Production Externalities and Expectations, Application to the economics of Climate Change.

Antoine Mandel

Centre d'Economie de la sorbonne, Université Paris 1, CNRS

Workshop Mathematics and the Environment
Banff, May 8th-13th 2007

Abstract

- We extend the problem of decentralization of Pareto Optima in a general equilibrium economy with production externalities to the case where there may be differences between:
- The production set Y upon which pareto Optimality is defined.
- The aggregate of the firms expectations about their production possibilities Z_j, $\{z \in \mathbb{R}^L \mid \exists (z_j) \in \prod_{j=1}^n Z_j(z_{-j}) \ s.t \ \sum_{j=1}^n z_j = z\}$.

Outline

1. Motivation
2. Model / decentralization results
3. Tentative interpretation

What is the standard G.E model with production externalities

- Arrow (1969) and Laffont (1978)
- An economy with L goods.
- Environment of firm j, $(z_{-j}) = (z_1, \cdots, z_{j-1}, z_{j+1}, \cdots z_n)$
- Production possibilities of firm j, $Z_j : z_{-j} \rightarrow Z_j(z_{-j}) \subset \mathbb{R}^L$
- Consumers: utility function $u_i : \mathbb{R}^L_+ \rightarrow \mathbb{R}$, no externality

Sandard G.E model with production externalities

- Sandard welfare theorems do not hold: commodities prices do not reflect the external effects.
- Decentralization of the Pareto Optima with regards to the Z_j or equivalently with regards to the aggregate production set: $\{z \in \mathbb{R}^L \mid \exists (z_j) \in \prod_{j=1}^n Z_j(z_{-j}) \; s.t \; \sum_{j=1}^n z_j = z\}$.
- Arrow's solution: when a firm produces a quantity $(y_j)_\ell$ of good ℓ, it emits simultaneously an external effect on every other firm and must buy the right to emit this effect at every firm k at the price $(p_{j,k})_\ell$.
- Opening of $L \times n \times (n-1)$ markets: A general equilibrium Coase theorem.

Why extending the standard model ?

- Based on an intuition on the economics of climate change.
- Many of the potential consequences of climate change, such as changes in agricultural yields and in localization of crops, disruption of ecosystems or increased vulnerability of physical capital affect the production possibilities of economies.
- Production sector also responsible for climate change (greenhouse gases emissions).
- A typical production externality.

Why extending the standard model ?

- Potentially damaged firms never claimed for a compensation, even less advocated the opening of markets of allowances
- Governmental concern. Kyoto, Allowance markets.
- Divergence between production sector and government concern.

Why extending the standard model ?

- Different expectations on the influence of climate change on future production possibilities.
- government intervenes because it is less optimistic than producers.
- Y, may be a strict subset of $\{z \in \mathbb{R}^L \mid \exists (z_j) \in \prod_{j=1}^n Z_j(z_{-j}) \; s.t \; \sum_{j=1}^n z_j = z\}$.
- Give a general equilibrium model for such a situation.
- Theoretic solution to the decentralization problem in this framework.

The Model 1

- Firm j production capacities, correspondence $z_{-j} \to Z_j(z_{-j})$.
- Consumers: standard utility maximizers, do not face externalities.
- initial resources ω.
- Standard assumptions for competitive behavior.

The Model 2

- Y : production plans the government considers as feasible in the aggregate (all externalities internalized). Criteria of efficiency for the production process.
- Y is a production set à la Arrow-Debreu.
- Possibly less-optimistic than the producers: $Y \subset \{z \in \mathbb{R}^L \mid \exists (z_j) \in \prod_{j=1}^n Z_j(z_{-j}) \ s.t \ \sum_{j=1}^n z_j = z\}$.
- Focus on the decentralization of the pareto Optima with regards to Y.

On the temporal structure of the model

- Time is not introduce time explicitly even though the problematic is clearly intertemporal ?
- General equilibrium model: time introduced implicitly by dated goods and complete contingent markets.
- The only rational that would remain to introduce explicitly time is to account for incompleteness of markets.

A schematic climate change economy

- L goods, two periods of time.
- Climate change is due to greenhouse gases emissions in the first period and affect the production possibilities in the second period.
- A single state of nature in the first period denoted by 0
- $s = 1 \cdots S$ states of nature in the second period (uncertainty about climate change.)

A schematic climate change economy

- CO2 emissions measured by a function f_j. Total emissions in the first period $E = \sum_{j=1}^{n} f_j(y_j^0))$
- True prod. possibilities: $Y_j(E = (Y_j^0, Y_j^1(E), \cdots, Y_j^S(E))$
- Gov: $Y = \{z \mid \exists (y_j) \in \prod_{j=1}^{n} Y_j(\sum_{j=1}^{n} f_j(y_j^0))\ , \sum_{j=1}^{n} y_j = z\}$.
- firms: $Z_j(E) = (Z_j^0, Z_j^1(E), \cdots, Z_j^S(E))$
- Over-optimism $Y_j \subset Z_j \Rightarrow$
  $Y \subset \{z \mid \exists (z_j) \in \prod_{j=1}^{n} Z_j(z_{-j})\ s.t\ \sum_{j=1}^{n} z_j = z\}$.

The Model 3

- Possible failures
- Externalities $\rightarrow$ Equilibrium price differs from aggregate marginal cost
- Externalities $\rightarrow$ Inefficiency $\rightarrow$ Production below ∂Y.
- Over-optimistic firms $\rightarrow$ Production above ∂Y $\rightarrow$ fail to deliver the production plans they have announced

Solution Concept

- Decentralization through a market of allowances.
- An "allowance function" $h_j : (\mathbb{R}^L)^n \to \mathbb{R}$,
- Firm j must use $h_j(z_j, \bar{z}_{-j})$ allowances as input in order to produce z_j within an environment $(\bar{z}_{-j})$.
- Firm j production correspondence:

$$G_j(\bar{z}_{-j}) = \{(z_j, u_j) \in \mathbb{R}^{L+1} \mid z_j \in Z_j(z_{-j}) \ , \ u_j \leq -h_j(z_j, \bar{z}_{-j})\}.$$

- Government supplies the economy with a quantity $A \in \mathbb{R}$ of allowances by initially allocating the agents.

Solution Concept

Definition (Price Equilibrium with Allowances)

A collection of production plans $(\bar{z}_j, \bar{u}_j), \in \prod_{j=1}^{n} G_j(z_{-j})$ *together with a collection of consumption plans* $(\bar{x}_i) \in (\mathbb{R}^L_+)^m$ *is a Price Equilibrium with Allowances if there exist a price* $(\overline{p}, \overline{q}) \in \mathbb{R}^{L+1}_+$ *and an assignment of wealth levels* $(w_1, \ldots, w_m)$ *with* $\sum_{i=1}^{m} w_i = (\overline{p}, \overline{q}) \cdot (\sum_{j=1}^{n} \bar{z}_j + \omega, \sum_{j=1}^{n} \bar{u}_j + A)$ *such that:*

1. *For all* j*,*$(\bar{z}_j, \bar{u}_j)$ *maximizes profit,* $(\overline{p}, \overline{q}) \cdot (z_j, u_j)$*, in* $G_j(\bar{z}_{-j})$
2. *For all* i $\bar{x}_i$ *maximizes* $u_i(x_i)$ *in the budget set* $\{x_i \in \mathbb{R}^L_+ \mid p \cdot x_i \leq w_i\}$
3. $\sum_{i=1}^{m} \bar{x}_i = \sum_{j=1}^{n} \bar{z}_j + \omega$
4. $\sum_{j=1}^{n} \bar{u}_j + A = 0$

When is an equilibrium with allowances Pareto Optimal ?

- The allowance function must compensate the differences between the aggregate (relative to Y) marginal costs of production and the individual ones (relative to Z_j).
- The distance to the efficiency frontier can be seen as a summary of the quantity of “bad” in the economy.

Construction of allowance functions

- The shortage function for Y, (Luenberger(1995)) $g(z) = min\{s \in \mathbb{R} \mid z - s\gamma \in Y\}$.
- $\gamma \in \mathbb{R}^L_{++}$, a reference commodity bundle.
- Intrinsic (in terms of commodities) measure of the distance to the frontier of Y.
- Characterize Y :
  1. $z \in Y$ if and only if $g(z) \leq 0$;
  2. For every $z \in \partial Y$, $N_Y(z) =< \partial g(z) >$.

Solution Concept 2: Allowance functions constructed on the shortage function.

- A share in the aggregate level of "bad"

$$h_j^1(z_j, \bar{z}_{-j}) = \frac{g(z_j + \sum_{k \neq j} \bar{z}_k)}{n}$$

- The difference between the aggregate level of "bad" when it produces and this when it does not produce

$$h_j^2(z_j, \bar{z}_{-j}) = g(z_j + \sum_{k \neq j} \bar{z}_k) - g(\sum_{k \neq j} \bar{z}_k).$$

- A convex and increasing transformation of the preceding:

$$h_j^3(z_j, \bar{z}_{-j}) = \phi(g(z_j + \sum_{k \neq j} \bar{z}_k)) - \psi(g(\sum_{k \neq j} \bar{z}_k)).$$

Decentralization results

h_1, h_2 and h_3 transfer the government views on efficiency at the individual level. They moreover satisfy:

Assumption (Compensation)

For every production plan z associated to a Pareto Optimum, there exist $(z_j) \in \prod_{j=1}^n Z_j(z_{-j})$ with $\sum_{j=1}^n z_j = z$ and $\lambda \geq 0$ such that for all j,

$$N_Y(\sum_{j=1}^n \bar{z}_j) \subset N_{Z_j(\bar{z}_{-j})}(\bar{z}_j) + \lambda \partial h_j(\cdot, \bar{z}_{-j})(\bar{z}_j).$$

Decentralization results

Proposition

Under assumption (Compensation), any Pareto Optimum can be decentralized as an equilibrium with allowances.

Proposition

Assume Y has a smooth boundary and one of the utility functions is smooth and strictly concave. If (Compensation) does not hold, there exist at least a Pareto Optimum which can not be decentralized as an equilibrium with allowances.

Decentralization results

Proposition

Under assumption (Compensation), any Pareto Optimum can be decentralized as an equilibrium with allowances.

Proposition

Assume Y has a smooth boundary and one of the utility functions is smooth and strictly concave. If (Compensation) does not hold, there exist at least a Pareto Optimum which can not be decentralized as an equilibrium with allowances.

First Welfare like results

- The allowance market drives the price to a Pareto Optimum supporting direction.
- But nothing guarantees that the equilibrium production always lies on the efficiency frontier ∂Y.

First Welfare like results

- The allowance market drives the price to a Pareto Optimum supporting direction.
- But nothing guarantees that the equilibrium production always lies on the efficiency frontier ∂Y.

- How to obtain a first welfare like theorem?
- 1) Choose adequately the initial allocation of allowances.
- 2) Government can exchange allowances against commodity bundles.

First Welfare like results: Intuition

- Aim of the government: prevent failures by the firms to deliver the production they had announced.
- All right if the government owns a stock of commodities.
- The building of this stock may be related to the allowance market

First Welfare like results 2

- The government exchange allowances against commodity bundles γ .
- Creates additional allowances thanks to its legal prerogatives.
- Exchange supernumerary allowances against commodities bundles it purchases on the market.
- Production correspondence of firm j is turned to $H_j(z_{-j}) =$
$$\{(z_j, u_j, v_j) \in \mathbb{R}^{L+1} \mid \exists y_j \in Z_j(z_{-j})\ z_j = y_j + v_j\gamma\ ,\ u_j + v_j \leq -h_j(y_j, z_{-j})\}$$

Definition

[Equilibrium with allowance clearance] *A collection of production plans* $(\bar{z}_j, \bar{u}_j, \bar{v}_j) \in \prod_{j=1}^{n} H_j(z_{-j} - s_{-j}\gamma)$ *together with a collection of consumption plans* $(\bar{x}_i) \in (\mathbb{R}_+^L)^m$ *form a price equilibrium with allowance clearance if there exist a price* $(\overline{p}, \overline{q}) \in \mathbb{R}_+^{L+1}$ *, a government extra supply of allowances* $g = -\sum_{j=1}^{n} \bar{v}_j$*, and an assignment of wealth levels* $(w_1, \dots, w_m)$ *with* $\sum_{i=1}^{m} w_i = (\overline{p}, \overline{q}) \cdot (\sum_{j=1}^{n} \bar{z}_j + \omega, \sum_{j=1}^{n} \bar{u}_j + \sum_{j=1}^{n} \bar{v}_j + \overline{A})$ *such that:*

1. *For all j,*$(\bar{z}_j, \bar{u}_j, \bar{v}_j)$ *maximizes profit,* $(\overline{p}, \overline{q}) \cdot (z_j, u_j + v_j)$*, in* $H_j(z_{-j} - s_{-j}\gamma)$
2. *For all i* $\bar{x}_i$ *maximizes* $u_i(x_i)$ *in the budget set* $\{x_i \in \mathbb{R}_+^L \mid p \cdot x_i \leq w_i\}$
3. $\sum_{i=1}^{m} \bar{x}_i = \sum_{j=1}^{n} \bar{z}_j + \omega$
4. $\sum_{j=1}^{n} \bar{u}_j + \sum_{j=1}^{n} \bar{v}_j + \overline{A} = 0$

First Welfare like results 2

- Choose an allowance function of the form

$$h_j(z_j, \bar{z}_{-j}) = \phi(g(z_j + \sum_{k \neq j} \bar{z}_k)) - \psi(g(\sum_{k \neq j} \bar{z}_k)).(\star\star)$$

where ϕ is a strictly convex function such that $\phi'(0) = 1$.

Proposition

Assume assumptions, P, C, G, (Strong Over Optimism) hold and the allowance function is of the form $(\star\star)$. Any equilibrium with allowance clearance is Pareto Optimal.

Proof:

the fundamental issue is that at equilibrium the price of the allowance is necessarily equal to this of the commodity bundle γ. That is:

$$q = p \cdot \gamma$$

Application to climate change

- The climate change economy fits in the framework of our results.
- Opening of a production allowance market suitable to decentralize Pareto optima.
- Problems with interpretation: the production allowance has two types of effects on the firms behavior:
- Prevents firms from emitting too much greenhouse gases in the first period
- Prevents them from setting up over optimistic production plans in the second period.

Application to climate change

- The climate change economy fits in the framework of our results.
- Opening of a production allowance market suitable to decentralize Pareto optima.
- Problems with interpretation: the production allowance has two types of effects on the firms behavior:
- Prevents firms from emitting too much greenhouse gases in the first period
- Prevents them from setting up over optimistic production plans in the second period.

Application to climate change

- The climate change economy fits in the framework of our results.
- Opening of a production allowance market suitable to decentralize Pareto optima.
- Problems with interpretation: the production allowance has two types of effects on the firms behavior:
- Prevents firms from emitting too much greenhouse gases in the first period
- Prevents them from setting up over optimistic production plans in the second period.

Application to climate change

- The climate change economy fits in the framework of our results.
- Opening of a production allowance market suitable to decentralize Pareto optima.
- Problems with interpretation: the production allowance has two types of effects on the firms behavior:
- Prevents firms from emitting too much greenhouse gases in the first period
- Prevents them from setting up over optimistic production plans in the second period.

- The climate change economy fits in the framework of our results.
- Opening of a production allowance market suitable to decentralize Pareto optima.
- Problems with interpretation: the production allowance has two types of effects on the firms behavior:
- Prevents firms from emitting too much greenhouse gases in the first period
- Prevents them from setting up over optimistic production plans in the second period.

Comparison with an emission allowance market.

- An emission allowance market or markets for external effects à la Arrow can also be used for the first task.
- Not sufficient to obtain an optimal outcome as they do not correct the anticipation errors of the producers.

Conclusion

- The production allowance market:a proxy for an adaptation policy.
- Emission allowance market or external effects markets à la Arrow:tools for a mitigation policy.
- Now emission allowance markets are not efficient unless production allowance markets also exist: recognizing the need for adaptation is a prerequisite for efficient mitigation.

Conclusion

- The production allowance market:a proxy for an adaptation policy.
- Emission allowance market or external effects markets à la Arrow:tools for a mitigation policy.
- Now emission allowance markets are not efficient unless production allowance markets also exist: recognizing the need for adaptation is a prerequisite for efficient mitigation.

Conclusion

- The production allowance market:a proxy for an adaptation policy.
- Emission allowance market or external effects markets à la Arrow:tools for a mitigation policy.
- Now emission allowance markets are not efficient unless production allowance markets also exist: recognizing the need for adaptation is a prerequisite for efficient mitigation.

references

- Arrow, K.J (1969) "The organization of economic activity : Issues pertinent to the Choice of Market versus Non-Market allocation", in Joint economic commitee, The analysis and evaluation of Public Expenditures ; The PPB System, Washington DC : Government Printing Office, pp 47-64.
- Debreu, G (1959) “Theory of Value. An axiomatic analysis of economic equilibrium ", Cowles Foundation Monograph 17.
- Laffont, J, J. (1978) Effets externes et théorie économique, Monographie du Séminaire d'économétrie. Centre National de la Recherche Scientifique, Paris.
- Luenberger D. (1995) Microeconomic Theory, McGraw-Hill, Inc., New York, .