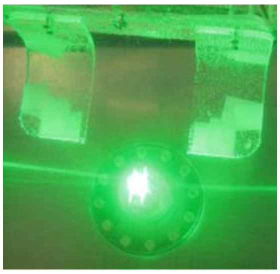
Image Reconstruction in Photoacoustic Tomography taking acoustic attenuation into account



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Banff International Research Station
for Mathematical Innovation and Discovery

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Outline

- Photoacoustic Imaging
- Acoustic attenuation
  - Stokes equation
  - Attenuation in tissue: power law dependence
  - Inversion
- Heat diffusion equation
  - Solution in k-space
  - Inversion
  - Regularization methods
  - Entropy production and information loss
- Conclusion and Outlook

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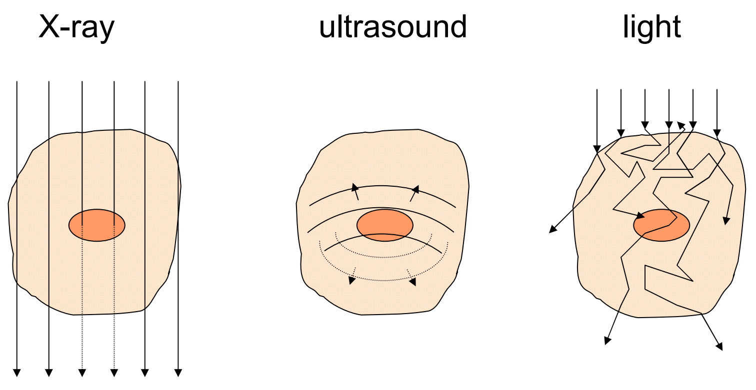
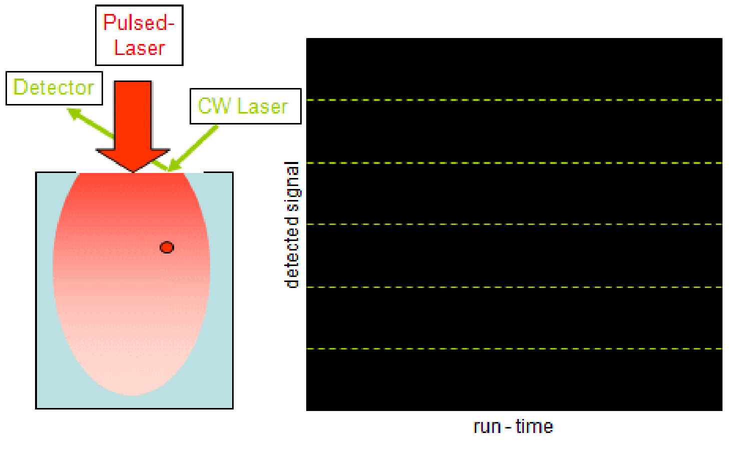
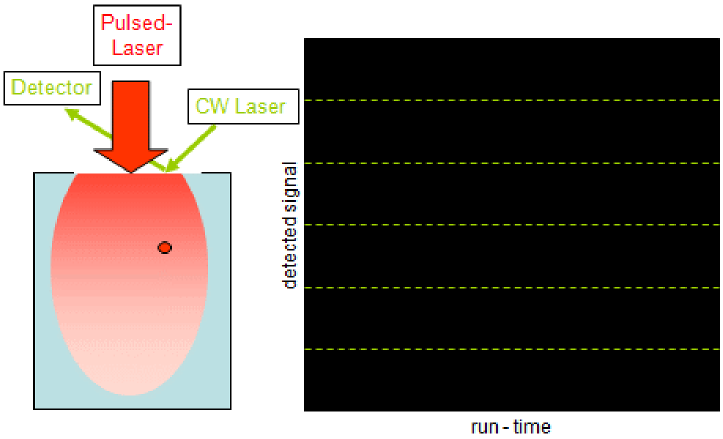
Imaging techniques



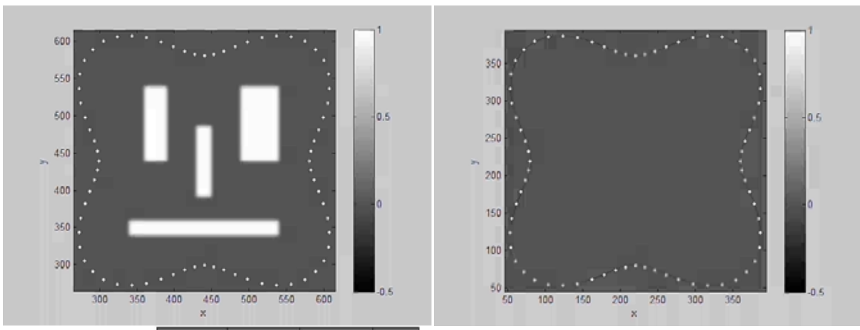
Photoacoustic Imaging

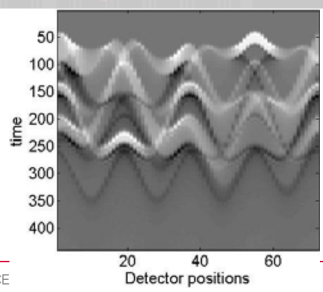


Photoacoustic Imaging



Model based time reversal method





P. Burgholzer, G. J. Matt, M. Haltmeier, and G. Paltauf, "Exact and approximative imaging methods for photoacoustic tomography using an arbitrary detection surface," Physical Review E 75, 2007

Stokes' equation

- plane waves: $p = p_0 e^{i(Kx-\omega t)} = p_0 e^{ikx} e^{-i\omega t} e^{-\alpha x}$
- With complex K(ω) = k(ω) + i α(ω) = ω/c(ω) + i α(ω)
- k(ω), α(ω) have to satisfy Kramers-Krönig-Relations
- e.g. Stokes equation: density change follows pressure change with a relaxation time τ

$$\Delta p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} + \tau\,\Delta\frac{\partial p}{\partial t} = \frac{p_0}{c^2}\frac{\partial \delta(t)}{\partial t}$$

Stokes equation
Relaxation time τ

$$\alpha^2 = \frac{\omega^2}{2c^2}\left(\frac{1}{\sqrt{A}} - \frac{1}{A}\right) \qquad for\ \omega\tau << 1:\quad \alpha \cong \frac{\omega^2\tau}{2c}$$

$$k^2 = \frac{\omega^2}{2c^2}\left(\frac{1}{\sqrt{A}} + \frac{1}{A}\right) \qquad k \cong \frac{\omega}{c}\left(1 - \frac{3}{8}\omega^2\tau^2\right)$$

with $A \equiv 1 + \omega^2\tau^2$

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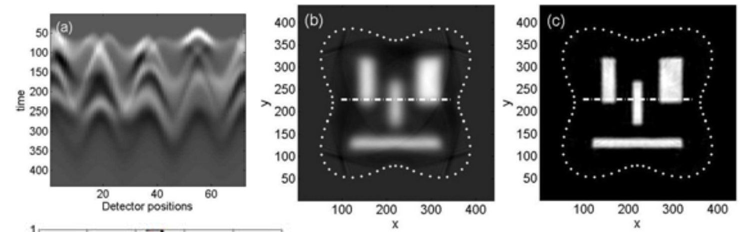
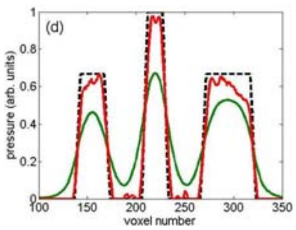
Photoacoustic imaging with time reversal accounting for acoustic attenuation





- (a) Pressure simulation results for τ = 0.2 and c=1
- (b), (c) Reconstructions with time reversal without and with compensation of attenuation
- (d) Reconstruction profile along horizontal dashed line

P. Burgholzer et al., "Compensation of acoustic attenuation for high-resolution photoacoustic imaging with line detectors using time reversal" Proc. SPIE 6437-75, Photonics West, BIOS 2007

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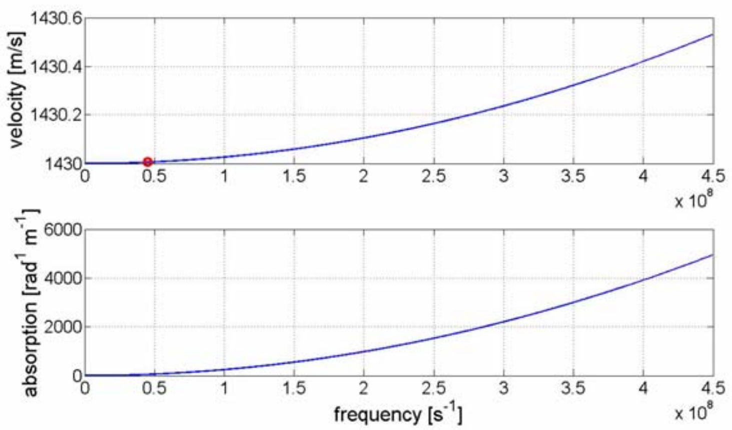
Example for Stokes' equation: Oil

τ = 70 ps

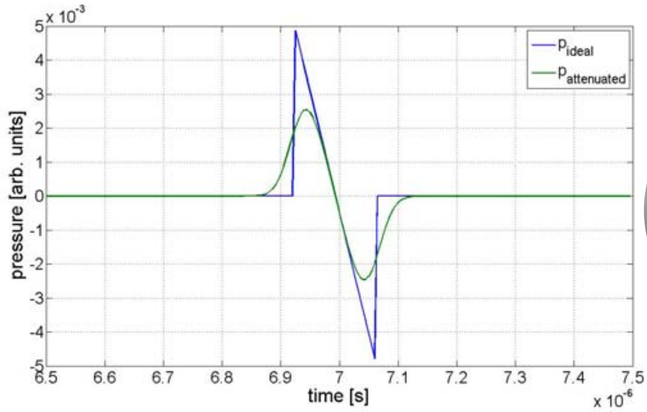


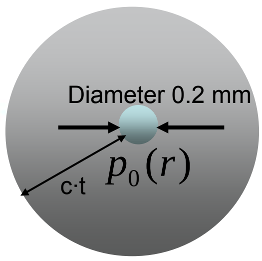
Michael J. Buckingham., "Causality, Stokes' wave equation, and acoustic pulse propagation in a viscous fluid", Phys. Rev. E 72, 2005

PA signals in oil 10 mm from inclusion

$$\tilde{p}(\mathbf{r},\omega)=\frac{\omega}{c_0K(\omega)}\int_{-\infty}^{\infty} p_{ideal}(\mathbf{r},t)\,e^{ic_0K(\omega)t}\,dt$$

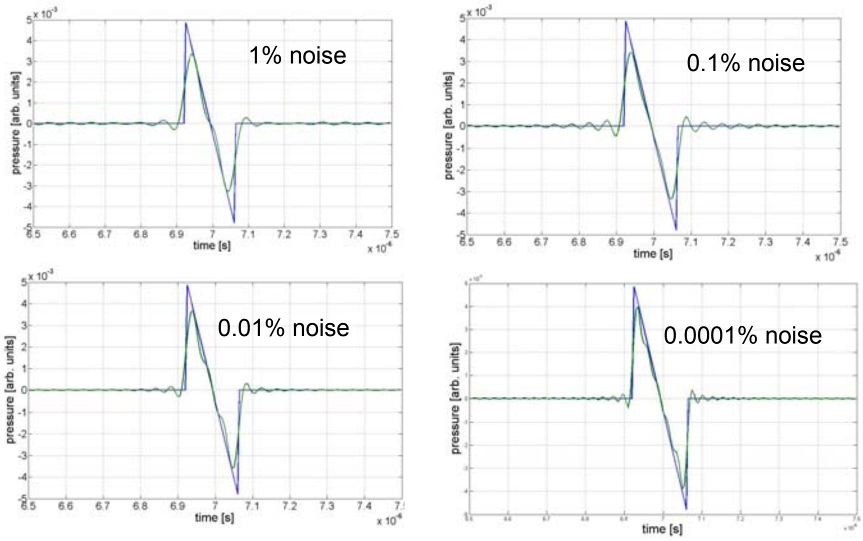
Riviere, Zhang, and Anastasio, Optics Letters (2006).





$p(r,t)$ 3D

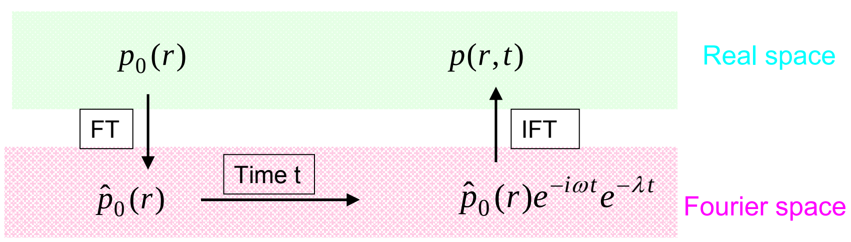
Inversion: regularization with SVD for noisy signals



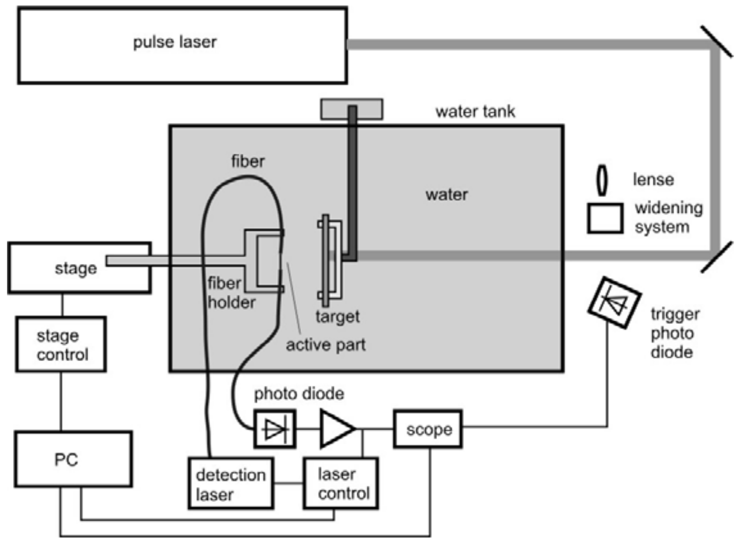
Time damped solutions

Two possible solutions of the wave equation are:

- ω real, K(ω) = k(ω) + i α(ω) = ω/c(ω) + i α(ω) complex, describes a stationary wave damped in space.
- k real, Ω (k) = ω(k) - i λ(k) complex, describes a standing wave (e.g. in a laser resonator) damped in time.



Experimental Determination of Attenuation



Attenuation in tissue

- For tissue: $\alpha(\omega) \approx \alpha_0 |\omega|^y \ with\ y \approx 1$

Kendall R. Waters, Michael S. Hughes, Joel Mobley & James G. Miller; Differential Forms of the Kramers-Krönig Dispersion Relations; IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, Vol. 50, No. 1, January 2003, 68-76

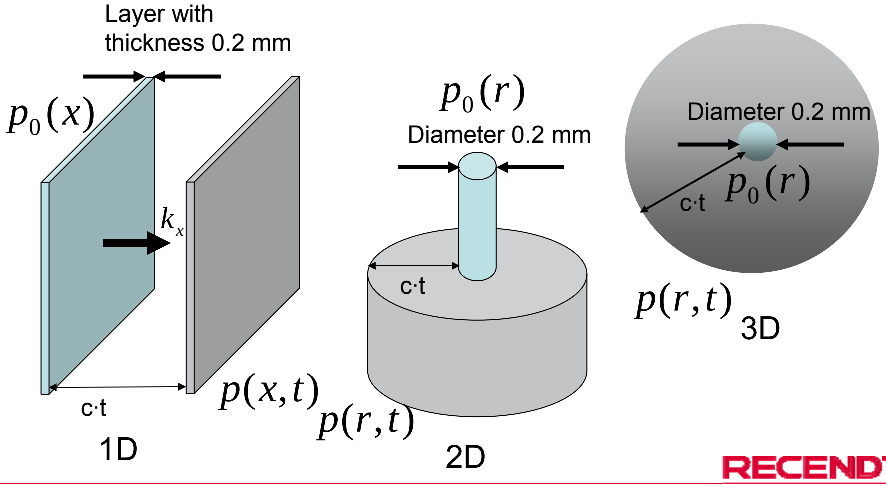
$$\frac{1}{c(\omega)} = \frac{1}{c(\omega_0)} + \alpha_0 \tan\left(\frac{\pi}{2} y\right)\left(|\omega|^{y-1} - |\omega_0|^{y-1}\right)$$

for y=1

$$\frac{1}{c(\omega)} = \frac{1}{c(\omega_0)} - \frac{2}{\pi} \alpha_0 \ln\left|\frac{\omega}{\omega_0}\right|$$

Influence of Attenuation in 1D, 2D and 3D

Attenuation in human fat: 0.6 dB MHz^{-1} cm^{-1} ; detector distance is 10 mm (3 dB MHz^{-1} cm^{-1} in human dermis)



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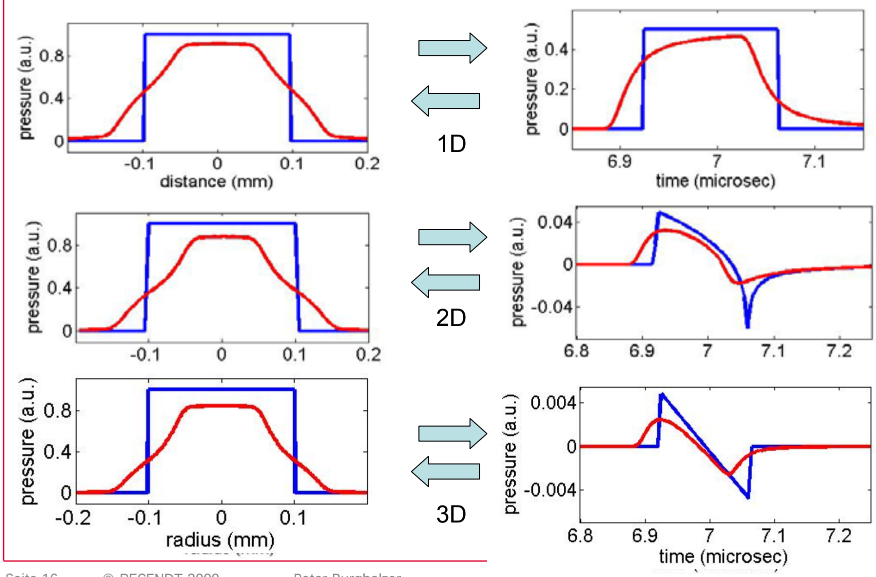
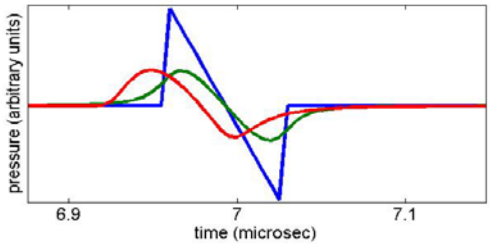
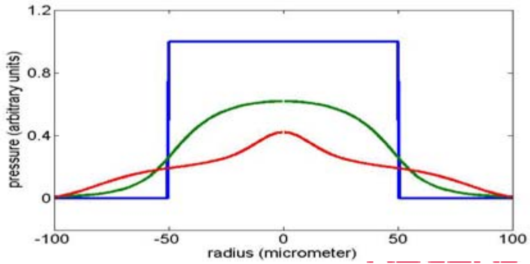
Acoustic attenuation in various dimensions



The influence of dispersion

- Initial pressure distribution: spherical absorber (diameter 0.1mm)
- Simulation results at a distance of 10 mm in human fat neglecting dispersion (green) and taking dispersion into account (red)
- Reconstruction of the initial pressure distribution from above detector signals neglecting dispersion (green) and taking dispersion into account (red)





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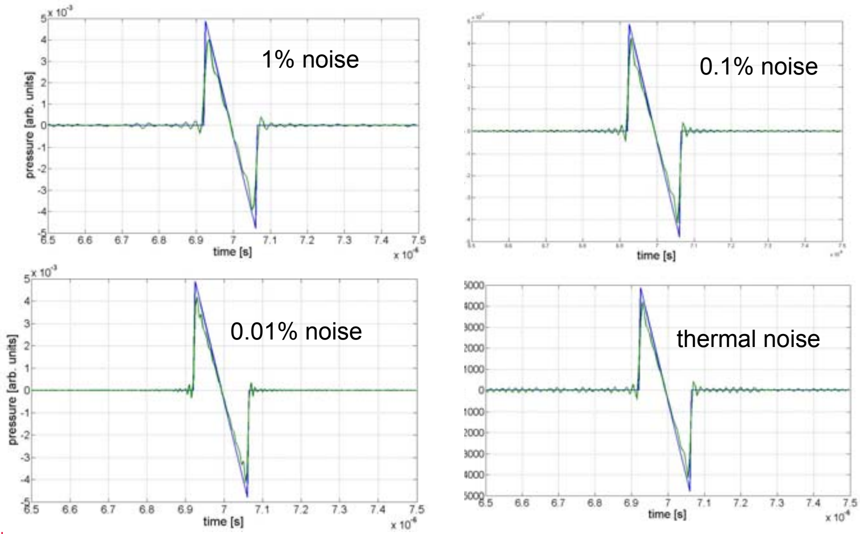
Inversion: regularization with SVD for noisy signals



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1 D heat diffusion equation

$$\frac{\partial}{\partial t} T = \alpha \, \Delta T$$

Fourier 1823,
or e.g. Mandelis et al.
α..thermal diffusivity

Initial values: $T(x,t=0) = T_0(x)$

Neumann boundary conditions: $\frac{\partial}{\partial x} T = 0 \; for \; x=0 \; and \; x=l$

Usually solved by temporal Fourier transform →

Helmholtz equation with solutions: $\tilde{T} \propto \cos(\pm\sqrt{\frac{\omega}{2\alpha}}x - \omega t)\, e^{\mp\sqrt{\frac{\omega}{2\alpha}}x}$

or spatial Fourier transform, (cos-transform) e.g. by Bronstein: $T(x,t) = \sum_{n=0}^{\infty} b_n e^{-k_n{}^2 \alpha t} \cos(k_n x), \; with \; k_n = \frac{\pi n}{l}$

$$b_0 = \frac{1}{l}\int_0^l T_0(x)dx \quad b_n = \frac{2}{l}\int_0^l T_0(x)\cos(kx)dx, \; n = 1,2,3,...$$

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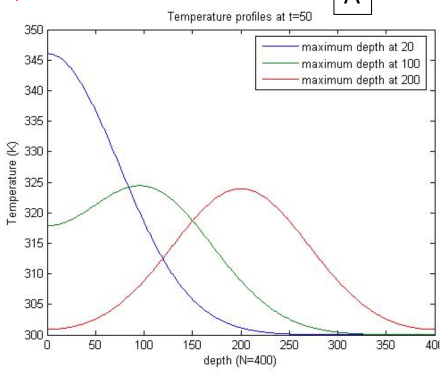
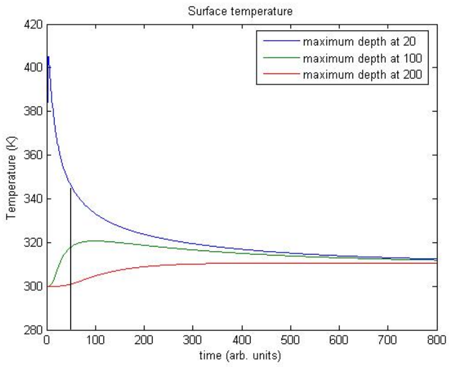
$T(x,t=0) = T_0(x)$ → U → $T(x,t)$ → $T(x=0,t) = T_S(t)$

Cos-transform ↓ b_n → Time t, A → $b_n e^{-k_n{}^2 \alpha t}, \; with \; k_n = \frac{\pi n}{l}$ → Inverse Cos-transform ↑ $T(x,t)$





1 D heat diffusion equation: "time reversal"

$T(x,t=0)=T_0(x)$ ← Inv(U) ← $T(x,t)$

Inverse Cos-transform ↑ ↓ Cos-transform

$b_n e^{+k_n^2 \alpha t}, \; with \; k_n = \frac{\pi n}{l}$ ← Time t, Inv(A) ← b_n

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SVD and Tikhonov regularization method in k-space

$$b_n(t) = e^{-k_n^2 \alpha t} b_n(0), \; with \; k_n = \frac{\pi n}{l}$$

$$\mathbf{b}_t = \mathbf{A}_t \mathbf{b}_0 \qquad \mathbf{A}_t = diag(\exp(-k_n^2 \alpha t))$$

SVD:

$$b_n(0) = \begin{Bmatrix} e^{+k_n^2 \alpha t} b_n(t), \; for \; n \le i \\ 0 \; else \end{Bmatrix}$$

Tikhonov:

$$\min((\mathbf{A}_t \mathbf{b}_0 - \mathbf{b}_t)^2 + \lambda \mathbf{b}_0^2) \Rightarrow \mathbf{A}_t{}^t \mathbf{b}_t = (\mathbf{A}_t{}^t \mathbf{A}_t + \lambda \mathbf{E}) \mathbf{b}_0$$

$$b_n(0) = \frac{e^{-k_n^2 \alpha t}}{e^{-2k_n^2 \alpha t} + \lambda} b_n(t)$$

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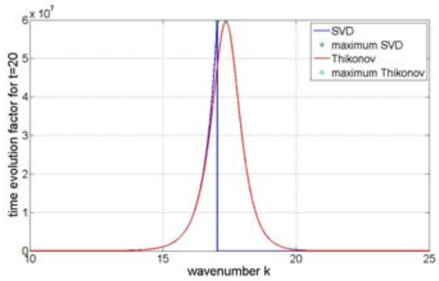
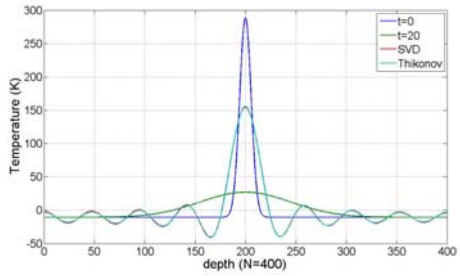
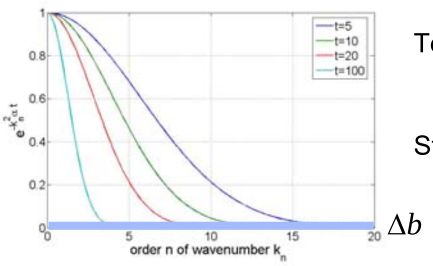
SVD and Tikhonov regularization method in k-space (2)







Temperature (and also pressure) are mean values

Statistical fluctuations: $\left\langle (\Delta T)^2 \right\rangle = \dfrac{k_B T^2}{C}$

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Conclusions

Dissipation causes:

- Entropy production
- Fluctuations: using these as “noise” level the reconstructed image shows a loss of information which is equal to the entropy production (at least for the 1D heat diffusion equation).

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Outlook

- Heat diffusion equation: 2D and 3D
- Pressure waves taking acoustic attenuation into account
- fluctuation – dissipation theorem from statistical physics describes in a very general way how fluctuations and entropy production are related. Therefore it should be possible to generalize the results found for 1 D temperature profiles.

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… thank you for your attention