Fluid models for complex systems

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Summary

1. Introduction

Complex system

- System with interacting agents without leaders
  - Spontaneous emergence of spatio-temporal coordination
  - morphogenesis

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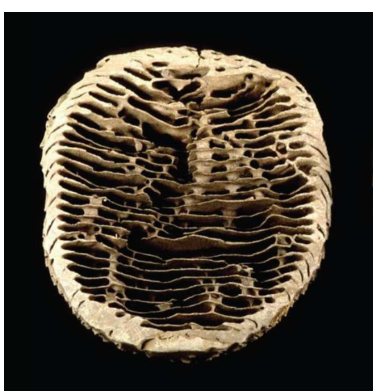

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- Classical micro-macro approach is bottom-up
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  - build macro models for large systems
- Complex systems require top-down approach
  - From macro models build macro observables
  - and test hypotheses about micro interactions
  - use model and data together to extract information

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  - Phase transitions can be encoded more easily
- This talk: micro-macro passage for two models
  - Vicsek (alignement interaction)
  - Persistent Turning Walker

2. From particles to mean-field model

Couzin-Vicsek model

- Alignement interaction ('moving spins')
  - Discrete model
  - X_k^n: position of k-th individual at time $t^n = n\Delta t$
  - ω_k^n: velocity with $|\omega_k^n| = 1$

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- During each Δt:
  - Particle moves a distance $\omega_k^n \, \Delta t$
  - ω_k^n changed to ω_k^{n+1}

    $=$ direction $\bar{\omega}_k^n$ of average neighbours' velocity

    $+$ noise
  - Noise accounts for inaccuracy of the perceptive system

➠ [Vicsek et al, PRL 95]:

$$X_k^{n+1} = X_k^n + \omega_k^n \, \Delta t$$

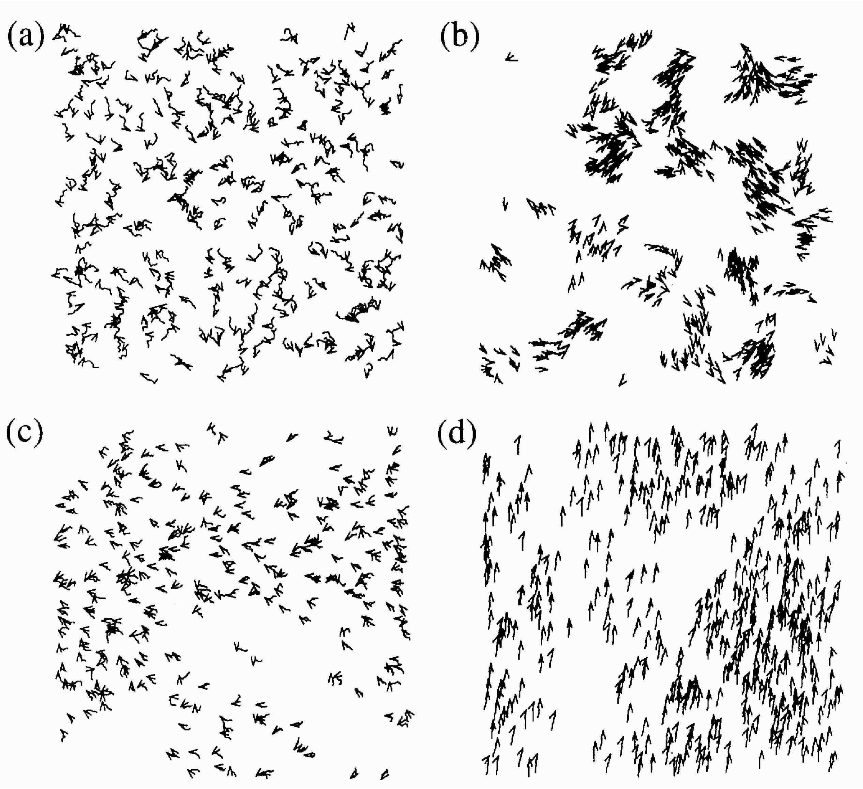
$$\omega_k^{n+1} = \bar{\omega}_k^n + \text{ noise}$$

$$\bar{\omega}_k^n = \frac{J_k^n}{|J_k^n|}, \qquad J_k^n = \sum_{j, \, |X_j^n - X_k^n| \leq R} \omega_j^n$$

noise $=$ uniform for angle in interval $[-\sigma, \sigma]$ in 2D

Phase transition

- Model shows 2 regimes [Vicsek et al, PRL 95]
  - Disorganized / Aligned
  - Phase transition to disorder



Time scale separation

- Two time scales are collapsed
  - Discretization step Δt and Mean interaction time τ

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  - Discretization step Δt and Mean interaction time τ
- After separating theses two time scales:

$$\frac{\omega_k^{n+1} - \omega_k^n}{\Delta t} = \frac{1}{\tau} (\mathrm{Id} - \omega_k^{n+1/2} \otimes \omega_k^{n+1/2})(\bar{\omega}_k^n - \omega_k^n) + \text{noise}$$

$$\omega_k^{n+1/2} = \frac{\omega_k^{n+1} + \omega_k^n}{|\omega_k^{n+1} + \omega_k^n|}$$

$$\bar{\omega}_k^n = \frac{J_k^n}{|J_k^n|}, \quad J_k^n = \sum_{j, |X_j^n - X_k^n| \leq R} \omega_j^n$$

- Letting $\Delta t \to 0$ gives

$$\dot{X}_k(t) = \omega_k(t)$$

$$d\omega_k(t) = (\mathrm{Id} - \omega_k \otimes \omega_k)(\nu(\bar{\omega}_k - \omega_k)dt + \sqrt{2D}dB_t)$$

$$\bar{\omega}_k = \frac{J_k}{|J_k|}, \quad J_k = \sum_{j, |X_j - X_k| \leq R} \omega_j$$

$$\nu = \tau^{-1} = \text{ interaction frequency}$$

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- $f(x, \omega, t)\, dx\, d\omega =$ probability of finding a particle in $dx\, d\omega$ at time t
  - satisfies a Fokker-Planck equation

Mean-field model

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$$\partial_t f + \omega \cdot \nabla_x f + \nabla_\omega \cdot (F f) = D \Delta_\omega f$$

$$F = \nu(\mathrm{Id} - \omega \otimes \omega)\bar{\omega}$$

$$\bar{\omega} = \frac{J}{|J|}, \quad J = \int_{|y-x|\leq R, |v|=1} v f(y, v, t)\, dy\, dv$$

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- Choice of time scale: $\nu = 1$

Rescaled mean-field model

- Passage to macroscopic time and space scales
  - $\tilde{x} = \varepsilon x, \quad \tilde{t} = \varepsilon t \quad$ with $\quad \varepsilon \ll 1$
  - Interaction radius is microscopic: $\quad \tilde{R} = \varepsilon R$

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$$\varepsilon(\partial_t f^\varepsilon + \omega \cdot \nabla_x f^\varepsilon) + \nabla_\omega \cdot (F^\varepsilon f^\varepsilon) = D \Delta_\omega f^\varepsilon$$

$$F^\varepsilon = (\mathrm{Id} - \omega \otimes \omega)\bar{\omega}^\varepsilon$$

$$\bar{\omega}^\varepsilon = \frac{J^\varepsilon}{|J^\varepsilon|}, \quad J^\varepsilon = \int_{|y-x| \leq \varepsilon R, \, |\upsilon|=1} \upsilon f^\varepsilon(y, \upsilon, t) \, dy \, d\upsilon$$

Equivalent mean-field model

➠ Expansion gives

$$\bar{\omega}^\varepsilon = \Omega^\varepsilon + O(\varepsilon^2)$$

$$\Omega^\varepsilon = \frac{j^\varepsilon}{|j^\varepsilon|}, \quad j^\varepsilon = \int_{|v|=1} v f^\varepsilon(x, v, t)\, dv$$

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- Rescaled model equivalent (up to HOT) to

$$\varepsilon(\partial_t f^\varepsilon + \omega \cdot \nabla_x f^\varepsilon) + \nabla_\omega \cdot (F_0^\varepsilon f^\varepsilon) = D \Delta_\omega f^\varepsilon$$

$$F_0^\varepsilon = (\mathrm{Id} - \omega \otimes \omega)\Omega^\varepsilon$$

3. From mean-field model to 'hydrodynamics'

Collision operator

- Model can be written

$$\partial_t f^\varepsilon + \omega \cdot \nabla_x f^\varepsilon = \frac{1}{\varepsilon} Q(f^\varepsilon)$$

with 'collision operator'

$$Q(f) = -\nabla_\omega \cdot (F_f\, f) + D \Delta_\omega f$$

$$F_f = (\mathsf{Id} - \omega \otimes \omega) \Omega_f$$

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- Problem: find the formal limit $\varepsilon \to 0$ of this model

1st step: find the equilibria

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  - For any arbitrary Ω, $\exists$ a unique normalized solution $f = M_\Omega$ s.t. $\Omega_f = \Omega$

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$$M_\Omega(\omega) = C_D \exp \frac{(\omega \cdot \Omega)}{D}, \qquad \int M_\Omega(\omega)\, d\omega = 1$$

Equilibria

➠ $Q(f)$ can be written

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$$H(f) = \int Q(f) \frac{f}{M_{\Omega_f}} \, d\omega = -D \int M_{\Omega_f} \left| \nabla_\omega \left(\frac{f}{M_{\Omega_f}} \right) \right|^2 \leq 0$$

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- $\mathcal{E} = \{ \rho M_\Omega(\omega)$ for arbitrary $\rho \in \mathbb{R}_+$ and $\Omega \in \mathbb{S}^2 \}$ (or $\mathbb{S}^1$ in dim 2)
  - dim $\mathcal{E} = 3 \quad (= 2$ in dim 2)

Limit of f^ε

- Particular cases:
  - $D = 0$ (no noise): all particles concentrate on velocity $\omega = \Omega$: $\quad M_\Omega(\omega) = \delta(\omega, \Omega)$
  - $D = \infty$ (large noise): velocity distribution is isotropic: $M_\Omega(\omega) = 1/4\pi \quad (= 1/2\pi$ in dim = 2)

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- Problem: find the dependence of ρ and $\Omega(x, t)$ upon (x, t)

Collision invariant (conserved quantity)

- Function $\psi(\omega)$ such that

$$\int Q(f)\psi \, d\omega = 0, \quad \forall f$$

  - Form a vector space $\mathcal{C}$

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- Use:
  - Multiply eq. by ψ: ε^{-1} term disappears
  - Find a conservation law
  - Problem fully determined if dim $\mathcal{C} =$ dim $\mathcal{E}$

Lack of collision invariants

- Here dim $\mathcal{C} = 1$ because $\mathcal{C} = \text{Span}\{1\}$
  - dim $\mathcal{E} = 3 >$ dim $\mathcal{C} = 1$
  - Only conservation of mass

$$\partial_t \rho + \nabla_x \cdot (c_1 \rho \Omega) = 0, \quad c_1 = |j_{M_\Omega}| < 1$$

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- Is the limit problem ill-posed ?
  - Answer = no
  - find eq. for Ω by weekening the concept of collision invariant

Generalized collision invariant

- Given Ω, find ψ_Ω a GCI, such that

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- $\vec{\psi}_\Omega(\omega) = \frac{\Omega\times\omega}{|\Omega\times\omega|} g(\Omega\cdot\omega)$ with $g(\mu)$ sol. of an elliptic eq.:

$$-(1-\mu^2)\partial_\mu(e^{\mu/d}(1-\mu^2)\partial_\mu g) + e^{\mu/d} g = -(1-\mu^2)^{3/2} e^{\mu/d}$$

Use of generalized collision invariant

- Multiply eq. by $\vec{\psi}_{\Omega_{f^\varepsilon}}$
  - $O(\varepsilon^{-1})$ terms disappear
  - Let $\varepsilon \to 0$: $\vec{\psi}_{\Omega_{f^\varepsilon}} \to \vec{\psi}_{\Omega}$
  - Get eq.

$$\int (\partial_t(\rho M_\Omega) + \omega \cdot \nabla_x(\rho M_\Omega)) \, \vec{\psi}_\Omega \, d\omega = 0$$

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- Not a conservation equation because of dependence of $\vec{\psi}_\Omega$ upon Ω

➠ $\rho(x,t)$ and $\Omega(x,t)$ evolve according to

$$\partial_t \rho + \nabla_x \cdot (c_1 \rho \Omega) = 0$$

$$\rho \, (\partial_t \Omega + c_2 (\Omega \cdot \nabla) \Omega) + D \, (\mathsf{Id} - \Omega \otimes \Omega) \nabla_x \rho = 0$$

$$|\Omega| = 1$$

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$$|\Omega| = 1$$

- c_2 defined as a particular moment of the GCI
  - $c_2 < c_1$

4. Properties of the hydrodynamic model

➠ By time rescaling

$$\partial_t \rho + \nabla_x \cdot (\rho \Omega) = 0$$
$$\rho \left(\partial_t \Omega + c (\Omega \cdot \nabla) \Omega \right) + d \, (\mathsf{Id} - \Omega \otimes \Omega) \nabla_x \rho = 0$$
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where $c = c_2 / c_1 < 1$, $\quad d = D / c_1$

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  - Non-conservative terms arise from the constraint

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$$
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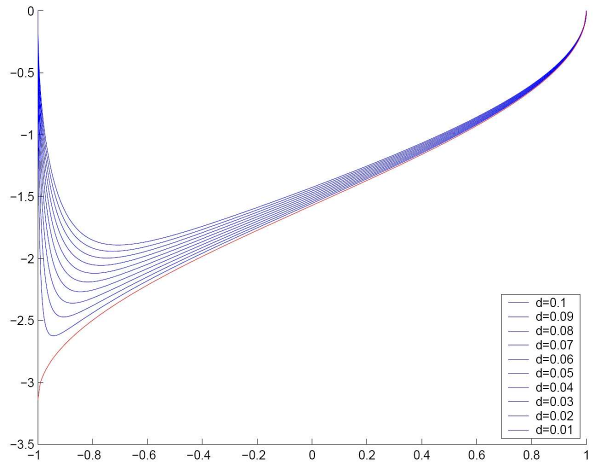
\end{aligned}
$$

where $c = c_2 / c_1 < 1, \quad d = D / c_1$

- Hyperbolic model with constraint
  - Non-conservative terms arise from the constraint
- Velocity waves are slower than density waves
  - Similar situation to traffic

Numerical simulation: GCI



Function g/D as a function of $\omega \cdot \Omega$ for small values of D

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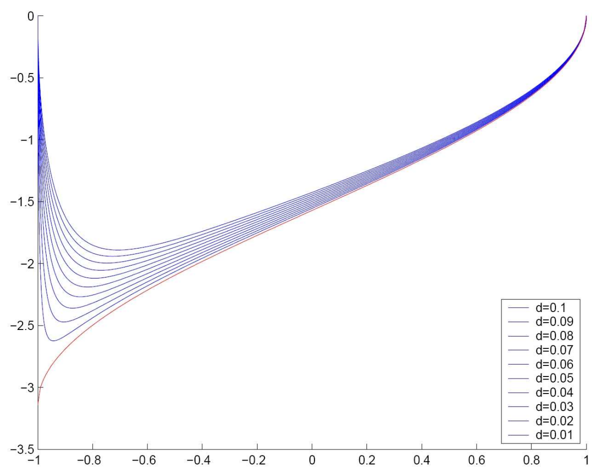

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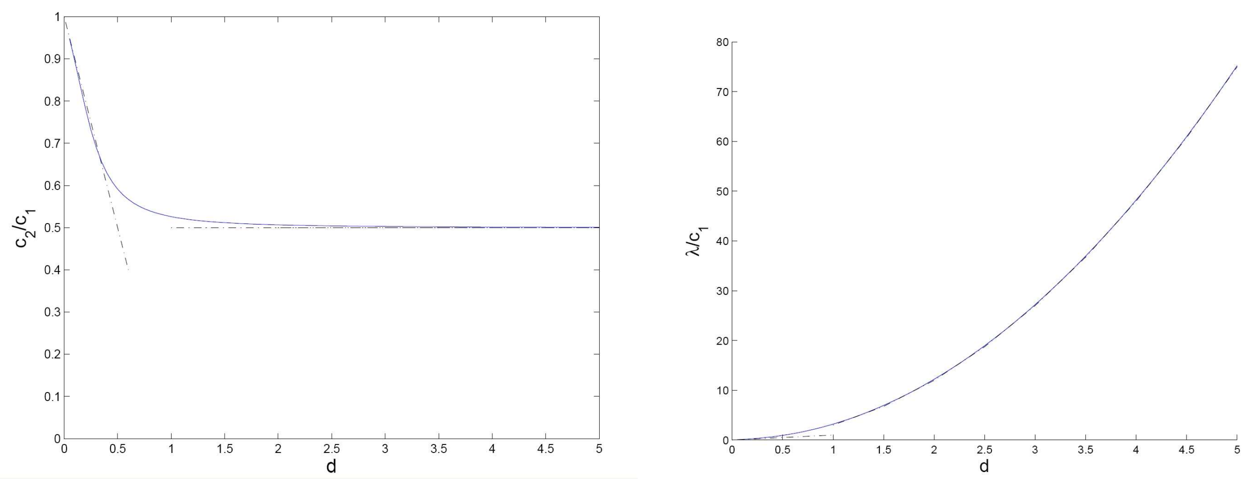
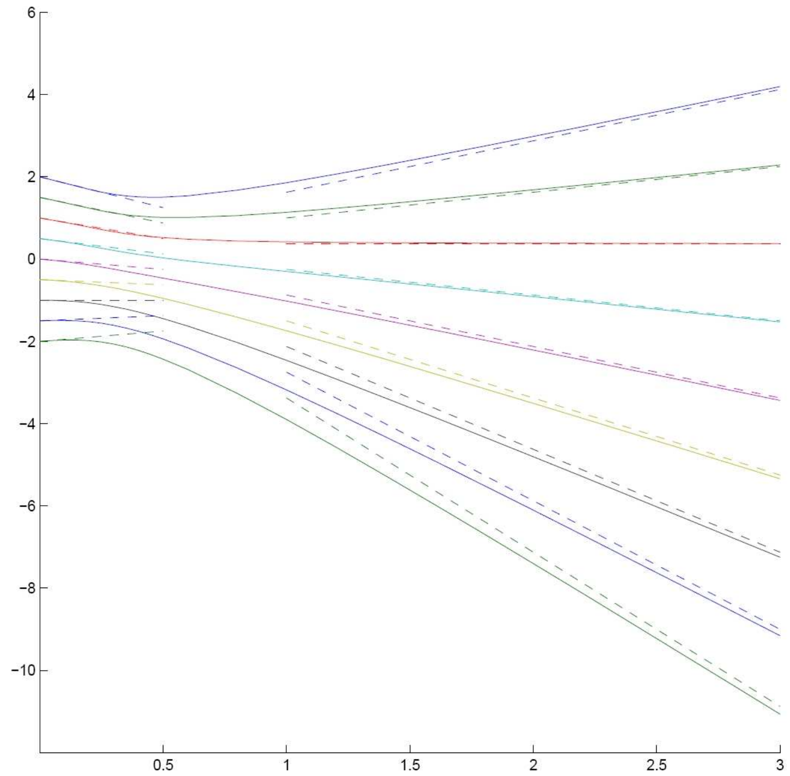


c and d as a function of noise level D

With cone of vision



c as a function of noise level D for various apertures of vision cone (2D case)

The more forward individuals look, the more backwards velocity waves propagate

Mills are stationary solutions

- Mills: $\rho = \rho(r)$, $\Omega = x^\perp / r$
  - are solutions of macro CVA model iff:

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{\frac{c}{d}}$$

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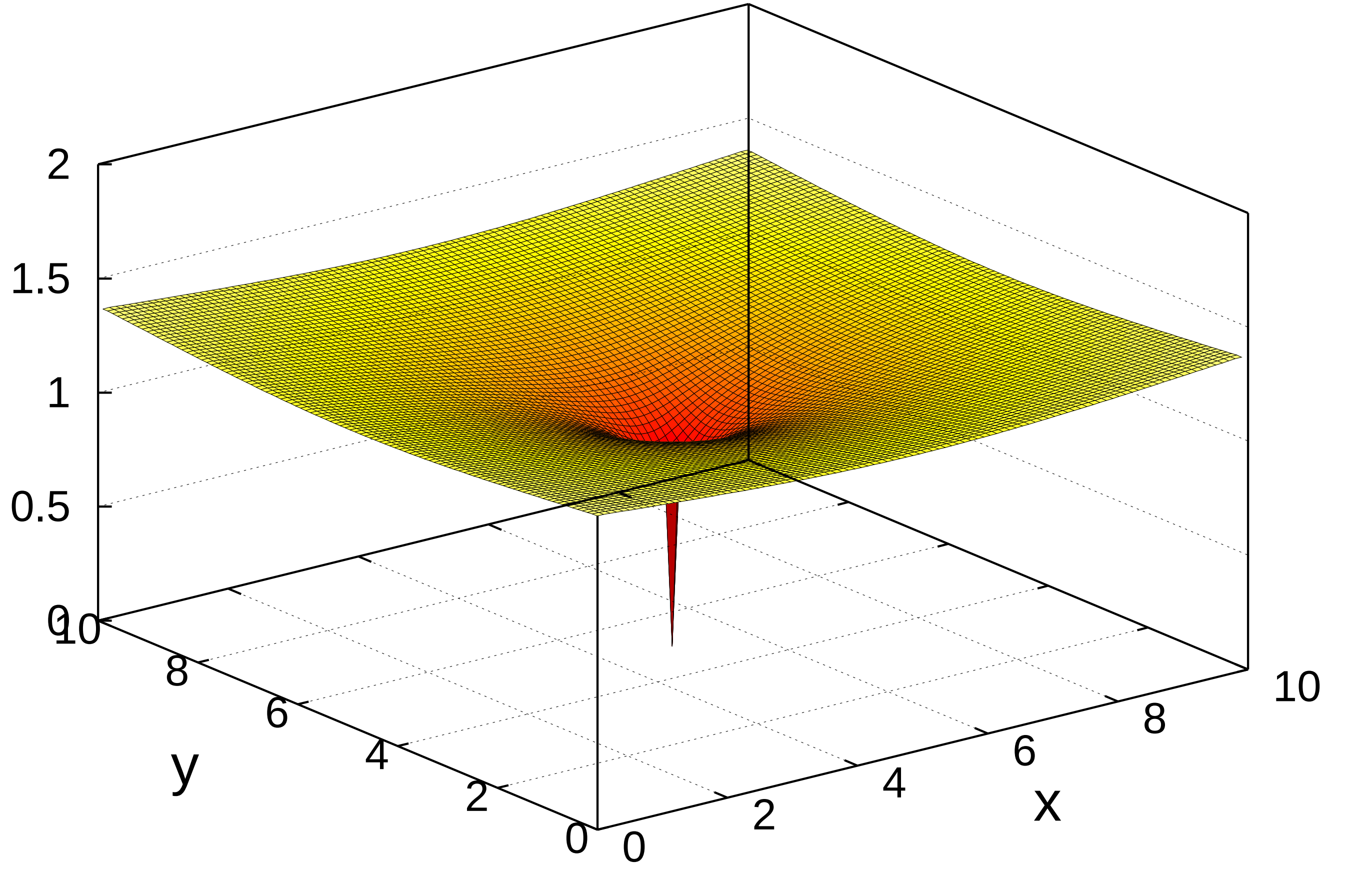
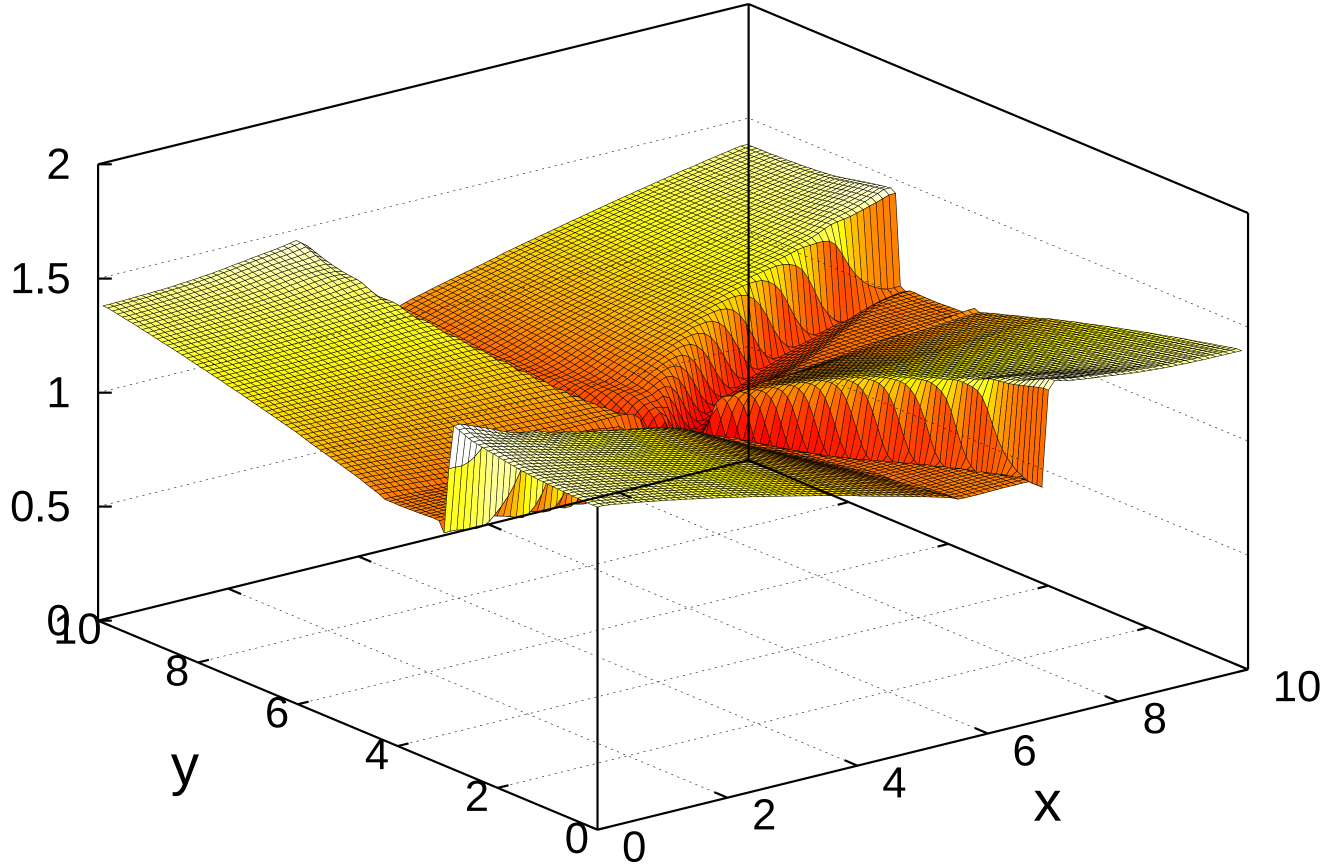
- Shape depends on noise level
  - Small noise: ρ convex function of r: sharp edged mills
  - Large noise: ρ concave function of r: fuzzy edges

Mills: numerical solutions



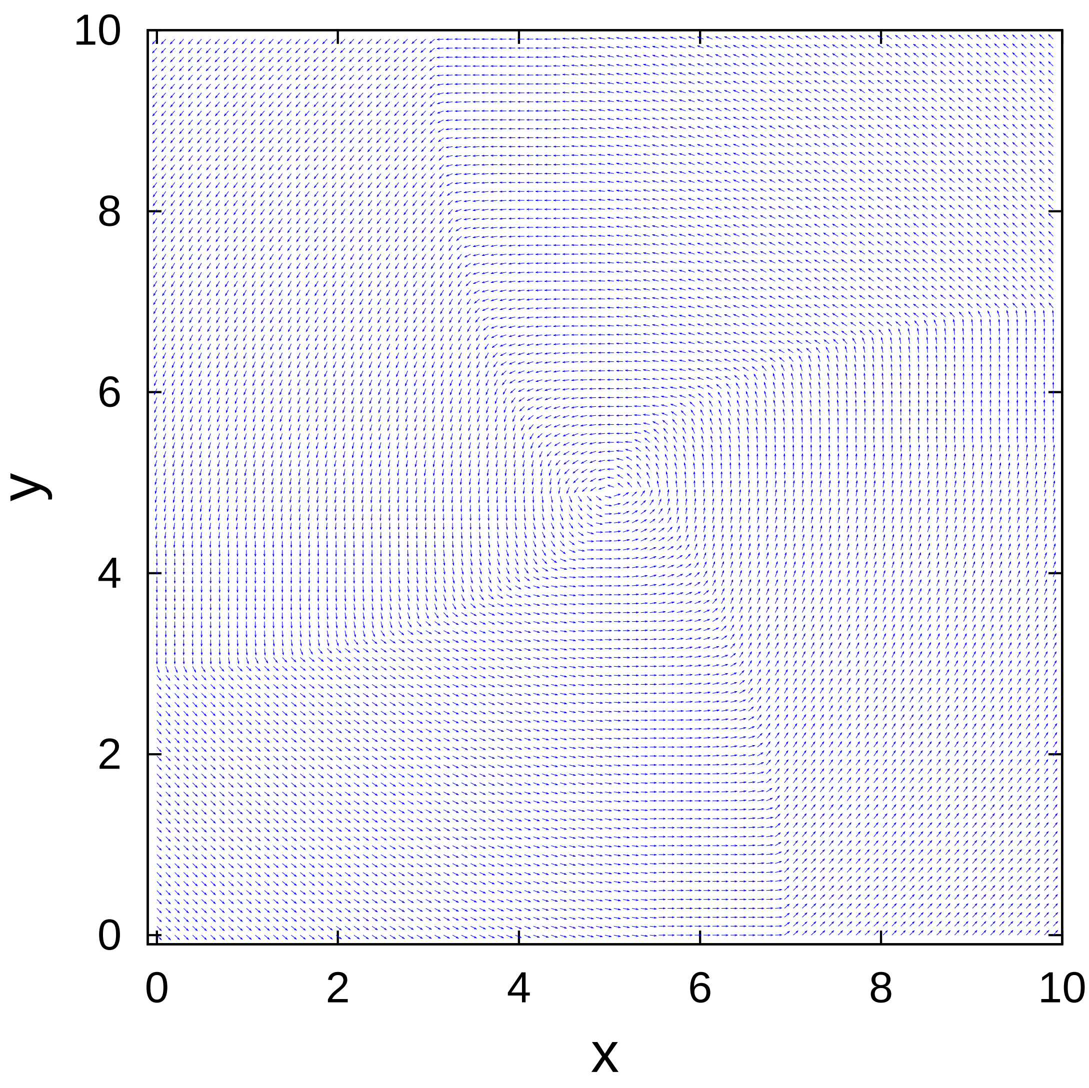
Initial density (mill solution)

Mills: numerical solutions



Density at $t = 5$

Mills: numerical solutions



Flux orientation at $t = 5$

Order parameter (after Vicsek)

- Coeff. c_1 measures the order / disorder

$$c_1 = |j_{M_\Omega}|$$

  - $c_1 \sim 1$: particle directions are aligned
  - $c_1 \sim 0$: particle directions are random

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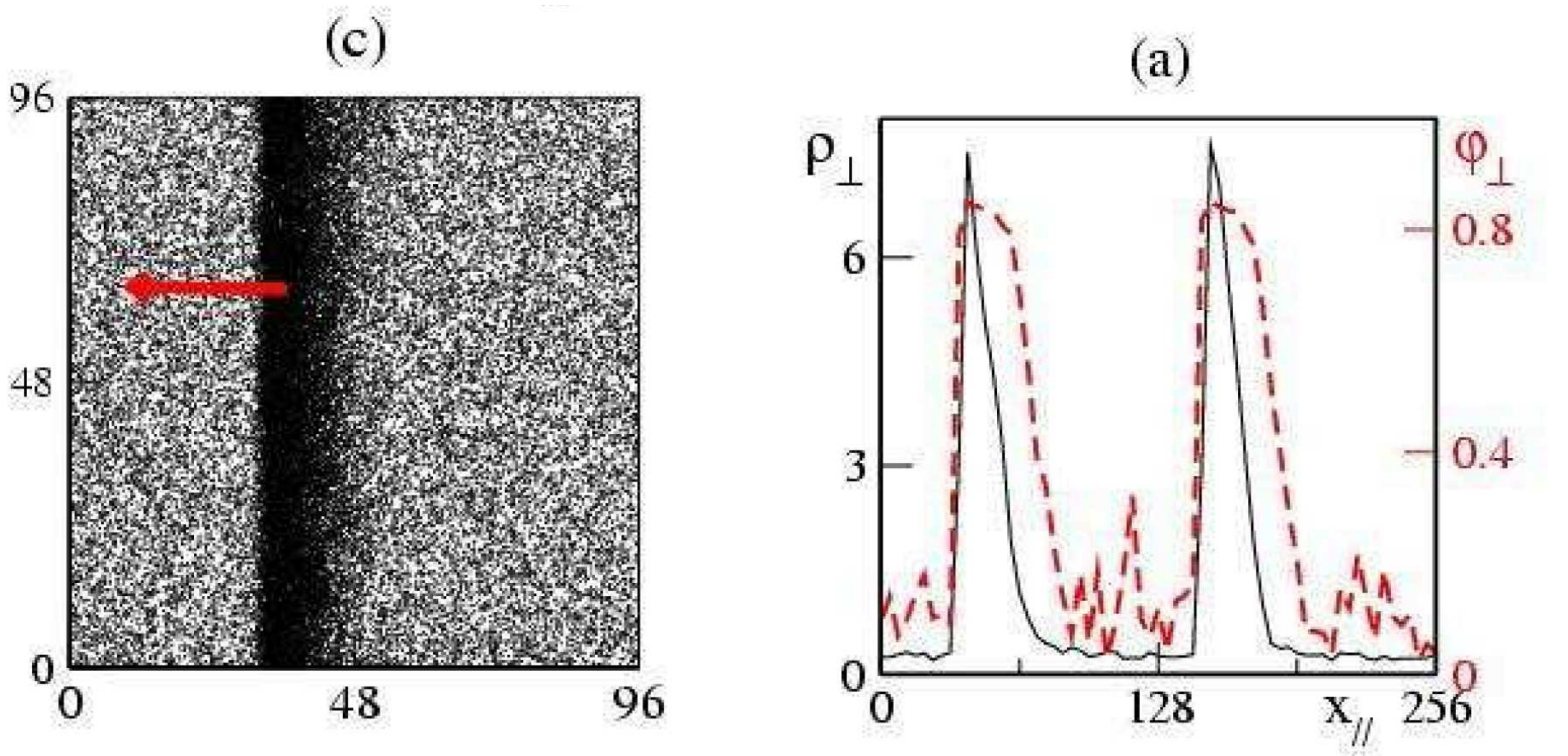
- $\neq$ simulations: higher order at higher density
  - Possible cure: make $D(\rho)$.
  - Justification: Fluctuations in the mean-field limit

Simulation of Vicsek particle model

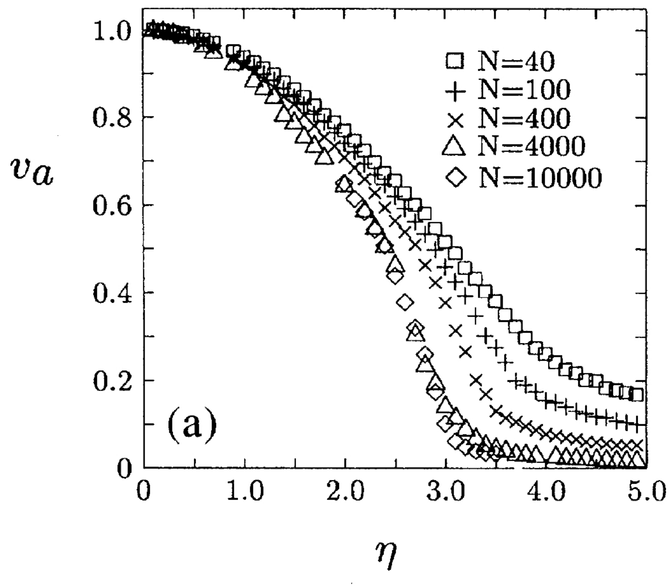


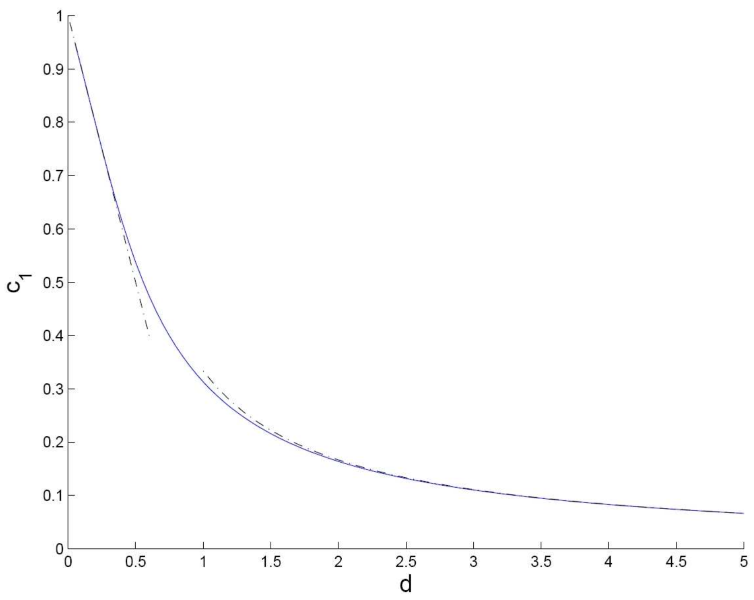
Left: Point position of the particles

Right: Density (black) and order parameter (red) profiles transverse to a band

After Chate et al, arXiv:0712.206.2V1

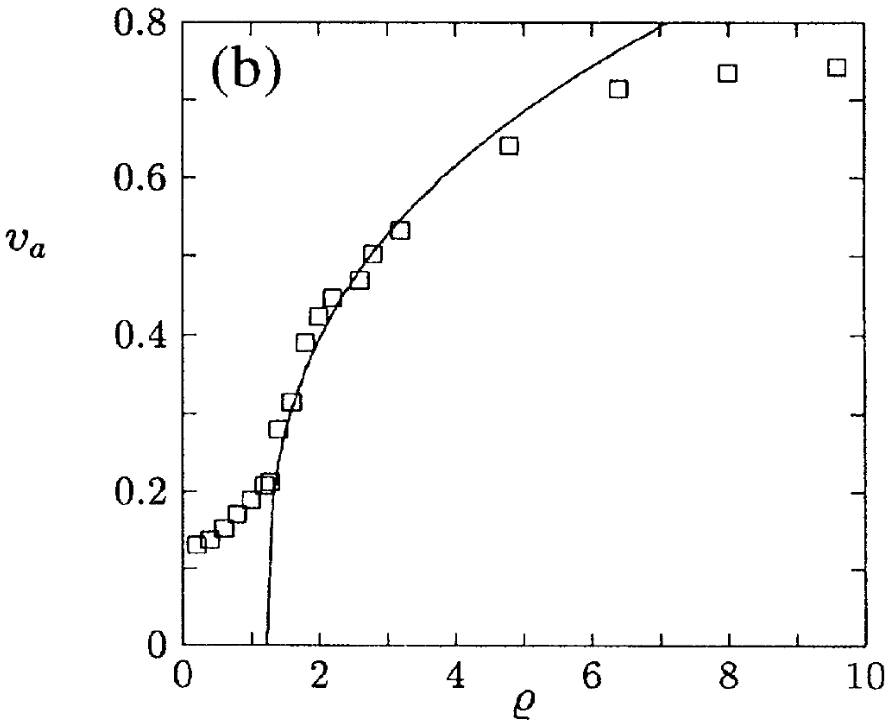
Phase transition as noise level varies





Left: Order parameter as a fct of noise level D (after Vicsek)

Right: Order parameter as a fct of noise level D (after hydro model)



Order parameter as a fct of density (after Vicsek)

In hydro model, order parameter does not depend on density

Phase transition

- Hydro model unable to reproduce phase transition of Vicsek particle model
  - Unique equilibria (no bi-stability)
  - Hyperbolicity (no instability)
  - Smooth variation of the coefficients wrt noise level D

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  - Unique equilibria (no bi-stability)
  - Hyperbolicity (no instability)
  - Smooth variation of the coefficients wrt noise level D
- Possible explanation:
  - Vicsek particle simulations are not in hydro regime
  - Interaction radius $R_{Vicsek} = O(1) \quad | \quad R_{Hydro} = O(\varepsilon)$
  - $\varepsilon_{Vicsek} \sim 0.03$ not very small
  - requires a non-local collision operator with account of fluctuations of particle number

4. Conclusion

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- Leads to the first derivation of a non-conservative model from kinetic theory
  - Published in [D. Motsch, M3AS, Vol. 18, (2008)]

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