Banff International Research Station
for Mathematical Innovation and Discovery

Geometric Inequalities
June 17-22, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Donald Cameron Hall, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Sunday–Thursday
Coffee Breaks: daily 10:00–10:30 am and 3:30–4:00 pm, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Donald Cameron Hall

Monday

	- Breakfast -
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager
9:00-9:50	**Alberto Bressan**: Optimal transportation metrics for nonlinear wave equations
	- Coffee -
10:30-11:20	**Stamatis Dostoglou**: Approximation of homogeneous measures in hydrodynamics
	- Lunch -
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:30-15:20	**Tony Carbery**: The Brascamp-Lieb inequalities: finiteness,structure, and extremals
	- Coffee -
16:00-16:25	**Martial Agueh**: Sharp Gagliardo-Nirenberg inequalities via p-Laplacian type equations
16:30-16:55	**Stefan Valdimarsson**: Optimizers for the Brascamp-Lieb inequality
	- Dinner -

Tuesday

- Breakfast -

9:00-9:50 **Ben Stephens**: Thread-wire surfaces: Near-wire minimizers and topological finiteness

- Coffee -

10:30-10:55 **Lotfi Hermi**: From differential inequalities for Riesz means to universal Weyl-type bounds for eigenvalues

11:00-11:50 **Amir Moradifam**: Optimal Hardy and Hardy-Rellich Inequalities

- Lunch -

14:30-15:20 **Jungchen Wei**: On an elliptic problem with supercritical negative exponent

- Coffee -

16:00-16:25 **Marcello Lucia**: Isoperimetric profile and uniqueness for semilinear problems

16:30-16:55 **Tobias Weth**: Radial symmetry of positive solutions to nonlinear polyharmonic Dirichlet problems

17:00-17:25 **Aaron Smith**: Ricci flow of asymmetric cigar manifolds towards the standard cigar

- Dinner-

Wednesday

- Breakfast -

9:00-9:50 **Dario Cordero-Erausquin**: Interpolations and geometric inequalities

- Coffee -

10:30-11:20 **Young-Heon Kim**: Curvature and the continuity of optimal transportation maps

- Lunch, free afternoon, dinner

Thursday

- Breakfast -

9:00-9:50 **Andrea Cianchi**: Elliptic equations and isocapacitary inequalities

- Coffee -

10:30-10:55 **Guillaume Carlier**: On a theorem of Alexandrov

11:00-11:25 **Antoine Henrot**: About the product of moments of inertia

11:30-11:55 **Franck Barthe**: Optimal transport, log-Sobolev and isoperimetric inequalities

- Lunch -

14:30-15:20 **Francesco Chiacchio**: Isoperimetric inequalities for factorized measures and applications

- Coffee -

16:00-16:25 **Adele Ferone**: Hardy and Hardy-Littlewood inequalities with remainder

16:30-17:20 **Rupert Frank**: The sharp constant in the Hardy-Littlewood-Maz'ya inequality in the three-dimensional upper half-space

- Dinner -

Friday

- Breakfast -

9:00-9:50 **Rafael Benguria**: Optimal Gagliardo-Nirenberg inequalities and fourth order elliptic equations in one dimension

- Coffee -

10:30-11:25 **Nicola Fusco**: The sharp Sobolev inequality in quantitative form

- Lunch -

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Geometric Inequalities
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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Martial Agueh** (University of Victoria, Canada)
Title: *Sharp Gagliardo-Nirenberg inequalities via p-Laplacian type equations*
Abstract: Sharp constants and optimal functions of Gagliardo-Nirenberg inequalities involving the L^p-norm of the gradient are investigated by studying a p-Laplacian type equation. By transforming the unknown of the equation via some change of function, we find explicitly the solution of this equation in all generalities when the space dimension is $n = 1$, and in some particular cases when $n > 1$. Then we derive the sharp constants and optimal functions of all the L^p-Gagliardo-Nirenberg inequalities when $n = 1$, and of a subclass of them when $n > 1$. Our analysis includes also the sharp L^p-Nash's inequalities.

Speaker: **Franck Barthe** (Institut de Mathématiques de Toulouse)
Title: *Optimal transport, log-Sobolev and isoperimetric inequalities*
Abstract: We push further the optimal transportation method for log-Sobolev type inequalities. This allows to recover precise concentration inequalities for log-concave measures, to extend Bobkov's isoperimetric inequalities in this case, as well as Wang's extension of the Bakry Emery criterion (for negative bounds on the curvature, which are balanced by an integrability assumption). (Joint work with A. Kolesnikov)

Speaker:**Rafael Benguria** (Universidad Católica de Chile)
Title: *Optimal Gagliardo-Nirenberg inequalities and fourth order elliptic equations in one dimension*
Abstract: Here we compare minimization problems of Gagliardo-Nirenberg type on finite intervals with periodic boundary conditions to the same problems set on the whole real line. Our main result is that the minimum is achieved on periodic intervals, does not depend on the period, and is not achieved in the whole line. The value of the infimum on the whole line is determined by the optimal solutions on the periodic interval. We compute the value of the infimum numerically. This is joint work with Isabelle Catto (Paris IX) and Jean Dolbeault (Paris IX).

Speaker: **Alberto Bressan** (Penn State University, USA)
Title: *Optimal transportation metrics for nonlinear wave equations*
Abstract: In this talk I shall discuss some examples of nonlinear wave equations where the solution does not depend continuously on the initial data, in any of the natural Sobolev norms. However, the flow can be rendered Lipschitz continuous by adopting a different distance functional, defined in terms of an optimal transportation problem. This can also be seen as a Riemannian distance, determined by a suitable family of norms on tangent vectors. Further possible applications of this approach will be discussed.

Speaker: **Tony Carbery** (School of Mathematics, Edinburgh)
Title: *The Brascamp–Lieb inequalities: finiteness, structure and extremals*
Abstract: We discuss recent work on the Brascamp–Lieb inequalities.

Speaker:**Guillaume Carlier** (Université Paris IX Dauphine)
Title: *On a theorem of Alexandrov*
Abstract: We will give an elementary variational proof of classical theorems of Minkowski and Alexandrov on the existence and uniqueness, up to translations, of a closed convex hypersurface with a given Gaussian curvature (as a function of the exterior unit normal) or with a given surface function. We will emphasize the analogy with the classical optimal transportation problem and consider some applications to shape optimization.

Speaker: **Francesco Chiacchio** (Università degli Studi di Napoli)
Title: *Isoperimetric inequalities for factorized measures and applications*
Abstract: Some improved Hardy type inequalities, with respect to the Gaussian measure, will be presented. Their relationship with the Gross' logarithmic Sobolev inequality will be also illustrated. Finally, it will be shown a family of factorized probability measures enjoying an isoperimetric inequality. Via Talenti's theorem, such inequalities can be used in order to get sharp estimates for solutions of elliptic problems with degeneracy at infinity.

Speaker: **Andrea Cianchi** (Università degli Studi di Firenze)
Title: *Elliptic equations and isocapacitary inequalities*
Abstract: A priori estimates for solutions to nonlinear elliptic Neumann problems in open subsets Ω of $\mathbb{R}^n$ are established via inequalities relating the Lebesgue measure of subsets of $\emptyset$ and their relative capacity. This is a joint work with V.G.Maz'ya.

Speaker: **Dario Cordero-Erausquin** ()
Title *Interpolations and geometric inequalities*

Speaker: **Stamatis Dostoglou** (University of Missouri, Columbia)
Title: *Approximation of homogeneous measures in hydrodynamics*
Abstract: The talk will describe how ideas from optimal transport connect with the approximation of statistical solutions of the Navier-Stokes equations, i.e. probability measures supported by weak solutions of the equations.

Speaker: **Adele Ferone** (Seconda Università di Napoli)
Title: *Hardy and Hardy Littlewood inequalities with remainder*
Abstract: Improved Hardy inequalities, involving remainder terms, are established both in the classical and in the limiting case. The relevant remainders depend on a suitable distance from the families of the "virtual" extremals. A key tool is a Hardy-Littlewood inequality with a remainder term.

Speaker: **Rupert Frank** (KTH Stockholm)
Title: *The sharp constant in the Hardy-Sobolev-Maz'ya inequality in the three dimensional upper half-space*
Abstract: It is shown that the sharp constant in the Hardy-Sobolev-Maz'ya inequality on the three dimensional upper half space is given by the Sobolev constant. This is achieved by a duality argument relating the problem to a Hardy-Littlewood-Sobolev type inequality whose sharp constant is determined as well. The talk is based on joint work with R. D. Benguria and M. Loss.

Speaker: **Nicola Fusco** (Universitá di Napoli)
Title: *The sharp Sobolev inequality in quantitative form*
Abstract: I will present a quantitative version of the sharp Sobolev inequality in $W^{1,p}(R^n)$, $1 < p < n$, with a remainder term involving the distance from extremals.

Speaker: **Antoine Henrot** (Institut Élie Cartan Nancy)
Title: *About the product of moment of inertia*
Abstract: This is a joint work with Gérard Philippin from Laval University, Québec. In this talk, we are interested in the product of moment of inertia with respect to the coordinate axes.

Let Ω be a domain in $\mathbb{R}^2$, Γ its boundary and let us introduce the following notation, where $X = (x, y)$:

$$J(\Omega) := \int_\Omega x^2\, dX \int_\Omega y^2\, dX \text{ and } j(\Gamma) := \int_\Gamma x^2\, d\sigma \int_\Gamma x^2\, d\sigma.$$

We are interested in minimizing the functionals J, j on the class of admissible domains with given volume: $\mathcal{O} := \{\Omega \subset \mathbb{R}^2,\ |\Omega| = c\}$. We show:

Theorem 1: The ellipses minimize J.
This result is due to W. Blaschke but can also be recovered by a simple topological derivative argument. It extends naturally to higher dimension.

Theorem 2: The disk minimizes j.
We will mainly focus on the proof of this result which seems to be original. Let us remark that the result is unknown in higher dimension.

A consequence of Theorem 2 is a new proof of the following Theorem due to J. Hersch, L.E. Payne, M.M. Schiffer:

Theorem 3: Let $0 = p_1(\Omega) \le p_2(\Omega) \le p_3(\Omega)$ be the first eigenvalues of the Steklov problem:

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ \frac{\partial u}{\partial n} = pu & \text{on } \Gamma\,, \end{cases}$$

where Ω is a bounded Lipschitz open set and Γ its boundary.
Then, the disk maximizes the product $p_2(\Omega)p_3(\Omega)$ among plane open sets of given area.

Speaker: **Lotfi Hermi** (University of Arizona)
Title: *From differential inequalities for Riesz means to universal Weyl-type bounds for eigenvalues*
Abstract: Trace identities of the type derived by Harrell-Stubbe, and later generalizations by Levitin-Parnovski, proved to be a very efficient procedure to produce universal Yang-type bounds for eigenvalues of the Dirichlet Laplacian.

In this talk we show how these identities can be used to produce new universal Weyl-type bounds for averages of eigenvalues, and provide alternative routes to the Berezin-Li-Yau inequality as viewed by Laptev and Weidl. This is work with joint work with Evans Harrell.

Speaker: **Young-Heon Kim** (University of Toronto)
Title: *Curvature and the continuity of optimal transportation maps*
We will discuss the continuity of optimal transport maps, in view of a semi-Riemannian structure which we have formulated recently. A necessary condition for the continuity is given as some non-negativity condition on the curvature of this semi-Riemannian metric, and this result gives a quite general geometric framework for the regularity theory of Ma, Trudinger, Wang and Loeper on the potential functions of optimal transport. This is joint work with Robert McCann.

Speaker:**Marcello Lucia** (University of Cologne)
Title: *Isoperimetric profile and uniqueness for semilinear problems*
Abstract: An inequality involving the isoperimetric profile of a manifold is presented. We apply it to derive several new uniqueness results for two-dimensional semilinear equations. We also show how such an inequality can give a lower bounds on the first eigenvalue of the manifold.

Speaker: **Amir Moradifam**
Title: *Optimal Hardy and Hardy-Rellich Inequalities*
Abstract: We give a necessary and sufficient condition on a radially symmetric potential V on Ω that makes it an admissible candidate for an improved Hardy inequality of the following form:

$$\int_\Omega |\nabla u|^2 dx - (\tfrac{n-2}{2})^2 \int_\Omega \tfrac{|u|^2}{|x|^2} dx \geq c \int_\Omega V(|x|)|u|^2 dx \quad \text{for all } u \in H_0^1(\Omega). \tag{1}$$

A characterization of the best possible constant $c(V)$ is also given. This result yields easily the improved Hardy's inequalities of Brezis-Vázquez, Adimurthi et al., and Filippas-Tertikas, as well as the corresponding best constants. Our approach clarifies the issue behind the lack of an optimal improvement, while yielding other interesting "dual" inequalities. Another consequence is the following substantial sharpening of known integrability criteria: If a positive radial function V satisfies $\liminf_{r\to 0} \ln(r) \int_0^r sV(s)ds > -\infty$, then there exists $\rho := \rho(\Omega) > 0$ such that the improved Hardy inequality (1) holds for the scaled potential $V_\rho(x) = V(\frac{|x|}{\rho})$. On the other hand, if $\lim_{r\to 0} \ln(r) \int_0^r sV(s)ds = -\infty$, then there is no $\rho > 0$ for which (1) holds for V_ρ. This shows for example, that $V(x) = \frac{1}{|x|^\alpha}$ is an admissible potential for an improved Hardy inequality when $\alpha < 2$, while it is not so for $\alpha \geq 2$. All these results have immediate applications to the corresponding Schrödinger equations. Analogous improvements are given for Hardy-Rellich inequalities.

Speaker: **Aaron Smith** (Queens University and Stanford)
Title:*Ricci flow of asymmetric cigar manifolds towards the standard cigar* Abstract: In this talk, we will explicitly compute the evolution under Ricci flow of a class of asymmetric conformally flat two-manifolds manifolds, which we call asymmetric cigars. These asymmetric cigars converge towards the standard cigar soliton under Ricci flow in the sense that there exist time-dependent bi-Lipschitz maps between the manifolds with both constants approaching one. We derive the optimal rate of convergence of these maps. Time permitting, we will briefly discuss the extension of our results to the Yamabe flow on asymmetric cigar manifolds in higher dimensions.

Speaker: **Ben Stephens** (University of Toronto)
Title: *Thread-wire surfaces: Near-wire minimizers and topological finiteness*
Abstract: Alt's thread problem asks for least-area surfaces bounding a fixed "wire" curve and a movable "thread" curve of length L. Near-wire minimizers are important to a finiteness conjecture. For near-wire minimizers on a generic wire we show $\mathcal{C}^1$-regularity where the thread and wire join at cusp-corners. Our result confirms a prediction of the normal vector limit at such points. It is an example of the local geometry dominating free influence in a free boundary problem. Finally, we use a weighted isoperimetric inequality to prove that all minimizers are near-wire minimizers when the thread is near the wire length.

Speaker: **Stefan Valdimarsson** (UCLA)
Title: *Optimizers for the Brascamp–Lieb inequality*
Abstract: We examine the heat flow argument for the Brascamp–Lieb inequality to find all optimizers for the inequality.

Speaker: **Juncheng Wei** (The Chinese University of Hong Kong, Shatin, Hong Kong)
Title: *On an elliptic problem with supercritical negative exponent*
Abstract: We consider the following problem (proposed and studied by Ghoussoub, Guo and Esposito)

$$\Delta u = \frac{\lambda}{u^2} \text{ in } \Omega, \quad u = 1 \text{ on } \partial\Omega, \quad 0 < u < 1 \text{ in } \Omega$$

where Ω is a rather symmetric domain in R^2. We prove that there exists a $\lambda_* > 0$ such that for $\lambda \in (0, \lambda_*)$ the minimal solution is unique. Moreover, we show that the branch of positive solutions must undergo

infinitely many bifurcations as the maximums of the solutions on the branch go to 0 (possibly only changes of direction). Central to our analysis is the monotonicity formula, one-dimensional Sobolev inequality

$$\int_{S^1} u^{-2} \int_{S^1} (\beta u_\theta^2 - u^2) \geq -1,$$

and classification of solutions to a supercritical problem

$$\Delta U = \frac{1}{U^2} \text{ in } R^2,\ U(0) = 1, U(z) \geq 1.$$

Speaker: **Tobias Weth** (University of Giessen)
Title: *Radial symmetry of positive solutions to nonlinear polyharmonic Dirichlet problems* Abstract: I will present a joint result with F. Gazzola and E. Berchio on radial symmetry of positive solutions to a class of semilinear polyharmonic Dirichlet problems in the unit ball. The result is obtained via a new variant of the moving plane method relying on pointwise inequalities for the Green function of the polyharmonic operator relative to Dirichlet boundary conditions. In some special cases our result implies uniqueness of positive solutions.