Banff International Research Station
for Mathematical Innovation and Discovery

Interactions Between Noncommutative Algebra
and Algebraic Geometry
Oct. 26 - 31, 2008

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:50	Jason Bell, "Centralizers in finitely generated algebras and in division algebras"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Daniel Chan, "A non-commutative Mori contraction"
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:10	Group Photo; meet on the front steps of Corbett Hall
14:10–15:00	Daniel Rogalski "Subalgebras of the Sklyanin Algebra"
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:20	Zinovy Reichstein "Essential dimension and algebraic stacks"
16:30–17:20	Brad Shelton "Noncommutative Koszul Algebras from Combinatorics and Topology"
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:50	David Saltman, "Quaternion algebras and their maximal subfields"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Valery Lunts "Categorical resolution of singularities"
11:30–13:30	Lunch
13:30–14:20	Susan Montgommery, "Recent progess in "Invariant Theory for Hopf algebras"
14:30–15:20	Michele D'Adderio "On isoperimetric profiles of algebras"
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	Osamu Iyama, "Cluster tilting in 2-Calabi-Yau categories"
17:30–19:30	Dinner

Wednesday

7:00–8:40	Breakfast
8:40–9:30	Agata Smoktunowicz, "GK-dimension of factor algebras of Golod-Shafarevich algebras"
9:40–10:30	Toby Stafford, "Applications of noncommutative geometry to cherednik algebras"
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:50	Birge Huisgen-Zimmermann "Generic representations of quivers with relations"
11:50–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:50	Ulrich Kraehmer "On the Hochschild (co)homology of quantum homogeneous spaces"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Max Lieblich "The period-index problem for surfaces over finite fields"
11:30–13:30	Lunch
13:30–14:20	Susan Sierra "The classification of birationally commutative surfaces"
14:30–15:20	Quanshui Wu "Non-commutative Castelnuovo-Mumford Regularity and AS-regular Algebras"
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	Hokuto Uehara "Tilting generators via ample line bundles"
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:50	Informal discussion
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Informal discussion
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Jason Bell** (Simon-Fraser University)
Title: *Centralizers in finitely generated algebras and in division algebras*
Abstract: We look at the problem of describing centralizers in algebras and in division algebras. Our main result is that if A is a finitely generated complex noetherian algebra of GK dimension strictly less than 4 then the centralizer of a non-scalar element satisfies a polynomial identity. We also look at the connection between transcendence degree of subfields and the size of centralizers in division algebras and formulate a few conjectures.

Speaker: **Daniel Chan** (University of New South Wales)
Title: *A non-commutative Mori contraction*
Abstract: One of M. Artin's conjectures can be loosely stated as: a non-commutative surface is, up to birational equivalence, either ruled or finite over its centre. This suggests that it would be nice to have a criterion for a non-commutative surface to be ruled. In the commutative case, there is a criterion based on Mori theory. Given a K-negative extremal ray C in the cone of curves with self-intersection zero, the surface is ruled. In this talk, we discuss the possibility of a non-commutative version of this result. We show that given a non-commutative smooth projective surface (appropriately defined) which contains an object like the ray C above, there is a non-commutative "Mori contraction" to a curve. This is a report on joint work with Adam Nyman.

Speaker: **Michele D'Adderio** (University of California at San Diego)
Title: *On isoperimetric profiles of algebras*
Abstract: Isoperimetric profile in algebras was first introduced by Gromov. In this talk we show the behavior of the isoperimetric profile under various ring theoretic constructions and its relation with amenability. We show that the isoperimetric profile is a finer invariant than the lower transcendence degree, and we use it to answer a question of J. J. Zhang.

Speaker: **Birge Huisgen-Zimmermann** (University of California at Santa Barbara)
Title: *Generic representations of quivers with relations*
Abstract: The irreducible components of varieties parametrizing the finite dimensional representations of a finite dimensional algebra A are explored, in terms of both their geometry and the structure of the modules they encode; expected close connections between the two aspects are rendered more explicit. In particular, we establish the existence and uniqueness (not up to isomorphism, but in a strong sense to be specified) of *generic modules*, that is, of modules which display <u>all</u> categorically defined generic properties of the modules parametrized by a given irreducible component. Our approach to existence is largely constructive, by way of minimal projective presentations. We follow with an investigation of the properties of such generic modules with regard to quiver and relations of A. The sharpest specific results on all fronts

are obtained for truncated path algebras, that is, path algebras of quivers modulo ideals generated by all paths of a fixed length.

Speaker: **Osamu Iyama** (Nagoya University)
Title: **Cluster tilting in 2-Calabi-Yau categories** Abstract: Cluster tilting theory reveals combinatorial structure of 2-Calabi-Yau triangulated categories, and is applied to categorify Fomin-Zelevinsky cluster algebras by many authors (Buan, Marsh, Reineke, Reiten Todorov, Caldero, Chapoton, Schiffler, Keller,...). In my talk, we will introduce cluster tilting theory in 2-Calabi-Yau triangulated category. In particular, a combinatorial description of the change of endomorphism algebras of cluster tilting objects via mutation process is given in terms of Fomin-Zelevinsky quiver mutation rule and Derksen-Weyman-Zelevinsky quiver with potential mutation rule.

Speaker: **Ulrich Kraehmer** (University of Glasgow)
Title: *On the Hochschild (co)homology of quantum homogeneous spaces*
Abstract: In this talk I will speak about a generalisation of a result by Brown and Zhang establishing Poincare duality in Hochschild (co)homology for a class of algebras that can be considered as noncommutative analogues of affine homogeneous spaces.

Speaker: **Max Lieblich** (Princeton University)
Title: *The period-index problem for surfaces over finite fields*
Abstract: I will discuss the period-index problem for Brauer groups of fields of transcendence degree 2 over finite fields, and how stacky techniques relate this problem to the geometry of moduli spaces of vector bundles and to the Hasse principle for geometrically rational varieties over global fields.

Speaker: **Valery Lunts** (Indiana University)
Title: *Categorical resolution of singularities*
Abstract: This is my work in progress. I want to propose the notion of a categorical resolution of singularities, compare it with the usual resolutions in algebraic geometry and discuss some examples and conjectures.

Speaker: **Susan Montgomery** (University of Southern California)
Title: *Recent progess in "Invariant Theory" for Hopf algebras*
Abstract: Let H be a finite-dimensional Hopf algebra acting on an algebra A over a field k. Recently a number of questions about the relationship between A, its subalgebra of invariants A^H, and the semi-direct product $A\#H$, open since the 1970's and 1980's, have been solved. For example: (1) if $H*$ is semisimple, then the Jacobson radical is always H-stable (this is work of Linchenko); (2) if H is semisimple and A is H-semiprime, then any H-stable left or right ideal of A intersects A^H non-trivially. I will survey some of these results and discuss their proofs.

Speaker: **Zinovy Reichstein** (University of British Columbia)
Title: *Essential dimension and algebraic stacks*
Abstract: The essential dimension of an algebraic object (e.g., of an algebra, a quadratic form, or an algebraic variety) is the minimal number of independent parameters required to define the underlying structure. This numerical invariant has been studied by a variety of algebraic, geometric and cohomological techniques. In this talk, based on joint work with P. Brosnan and A. Vistoli, I will discuss a new approach based on the notion of essential dimension for an algebraic stack.

Speaker: **Daniel Rogalski** (University of California at San Diego)
Title: *Subalgebras of the Sklyanin Algebra*
Abstract: We study subalgebras of the 3-dimensional sklyanin algebra S, particularly those generated in degree 3. We classify such algebras which are also maximal orders. Geometrically, each such algebra

behaves like a blowup of the Sklaynin projective plane along a divisor of degree at most 7 on the elliptic curve.

Speaker: **David Saltman** (University of Texas at Austin) Title: *Quaternion algebras and their maximal subfields*
Abstract: Using the generic splitting field of Amitsur, we know that two division algebras with the same splitting fields are quite close: powers of each other in the Brauer group. However, if you restrict to finite dimensional splitting fields, or even maximal subfields, the situation is much different. Over global fields, two division algebras of degree greater than 2 can have the same finite dimensional splitting fields and not be powers of each other. However, two quaternion algebras over a global field with the same maximal subfields are isomorphic. Several people asked whether this is true in general. We will show that if you assume the center field F has zero unramified Brauer group, and D/F, D'/F are quaternion algebras with the same maximal subfields, then $D \cong D'$. We will also discuss a generalization to higher cohomology due to Skip Garibaldi.

Speaker: **Brad Shelton** (University of Oregon)
Title: *Noncommutative Koszul Algebras from Combinatorics and Topology*
Abstract: We consider a construction given by Gelfand, Retakh, Serconek and Wilson, that builds non-commutative graded quadratic algebras from finite layered graphs. A mistake in the literature suggests that all such algebras are Koszul. We give some results on the Koszul property of these algebras when the associated graph is related to a regular CW-complex.

Speaker: **Susan Sierra** (University of Washinton)
Title: *The classification of birationally commutative surfaces.*
Abstract: We give a complete classification of birationally commutative projective surfaces (connected N-graded noetherian domains of GK-dimension 3 that are birational to a commutative surface in Artin's sense), and show that all such algebras fall into four families determined by geometric data. We relate the geometry of the underlying data and the algebraic properties of the algebras, and discuss potential generalizations to higher dimensions. This extends results of Rogalski and Stafford in the case that the algebra is generated in degree 1.

Speaker: **Agata Smoktunowicz** (University of Edinburgh)
Title: *GK-dimension of factor algebras of Golod-Shafarevich algebras*
Abstract: It is known that Golod-Shafarevich algebras have exponential growth. In this talk it is shown that also all non-nilpotent factor rings of generic Golod-Shafarevich algebras over fields of infinite transcendence degree have exponential growth, provided that the number of defining relations of degree less then n grows exponentially with n. This answers a question stated by Efim Zelmanov in the paper "Some open problems in the theory of infinite dimensional algebras".

Speaker: **Toby Stafford** (University of Manchester)
Title:*Applications of noncommutative geometry to cherednik algebras*
Abstract: This talk will report on joint work with Victor Ginzburg and Iain Gordon. We establish a link between two geometric approaches to the representation theory of rational Cherednik algebras of type A: one based on noncommutative geometry; the other involving quantum hamiltonian reduction of an algebra of differential operators. This link is achieved by showing that the process of hamiltonian reduction intertwines a naturally defined geometric twist functor on D-modules with the shift functor for the Cherednik algebra. If I have time I will also apply this to the structure of characteristic cycles of modules over the Cherednik algebra.

Speaker: **Hokuto Uehara** (Tokyo Metropolitan University)
Title: *Tilting generators via ample line bundles*

Abstract: We construct tilting generators by ample line bundles under some assumptions. My talk is based on a joint paper arXiv:0804.4256 with Yukinobu Toda.

Speaker: **Quanshui Wu** (Fudan University)
Title: *Non-commutative Castelnuovo-Mumford Regularity and AS-regular Algebras*
Abstract: Let A be a connected graded k-algebra generated in degree 1, with a balanced dualizing complex. I will talk about the Castelnuovo-Mumford regularity and the Ext-regularity and prove that these regularities coincide for all finitely generated A-modules if and only if that A is a Koszul AS-regular algebra. By using Castelnuovo-Mumford regularity, we also prove that any Koszul standard AS-Gorenstein algebra is AS-regular.