

Theory and Applications of Classical  
and Quantum Kinetic Theory  
Jun 21 – Jun 26, 2009

PRELIMINARY PROGRAM

Monday, June 22, 2009

8:45	-	9:00	Welcome (Brenda Williams)
9:00	-	9:30	Aoki, Kazuo
9:40	-	10:10	Desvilletes, Laurent
10:10	-	10:30	<i>Break</i>
10:30	-	11:00	Panferov, Vladislav
11:10	-	12:00	Carlen, Eric
12:00	-	1:00	<i>Break</i>
1:00	-	2:00	<i>Tour of Banff Centre</i>
2:00	-	2:30	Carvalho, Maria
2:40	-	3:10	Kuo, Hung-Wen
3:10	-	3:30	<i>Break</i>
3:30	-	4:00	Nouri, Anne
4:10	-	4:40	<i>Open</i>

Tuesday, June 23, 2009

9:00	-	9:30	Morrison, Philip
9:40	-	10:10	Wennberg, Bernt
10:10	-	10:30	<i>Break</i>
10:30	-	11:00	Schmeiser, Christian
11:10	-	12:00	Mehats, Florian
12:00	-	2:00	<i>Break</i>
2:00	-	2:30	Rein, Gerhard
2:40	-	3:10	Strain, Robert
3:10	-	3:30	<i>Break</i>
3:30	-	4:00	Wagner, Wolfgang
4:10	-	4:40	<i>Open</i>

Wednesday, June 24, 2009

*Schedule Open*

Thursday, June 25, 2009

9:00	-	9:30	Struchtrup, Henning
9:40	-	10:10	Tzavaras, Athanasios
10:10	-	10:30	<i>Break</i>
10:30	-	11:00	Gamba, Irene M.
11:10	-	12:00	Guo, Yan
12:00	-	2:00	<i>Break</i>
2:00	-	2:30	Anton, Arnold
2:40	-	3:10	Matthes, Daniel
3:10	-	3:30	<i>Break</i>
3:30	-	4:00	Vasseur, Alexis
4:10	-	4:40	Agueh, Martial

Friday, June 26, 2009

9:00	-	9:30	<i>Open</i>
9:40	-	10:10	Juengel, Ansgar
10:10	-	10:30	<i>Break</i>
10:30	-	11:00	Liu, Tai-Ping
11:10	-	12:00	Yu, Shih-Hsien
12:00	-		<i>Open</i>

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ABSTRACTS

**AGUEH, MARTIAL**

A variational approach to uniqueness of ground states for certain quasilinear PDEs

We used a variational method based on optimal transport arguments to prove uniqueness of radial ground states for certain quasilinear elliptic equations, and we give the explicit expressions of the solutions. Our variational approach relies on a correspondence between the ground states of these equations and the equilibrium solutions of Fokker-Planck type equations.

**AOKI, KAZUO**

Fluid dynamics for a vapor-gas mixture derived from kinetic theory

When a vapor of a substance is in contact with its condensed phase, evaporation and condensation (or sublimation) take place on the interface between the vapor and the condensed phase. If we try to describe flows of the vapor with evaporation and/or condensation, we have to rely on kinetic theory even in the continuum limit, since the vapor is not in local equilibrium at the interface. In other words, even if the mean free path of the vapor molecules (or the Knudsen number based on it) is very small, we cannot derive correct fluid dynamics by macroscopic considerations. We can construct correct fluid-dynamic systems for small Knudsen numbers (including the continuum limit) only by considering the zero Knudsen number limit and its neighborhood on the basis of kinetic theory. Although such systems had been derived some time ago for a single component system composed of a vapor and its condensed phase, their extension to multi-component systems was made rather recently. Here, we consider the fluid-dynamic system for a vapor in the presence of a noncondensable gas (another component that neither evaporates nor condenses on the interface). Starting from the Boltzmann equation for a binary mixture of gases and its kinetic boundary conditions, we derive a system consisting of fluid-dynamic-type equations and their boundary conditions by a systematic asymptotic analysis for small Knudsen numbers. The type of the fluid-dynamic system is different

depending on the amount of the noncondensable gas contained in the system. In the present talk, we will focus our attention on the case where the amount of the noncondensable gas is of the same order of magnitude as that of the vapor. In this case, the flow speed becomes slow, with Mach number being of the order of the Knudsen number, and the fluid-dynamic-type equations describing this flow contains non-Navier--Stokes terms originating from the thermal stress and concentration stress. The boundary conditions for the fluid-dynamic-type equations contain the velocity slip caused by the temperature gradient along the interface as well as that caused by the concentration gradient there. We show the outline of the derivation of the fluid-dynamic system, together with some numerical examples. The talk is based on the works in collaboration with S. Takata, S. Yasuda, and C.-J. T. Laneryd.

## **ARNOLD, ANTON**

Asymptotically correct finite difference schemes for highly oscillatory ODEs

We are concerned with the numerical integration of ODEs of the form  $\epsilon^2 \psi_{xx} + a(x)\psi = 0$  for given  $a(x) \geq \alpha > 0$  in the highly oscillatory regime  $0 < \epsilon \ll 1$  (appearing as a stationary Schrödinger equation, e.g.). In two steps we derive an accurate finite difference scheme that does not need to resolve each oscillation: 1) With a WKB-ansatz the dominant oscillations are "transformed out", yielding a much smoother ODE. 2) For the resulting oscillatory integrals we devise an asymptotic expansion both in  $\epsilon$  and  $h$ . In contrast to existing strategies, the presented method has (even for a large spatial step size  $h$ ) the same weak limit (in the classical limit  $\epsilon \rightarrow 0$ ) as the continuous solution. Moreover, it has an error bound of the order  $O(\epsilon^2 h^2)$ . Ref: A. ARNOLD, N. BEN ABDALLAH and C. NEGULESCU: WKB-based schemes for the Schrödinger equation in the semi-classical limit, preprint 2009.

## **CARLEN, ERIC**

Entropy and Chaos in the Kac Model

We investigate the behavior in  $N$  of the  $N$ -particle entropy functional for Kac's stochastic model of Boltzmann dynamics, and its relation to the entropy function for solutions of Kac's one dimensional nonlinear model Boltzmann equation. We prove a number of results that bring together the notion of propagation of chaos, which Kac introduced in the context of this model, with the problem of estimating the rate of equilibration in the model in entropic terms. Joint work with Carvalho, Le Roux, Loss and Villani.

## **CARVALHO, MARIA CONCEICAO**

On Strong Convergence to Equilibrium for the Boltzmann Equation with Soft Potentials

The paper concerns  $L^1$ -convergence to equilibrium for weak solutions of the spatially homogeneous Boltzmann Equation for soft potentials  $(-\infty < \gamma < 0)$ , with and without angular cutoff. We prove the time-averaged  $L^1$ -convergence to equilibrium for all weak solutions whose initial data have finite entropy and finite moments up to order greater than  $2 + |\gamma|$ . For the usual  $L^1$ -convergence we prove that the convergence rate can be controlled from below by the initial energy tails, and hence, for initial data with long energy tails, the convergence can be arbitrarily slow. We also show that under the integrable angular cutoff on the collision kernel with  $(-\infty < \gamma < 0)$ , there are algebraic upper and lower bounds on the rate of  $L^1$ -convergence to equilibrium. Our methods of proof are based on entropy inequalities and moment estimates. This is joint work with E. A. Carlen and Xuguang Lee.

## **DESVILLETES, LAURENT**

Collisions in the context of sprays

Sprays are constituted of droplets dispersed in a surrounding gas. Those droplets collide in an inelastic way, and the lost kinetic energy is transformed in internal energy (temperature), which can also be exchanged during collisions. This leads to an original collision kernel which is reminiscent of kernels used in granular media. We describe some mathematical properties of this kernel, and some of the issues related to its numerical implementation. Comments: The short talk that I propose describes works in common with Julien Mathiaud (CEA, Paris).

## **GAMBA, IRENE M.**

Generation and propagation of exponential weighted estimates to solutions of non-linear collisional equations

We focus on non-linear non-conservative collisional models of Boltzmann type associated with the evolution of probability densities in the space homogeneous setting. We will describe the connections between moment angular averaging estimates, conservation properties, moment summability properties, and interactions potentials in the study of generation and propagation of exponentially weighted estimates in  $L^1$  and  $L^\infty$ .

## **GUO, YAN**

Phase Transition in a Vlasov-Boltzmann model for binary fluid

In a joint project with Esposito and Marra, we study the phase transition phenomenon in a Vlasov-Boltzmann model for a binary fluid interacting with repulsive self consistent potential as well as short-range collisions. We establish that below a critical temperature, a spatially non-homogeneous 'front' steady solution is dynamically stable while the homogeneous steady state becomes dynamically unstable. On the other hand, above the critical temperature, the unique homogeneous steady state is stable. Our stability proof relies on a recent  $L^p$ - $L^\infty$  framework which controls the field with a large amplitude, and our instability proof relies on a continuity argument from the Penrose type criterion for the corresponding collisionless Vlasov-Boltzmann model.

## **JÜNGEL, ANSGAR**

Macroscopic quantum models with diffusion

Macroscopic quantum models have been derived by Degond, Mehats, and Ringhofer from the Wigner-BGK equation by a moment method with a quantum Maxwellian closure. This leads to nonlocal quantum diffusion or quantum hydrodynamic equations. In this talk, we consider first two local approximations of these models, the fourth-order and the sixth-order quantum diffusion equations. New results on the global-in-time existence of solutions for the initial-boundary-value problems and the explicit decay rates to equilibrium will be given. The proofs are based on entropy dissipation methods, the algorithmic entropy technique by Juengel and Matthes, and exponential variable transformations. Furthermore, analytical results for quantum hydrodynamic equations with diffusion will be presented.

## **KUO, HUNG-WEN**

Short time behavior for Rayleigh problem

Rayleigh's problem of an infinite flat plate set into uniform motion impulsively in its own plane is studied by using the BKW model, linearized Boltzmann equation and full Boltzmann equation, respectively. For a small impulsive velocity (small Mach number) and short time, the flow behaves like a free molecule flow. Our analysis is based on certain pointwise estimates for the solution of the problem and flow velocity.

## **MATTHES, DANIEL**

Random kinetic models on the real axis

A class of one-dimensional homogeneous Boltzmann equations on  $\mathbb{R}$  is considered. The (random) interaction coefficients inside the collision kernel are designed in such a way that the sum of the particle momenta in individual interactions is not conserved, but the total momentum of the system is. We prove that by adjusting the random coefficients suitably, one can achieve stationary solutions with arbitrarily fat high energy tails. Moreover, we investigate the propagation of regularity and estimate the rate of strong equilibration of transient solutions.

## **MEHATS, FLORIAN**

A new variational approach to the stability of gravitational systems

We consider the three dimensional gravitational Vlasov Poisson system which describes the mechanical state of a stellar system subject to its own gravity. A well-known conjecture in astrophysics is that the steady state solutions which are nonincreasing functions of their microscopic energy are nonlinearly stable by the flow. This was proved at the linear level by Antonov in 1961. Since then, standard variational techniques based on concentration compactness methods as introduced by P.-L. Lions in 1983 have led to the nonlinear stability of subclasses of stationary solutions of ground state type. We propose here a new variational approach based on the minimization of the Hamiltonian under equimeasurable constraints which are conserved by the nonlinear transport flow, and recognize any anisotropic steady state solution which is a decreasing function of its microscopic energy as a local minimizer. The outcome is the proof of its nonlinear stability under radially symmetric perturbations. This work has been done in collaboration with Mohammed Lemou and Pierre Raphael.

## **MORRISON, PHILIP J.**

A discontinuous Galerkin method for the Vlasov-Poisson system

A discontinuous Galerkin method for approximating the Vlasov-Poisson system of equations describing the time evolution of a Vlasov plasma is proposed. The method is mass conservative and, in the case where piecewise constant functions are used as a basis, the method preserves the positivity of the electron distribution function. The performance of the method is investigated by computing five example problems. In particular, computed results are benchmarked against established theoretical results for linear advection and the

phenomenon of linear Landau damping for both the Maxwell and Lorentz distributions. Moreover, a nonlinear two-stream instability problem is computed. It is verified that the method conserves 'enstrophy', mass, momentum, and total energy. A final BGK state is obtained and investigated. Because of the high resolution, interesting details of the trapped particle population can be ascertained. The obtained results demonstrate that the discontinuous Galerkin method accurately approximates the Vlasov-Poisson system.

## **NOURI, ANNE**

Stability for Rayleigh-Benard convective solutions of the Boltzmann equation

Nonlinear stability of a solution to the Boltzmann equation in a Rayleigh-Benard setting is proven. Emphasis is put on the control of the hydrodynamic part of the solution, which is one of the major difficulties of the problem.

## **PANFEROV, VLADISLAV**

On some kinetic models of flocking

I will introduce a kinetic model for a dynamical system involving pairwise interactions and self-propulsion, that can be used to describe certain types of self-organization in biological systems (flocks of birds, schools of fish, swarms of insects...). The kinetic theory approach leads to the identification of macroscopic structures otherwise not recognized as solutions of the hydrodynamic equations, such as double rotating mills. Other macroscopic patterns, for instance spatially localized flocks and single rotating mills can also be obtained as particular steady solutions at the kinetic level.

## **REIN, GERHARD**

Gravitational collapse for the Einstein-Vlasov system

The Einstein-Vlasov system describes in the context of general relativity the time evolution of a large collisionless ensemble of particles which interact only through gravity. We give explicit conditions on regular, spherically symmetric initial data such that the corresponding solutions undergo a gravitational collapse and a black hole forms. In particular, the cosmic censorship conjecture holds for these solutions. Among the data there are such where the solution exists globally in Schwarzschild time.

## **SCHMEISER, CHRISTIAN**

A contribution to hypocoercivity

A simple method will be presented for proving hypocoercivity for kinetic equations involving a confining potential and a linear collision operator with one conservation law. The method is based on the construction of a Lyapunov functional, using coercivity of the collision operator and a spectral gap inequality for the macroscopic limit equation. Examples will be presented, where previously known results are improved (joint work with J. Dolbeault and C. Mouhot).

## **STRAIN, ROBERT**

Recent global results for the relativistic Boltzmann equation

We will discuss some recent stability results regarding global solutions of the relativistic Boltzmann equation coupled with their internally generated electric and magnetic forces. Despite its importance, no global in time solutions have been constructed so far for this Lorentz invariant model. We construct the first global in time classical solutions. This is joint work with Yan Guo.

## **STRUCHTRUP, HENNING**

Beyond Hydrodynamics: Macroscopic Transport Equations for Rarefied Gas Flows

Classical hydrodynamics—the laws of Navier-Stokes and Fourier—fail in the description of processes in rarefied gases. The Boltzmann equation, on the other hand, describes a gas on the microscopic level and gives a proper description for all gas processes; its numerical solution, however, is rather expensive. Macroscopic transport equations can be derived from the Boltzmann equation by averaging in velocity (moment method), and expansion in the Knudsen number (the ratio between the mean free path of a gas particle and a characteristic length of the process). Classical hydrodynamics result from expansion to first order, and higher order expansions promise to describe rarefied gases at lower cost than the Boltzmann equation. It will be shown that higher order Knudsen number expansions give meaningful equations sufficiently away from the wall, while the proper description of Knudsen boundary layers—which are dominant in slow rarefied flows—is not tied to the Knudsen number in a simple manner. Nevertheless, tests with moment systems show that a small number of moments can catch the most important Knudsen layer phenomena for Knudsen numbers below unity in sufficient accuracy. The regularized 13 moment equations are obtained by the order of magnitude method and are of third order accuracy in the bulk, but they also contain enough information to describe Knudsen layers. Analytical and numerical results for Couette, Poiseuille, and Transpiration flows and other processes will give evidence of the above statements.



## **TZAVARAS, ATHANASIOS**

Kinetic hierarchies and macroscopic limits for crystalline steps

We apply methods of kinetic theory to study the passage from particle systems to nonlinear partial differential equations (PDEs) in the context of deterministic crystal surface relaxation. Starting with the near-equilibrium motion of  $N$  line defects ("steps") with atomic size  $a$ , we derive coupled evolution equations ("kinetic hierarchies") for correlation functions,  $F_n^a$ , which express correlations of  $n$  consecutive steps. We investigate separately the evaporation-condensation and the surface diffusion dynamics in 1+1 dimensions when each step interacts repulsively with its nearest neighbors. In the limit  $a \rightarrow 0$  with  $Na = \mathcal{O}(1)$ , where  $a$  is appropriately nondimensional, the first equations of the hierarchies reduce to known evolution laws for the surface slope profile. The remaining PDEs take the form of simple continuity equations, which we solve exactly and thereby connect continuous limits of  $F_n^a$  with the slope profile.

## **VASSEUR, ALEXIS F.**

Global regularity of solutions to systems of reaction-diffusion with Sub-Quadratic Growth in any dimension

In this talk, we present the study of the regularity of solutions to some systems of reaction--diffusion equations, with reaction terms having a subquadratic growth. We show the global boundedness and regularity of solutions, without smallness assumptions, in any dimension  $N$ . The proof is based on blow-up techniques. The natural entropy of the system plays a crucial role in the analysis. It allows us to use of De Giorgi type methods introduced for elliptic regularity with rough coefficients. In spite these systems are entropy supercritical, it is possible to control the hypothetical blow-ups, in the critical scaling, via a very weak norm.

## **WAGNER, WOLFGANG**

Fragmentation models

Fragmentation processes (breaking of various objects into pieces) occur in many sciences and applications such as polymer chemistry, nuclear physics, biology, or mining industry. Kinetic equations are a common tool for describing the behavior of the size distribution of fragmenting particle systems. Solutions of

those equations may be non-conservative (losing mass), if fragmentation rate grows sufficiently fast at zero. This corresponds to a phase transition into dust ("zero size particles"). On the other hand, random fragmentation models can be used both for analytical studies and for numerical purposes. The transformation into dust corresponds to the explosion phenomenon of those models. In the talk we discuss explosion properties of two random fragmentation models based on a general criterion for explosion of jump processes. Some special cases and examples are presented.

### **WENNBERG, BERNT**

A model for sympatric speciation

Sympatric speciation is the process where one species separates into two or more different species which then exist within one common geographical area. I will describe a Markov process that illustrates this phenomenon, and show the results of some computer simulations.

### **YU, SHIH-HSIEN**

Pointwise time-asymptotic stability of a Boltzmann shock profile

The abstract is, "In this talk a scheme to construct the wave propagation around a Boltzmann shock profile will be surveyed. This is a parallel processes to decompose a variable coefficient problems in constant coefficient problems around the far field of a shock wave and an essential scalar equation to analyze the global wave interactions. This reduction relies heavily on the Green's function for the constant coefficient problem so that one can show the convergence of the scheme in a exponential pointwise estimate".