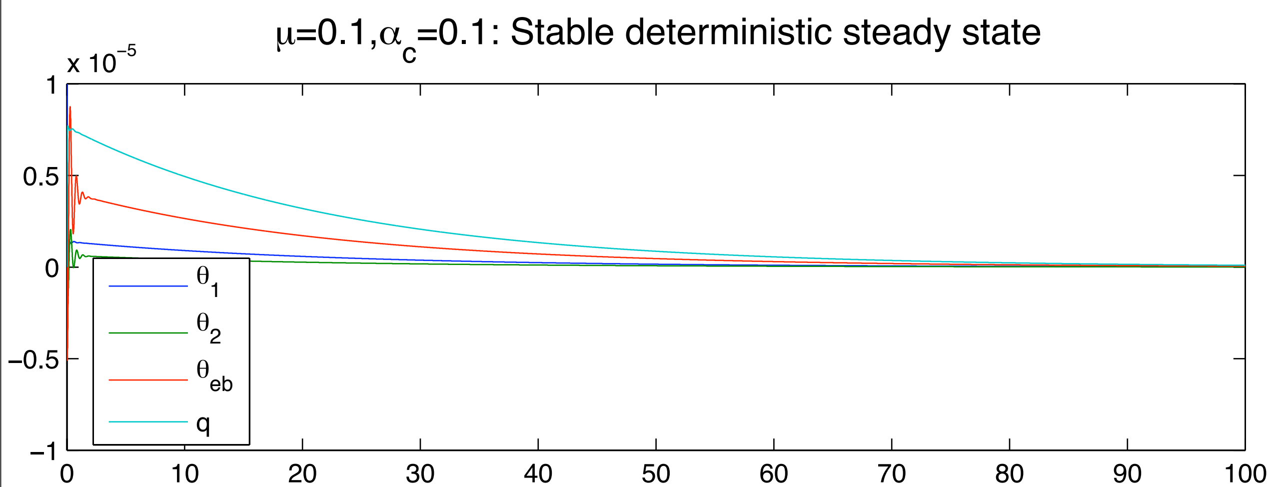
Transition time scales

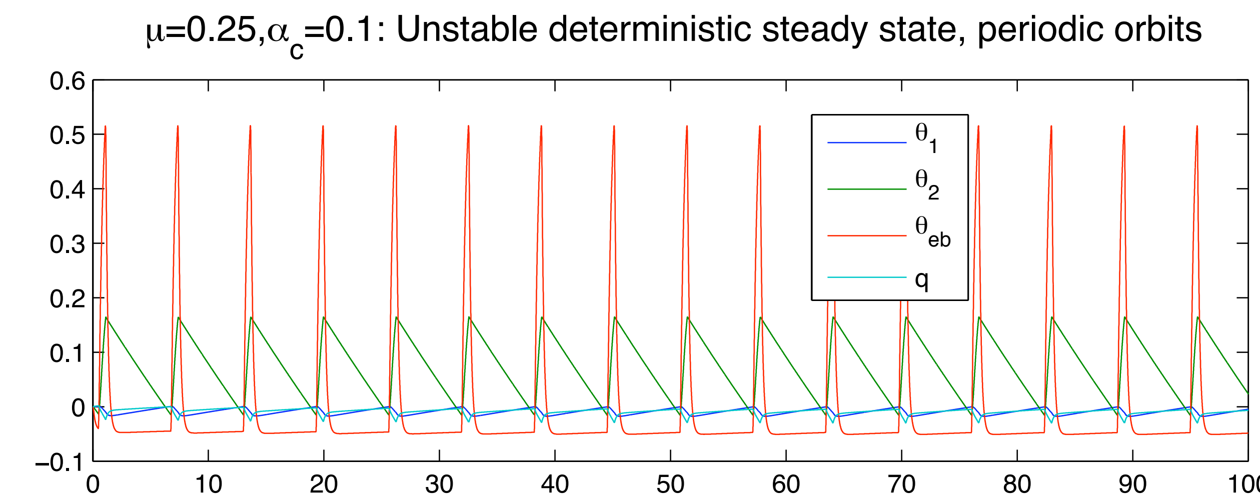
τ_{01}	free formation of congestus	1 hour
τ_{02}	free formation of deep	4 hours
τ_{12}	conversion of congestus to deep	1 hour
τ_{23}	conversion of deep to stratiform	1.5 hours
τ_{10}	free decay of congestus	5 hours
τ_{20}	free decay of deep	5 hours
τ_{30}	free decay of stratiform	5 hours

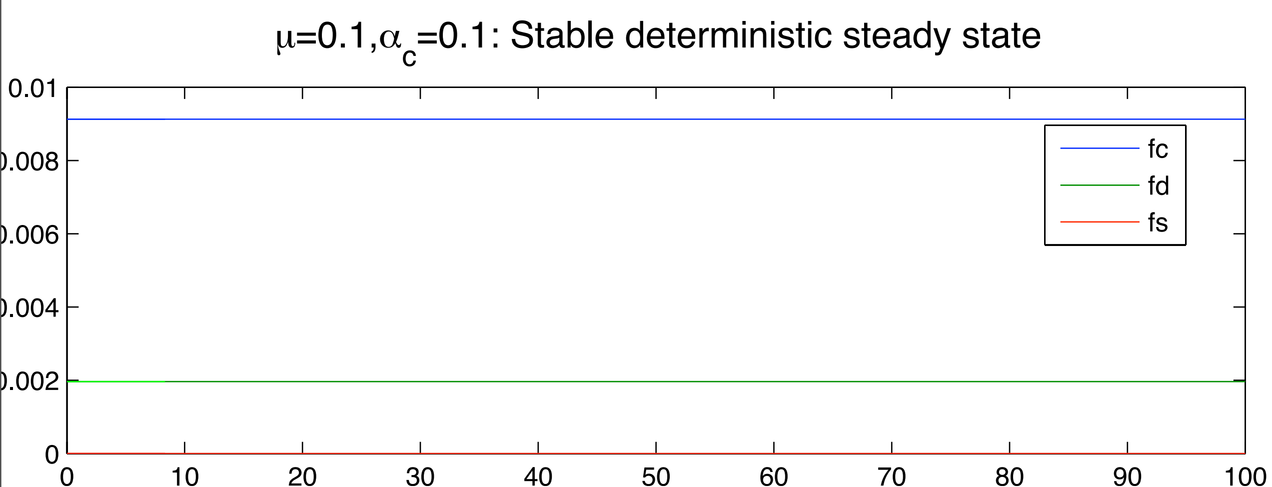
Dynamics of deterministic equations

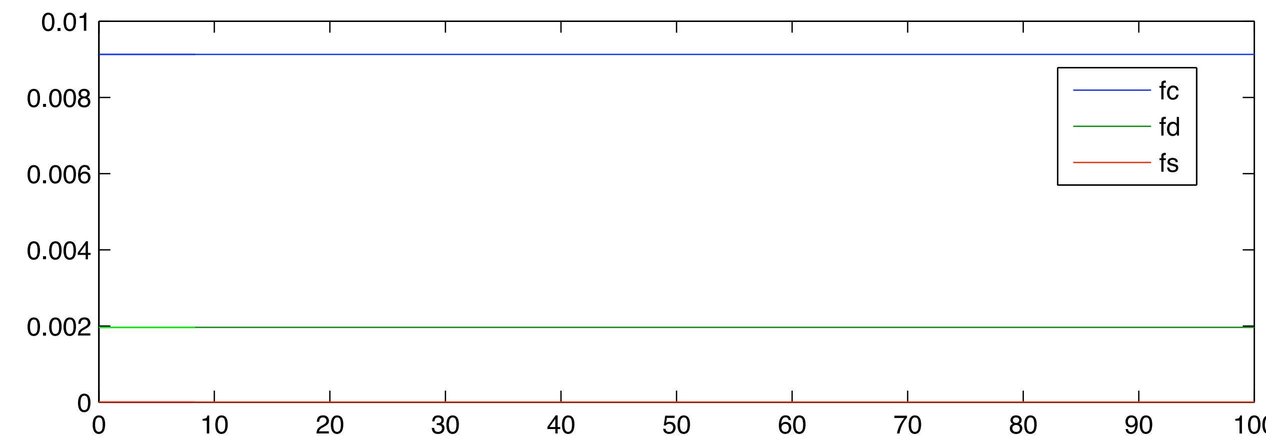
Stable for mu=0.1

Periodic orbits at mu=0.25









fixed area fractions