Boundary layer dynamics in a simple model for convectively coupled gravity waves

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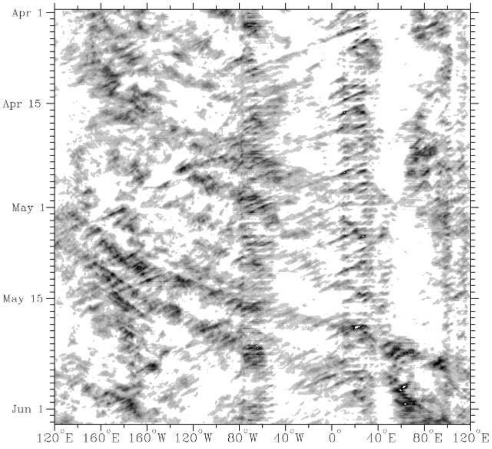
With thanks to: Andy Majda, Norm McFarlane, Adam Monahan, Bjorn Stevens

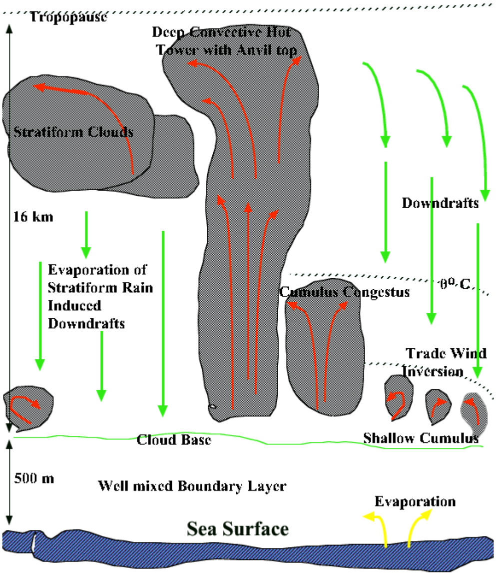
Convectively coupled Kelvin waves



(CLAUS brightness temperature from Kiladis *et al.* 2009)

- Eastward propagation
- Wavelength: $\lambda \sim O(1000)$ km
- Phase speed: $c \sim$ 10-20 m/s

Vertical structure and downdrafts

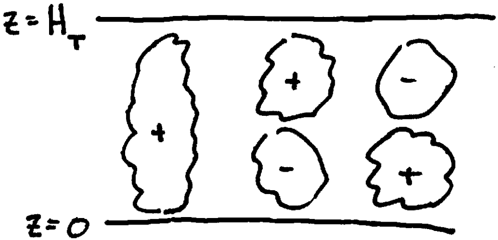


(Khouider & Majda 2008)

Outline

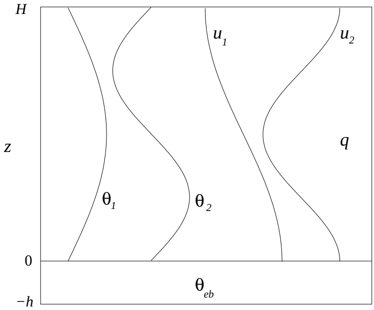
Outline

Idealized models for convectively coupled waves



- $n = 1$: Deep: e.g. Emanuel (1987), Yano *et al.* (1995)
- $n = 2$: Deep + stratiform: Mapes (2000), Majda & Shefter (2001)
- $n = 2$: Deep + stratiform + congestus: Khouider & Majda (2006, 2008)

Khouider–Majda multicloud model structure



- Free troposphere: $0 \leq z \leq H$
- Boundary layer: $-h \leq z < 0$
- $H = 16$ km
- $h = 500$ m

$$q = \frac{1}{H}\int_0^H (q_v - \overline{q}_v)\,\mathrm{d}z$$

$$(\cdot)_b = \frac{1}{h}\int_{-h}^0 (\cdot)\,\mathrm{d}z$$

$$\frac{\partial \boldsymbol{u}_j}{\partial t} = \boldsymbol{\nabla}\theta_j + \ldots$$

$$\frac{\partial \theta_1}{\partial t} = \boldsymbol{\nabla}\cdot\boldsymbol{u}_1 + H_d + \xi_s H_s + \xi_c H_c + \ldots$$

$$\frac{\partial \theta_2}{\partial t} = \frac{1}{4}\boldsymbol{\nabla}\cdot\boldsymbol{u}_2 - H_s + H_c + \ldots$$

Boundary layer

- Boundary layer in Khouider & Majda (2006, 2008) is purely thermodynamical:

$$\frac{\partial \theta_{eb}}{\partial t} = S - D,$$

- S is moistening by surface fluxes
- D is drying by *convective* downdrafts: $D \propto m_0 \left(1 + \mu(H_s - H_c)\right)^+$
- No environmental downdrafts because $w = 0$ at the top of the boundary layer.
- Environmental downdrafts require boundary layer dynamics.

Outline

1. Background
   - Khouider–Majda multicloud model
2. Multicloud model with boundary layer dynamics
   - Boundary layer equations
   - Coupling to free troposphere
   - Free troposphere equations
3. Waves and stability
   - Basic state
   - Moist basic state instabilities
   - Dry basic state instabilities
4. Conclusions

Boundary layer equations

- Derive bulk equations following Stevens (TCFD 2006).
- Average Reynolds equations over $-h \le z \le 0$ to get equations for $\boldsymbol{u}_b$, θ_b, θ_{eb}.
- Assume constant h

- Write turbulent fluxes as:
  - $\langle w'\boldsymbol{u}'\rangle_s = -c\,U\,\boldsymbol{u}_b$
  - $\langle w'\phi'\rangle_s = \dfrac{h}{\tau_e}(\phi_s - \phi_b)$
  - $\langle w'\boldsymbol{u}'\rangle_t = \dfrac{h}{\tau_T}(\boldsymbol{u}_b - \boldsymbol{u}_t)$
  - $\langle w'\phi'\rangle_t = M_u\,(\phi_b - \phi_t) - M_d\,(\phi_m - \phi_t)$

- Subscripts denote reference levels:
  - s: surface $z = -h$
  - t: top of boundary layer $z \to 0^+$
  - m: mid-troposphere

(Following Raymond 1995)

Boundary layer equations

$$\frac{\partial \boldsymbol{u}_b}{\partial t} + \boldsymbol{u}_b \cdot \boldsymbol{\nabla} \boldsymbol{u}_b = -\boldsymbol{\nabla} p_b - c\frac{U}{h}\boldsymbol{u}_b - \frac{1}{h}\left(\frac{h}{\tau_T} + h\boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)\Delta_t \boldsymbol{u}$$

$$\frac{\partial \theta_b}{\partial t} + \boldsymbol{u}_b \cdot \boldsymbol{\nabla}\theta_b = -Q_{Rb} + \frac{\Delta_s\theta}{\tau_e} - \frac{M_d}{h}\Delta_m\theta - \frac{1}{h}\left(M_u - M_d + h\boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)\Delta_t\theta$$

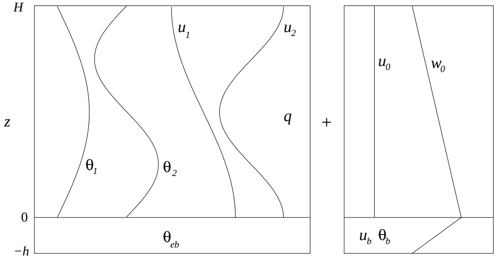
$$\frac{\partial \theta_{eb}}{\partial t} + \boldsymbol{u}_b \cdot \boldsymbol{\nabla}\theta_{eb} = -Q_{Rb} + \frac{\Delta_s\theta_e}{\tau_e} - \frac{M_d}{h}\Delta_m\theta_e - \frac{1}{h}\left(M_u - M_d + h\boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)\Delta_t\theta_e$$

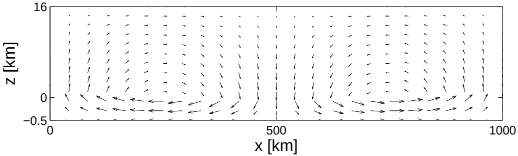
- surface, downdrafts, entrainment, fluxes (e.g. Stevens 2006)
- Downward differences: $\Delta_s\phi \equiv \phi_s - \phi_b, \quad \Delta_t\phi \equiv \phi_b - \phi_t, \quad \Delta_m\phi \equiv \phi_b - \phi_m$
- Redefine entrainment velocities:

$$E_u \equiv \left(\frac{h}{\tau_T} + h\boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)^+ \qquad E \equiv \left(M_u - M_d + h\boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)^+$$

Continuity of *w*

- Discontinuity of *w* at interface between boundary layer and free troposphere:
- Introduce barotropic $\boldsymbol{u}_0$ in troposphere with $\boldsymbol{\nabla} \cdot \boldsymbol{u}_0 \equiv -\delta \, \boldsymbol{\nabla} \cdot \boldsymbol{u}_b$, where $\delta \equiv h/H$





Pressure

- Need expressions for p_b and p_0.
- Assume BL pressure is
  - linear in z: $p_b = \frac{1}{2}(p_s + p_t) = \frac{1}{2}\left(p_s + p_0 - \sqrt{2}(\theta_1 - \theta_2)\right)$
  - and hydrostatic: $p_s = p_0 - \sqrt{2}(\theta_1 + \theta_2) - \pi\delta\theta_b$
- Divergence of $\boldsymbol{u}_b$ and $\boldsymbol{u}_0$ equations yield diagnostic equation for p_b:

$$-(1+\delta)\,\nabla^2 p_b = \nabla^2\left(\sqrt{2}\,(\theta_1 + \theta_2) + \frac{\pi}{2}\,\delta\,\theta_b\right) + c\,\frac{U}{H}\,\boldsymbol{\nabla}\cdot u_b + \mathrm{NL},$$

- For linear waves, get

$$p_b \approx -\sqrt{2}(\theta_1 + \theta_2) - \frac{\pi}{2}\,\delta\,\theta_b$$

Downdrafts

- Need expressions for M_u and M_d.
- Parameterize M_d with convective and environmental contributions:

$$M_d \equiv (D_c + h\,\boldsymbol{\nabla}\cdot\boldsymbol{u}_b)^+, \quad \text{where} \quad D_c \equiv m_0\,(1 + \mu(H_s - H_c))^+$$

- Note: neglecting $\boldsymbol{\nabla}\cdot\boldsymbol{u}_b$ in M_d leads to catastrophic short-wave instability! Recall

$$E \equiv (M_u - M_d + h\boldsymbol{\nabla}\cdot\boldsymbol{u}_b)^+$$

- Assume $M_u \propto D_c$ (Raymond 1995). Define $\alpha_m \equiv D_c/M_u$. Use $\alpha_m = 0.2$.

Free troposphere equations

$$\frac{\partial \boldsymbol{u}_0}{\partial t} + \boldsymbol{u}_0 \cdot \boldsymbol{\nabla} \boldsymbol{u}_0 = -\boldsymbol{\nabla} p_0 - \boldsymbol{\nabla} \cdot \boldsymbol{u}_0 \left(\boldsymbol{u}_0 - \boldsymbol{u}_b \right) + \frac{E_u}{H} \Delta_t \boldsymbol{u}$$

$$\frac{\partial \boldsymbol{u}_j}{\partial t} + \boldsymbol{u}_0 \cdot \boldsymbol{\nabla} \boldsymbol{u}_j + \boldsymbol{u}_j \cdot \boldsymbol{\nabla} \boldsymbol{u}_0 = \boldsymbol{\nabla} \theta_j + \frac{\sqrt{2}\, \delta}{\tau_T} \Delta_t \boldsymbol{u}$$

$$\frac{\partial \theta_1}{\partial t} + \boldsymbol{u}_0 \cdot \boldsymbol{\nabla} \theta_1 + \sqrt{2}\, \boldsymbol{\nabla} \cdot \boldsymbol{u}_0 - \boldsymbol{\nabla} \cdot \boldsymbol{u}_1 = H_d + \xi_s H_s + \xi_c H_c - Q_{R1}$$

$$\frac{\partial \theta_2}{\partial t} + \boldsymbol{u}_0 \cdot \boldsymbol{\nabla} \theta_2 + \frac{\sqrt{2}}{4} \boldsymbol{\nabla} \cdot \boldsymbol{u}_0 - \frac{1}{4} \boldsymbol{\nabla} \cdot \boldsymbol{u}_2 = -H_s + H_c - Q_{R2}$$

$$\frac{\partial q}{\partial t} + \boldsymbol{u}_0 \cdot \boldsymbol{\nabla} q + \boldsymbol{\nabla} \cdot \left(\left(\boldsymbol{u}_1 + \tilde{\lambda} \boldsymbol{u}_2 \right) \tilde{Q} - \boldsymbol{u}_0 \, \tilde{Q}_0 \right) = -P + \frac{E}{H} \Delta_t \theta_e + \frac{1}{H} \left(M_d - h \boldsymbol{\nabla} \cdot \boldsymbol{u}_b \right) \Delta_m \theta_e$$

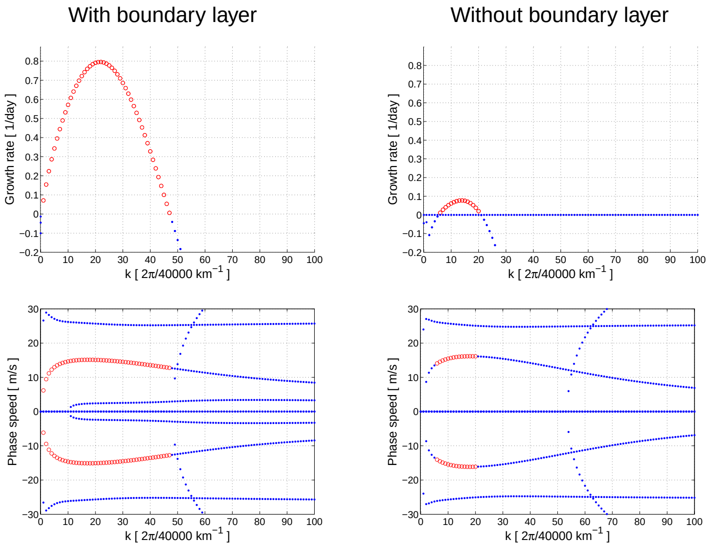
- Note: we've neglected:
  - Baroclinic–baroclinic advection
  - Barotropic vertical advection of baroclinic modes
  - Heating of θ_1 and θ_2 by boundary layer (but included in q equation for conservation)

Outline

Basic state

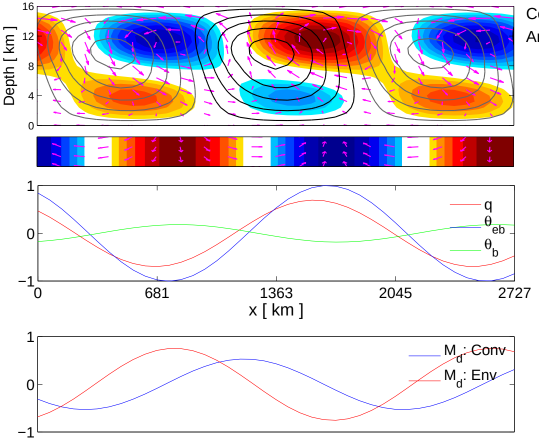
- Let $f = 0$, $v = 0$, $\partial_y = 0$
- Linearize around basic state in radiative–convective equilibrium (RCE).
- RCE is determined by specifying Q_{R1} and profiles of $\overline{\theta}_e$, $\overline{\theta}$. Solve for heating/cooling and some parameters.
- Regime characterized by $\overline{\theta}_{eb} - \overline{\theta}_{em}$
  - Moist troposphere: $\overline{\theta}_{eb} - \overline{\theta}_{em} = 11$ K
  - Dry troposphere: $\overline{\theta}_{eb} - \overline{\theta}_{em} = 19$ K
- RCE fixes values for parameters Q_{Rb} and τ_e. Typical values:
  - $Q_{Rb} \approx 5$ K/day
  - $\tau_e \approx 7$ h
  - $\overline{M}_d \approx 0.5$ cm/s

Moist basic state: moist gravity wave stability



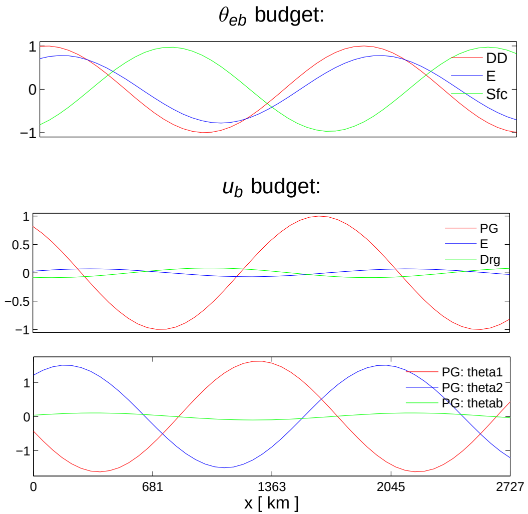
Moist basic state: moist gravity wave structure

Most unstable wave: $k = 14$, $\lambda = 1818$ km, $c = +15.0$ m/s



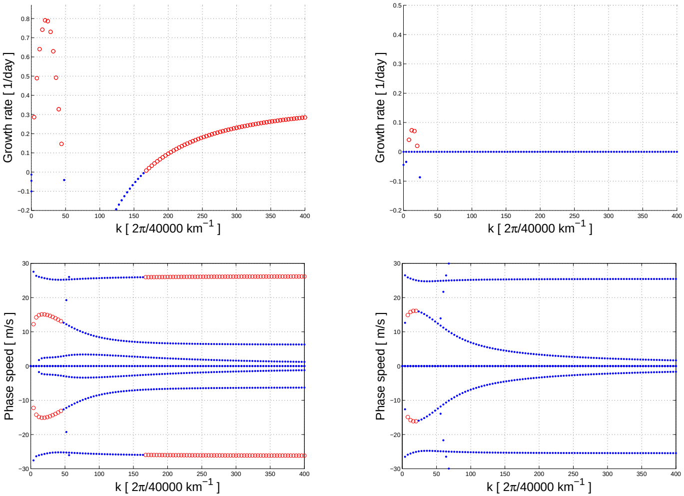
Contours: heating
Colours: θ
Arrows: (u, w)

Moist basic state: moist gravity wave budget



Moist basic state: fast moist gravity wave stability

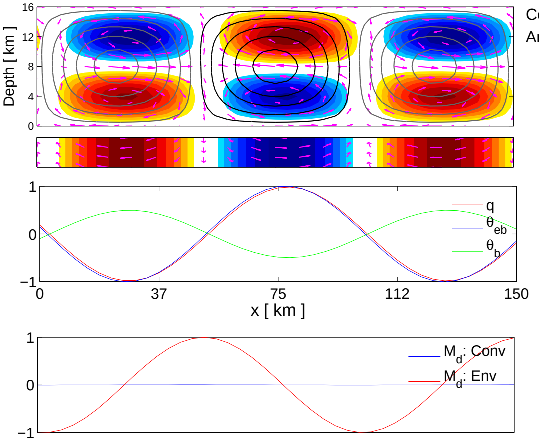
With boundary layer | Without boundary layer



Moist basic state: fast moist gravity wave structure

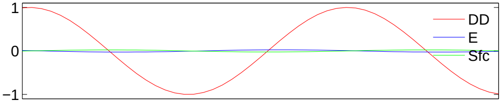
$k = 400$, $\lambda = 100$ km, $c = +26.2$ m/s

Contours: heating
Colours: θ
Arrows: (u, w)

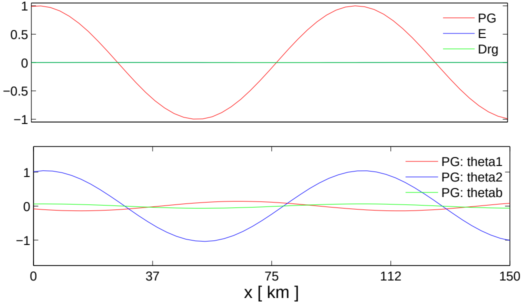


Moist basic state: fast moist gravity wave budget

θ_{eb} budget:

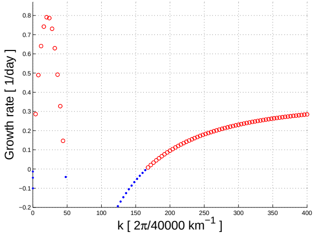


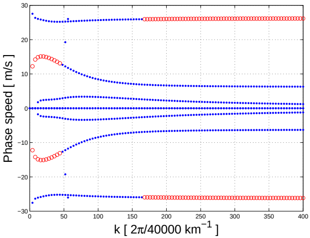
u_b budget:



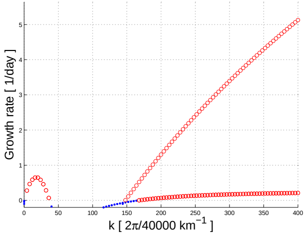
Moist basic state: environmental downdrafts

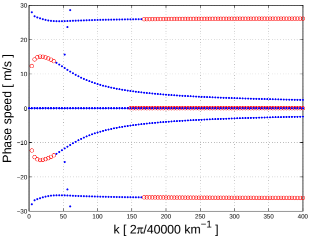
With $\nabla \cdot \boldsymbol{u}_b$ in M_d



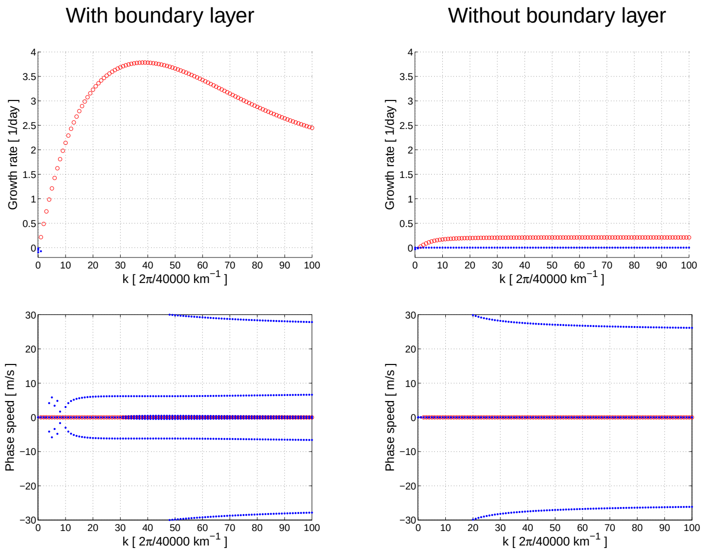


Without $\nabla \cdot \boldsymbol{u}_b$ in M_d





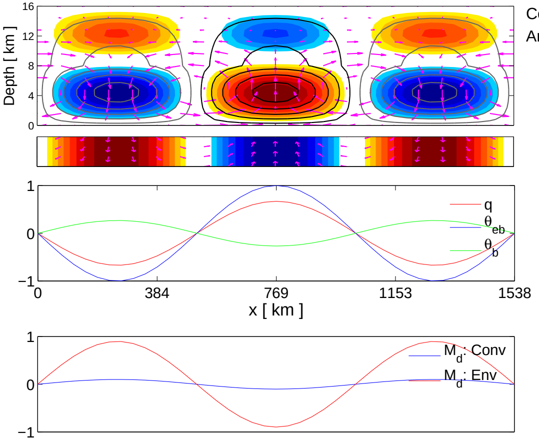
Dry basic state: congestus mode stability



Dry basic state: congestus mode structure

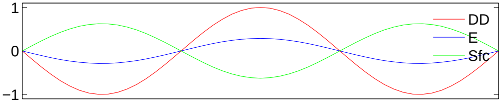
Most unstable wave: $k = 39$, $\lambda = 1026$ km, $c = 0$ m/s

Contours: heating
Colours: θ
Arrows: (u, w)

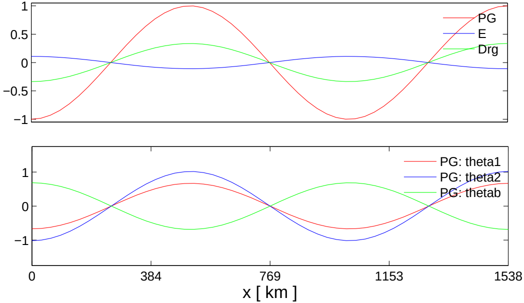


Dry basic state: congestus mode budget

θ_{eb} budget:



u_b budget:



Outline

Conclusions

Boundary layer dynamics in the multicloud model:

- *Enhance* instability of synoptic-scale moist gravity waves;
- *Destabilize* fast mesoscale gravity waves;
- *Focus* congestus instability at synoptic scales.

Details: Waite & Khouider 2009, *J. Atmos. Sci.* (in press)