Valuation of Energy Storage: An Optimal Switching Approach

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Energy Storage

- Unlike financial “paper” assets, commodities must be physically stored.
- Storage infrastructure is a major component of the energy industry.
- Large-scale needs; very capital intensive.
- Storage allows intertemporal transfer.
- Financially, storage is a straddle on calendar prices.
- Can be used to speculate: commodity prices fluctuate, aim is to buy low and sell high (and store in between).

Examples

- Natural Gas Storage: salt domes, pipelines, depleted reservoirs, aquifers.
- This is already a multi-billion industry with active trading.
- Poised for further growth with rolling-out of Liquified Natural Gas on world-wide basis.
- Hydroelectric Pumped Storage.
- The most scalable method of storing electricity; about 75% efficiency; 38 plants in the US, worldwide capacity of almost 50 GWh.
- Resource management. Metal/fossil fuel is “stored” in the ground, with one-way inventory depletion. Further exploration permits possibility of “replenishment”.

Complex Problem:

- Commodity prices are stochastic
- Strong seasonality effects
- Possibility of both forward and spot trades
- Engineering constraints/exogenous events
- Margin requirements on borrowed funds
- Inventory-dependent Injection/withdrawal rates
- Storage costs/switching costs

Model Setup

- We focus on the timing optionality within a real-options framework.
- The presence of inventory makes the problem highly path-dependent!
- Concentrate on the gas storage application.
- The two key state variables are (G_t) gas prices (stochastic).
- (C_t) current inventory of gas (a function of manager's policy).

Basic Ingredients

- (G_t) is exogenously given and is a d-dimensional Markov process.
- Inventory constraints: $c_{min} \leq C_t \leq c_{Max}$.
- At each instant t, choose an operating regime: u_t —rate for amount of gas to inject/withdraw.
- Transmission constraints: $a_{min} \leq u_t \leq a_{Max}$.
- Resulting *inventory path* $\bar{C}_t(u)$:

$$d\bar{C}_s(u) = a_{u_s}(\bar{C}_s(u))\, ds, \qquad \bar{C}_0(u) = c.$$

- Fixed horizon T: typically the facility is rented from the owner and must be returned at a later date.

$\Longrightarrow$ Set of admissible policies $u \in \mathcal{U}(c)$.

Control Problem

- The manager maximizes total revenue on $[0, T]$.
- When operating regime is i, rate of revenue is $\psi_i(t, G_t, C_t)$.
- When operating regime is changed from i to j, switching costs $K_{i,j}(G_t, C_t)$ are paid.
- Let $V(t, g, c, i)$ denote maximum expected future profit given the initial conditions.
- Wish to find

$$V(0, g, c, i) = \sup_{u \in \mathcal{U}} \mathbb{E}\left[\int_0^T \psi_{u(t)}(G_t, C_t)\, dt - \sum_{t \leq T} K_{u_{t-}, u_t}\right].$$

- Bellman Principle:

$$V(0, g, c, i) = \sup_{u} \mathbb{E}\left[\int_0^t \psi_{u(s)}(G_s, C_s)\, ds - \sum_{s \leq t} K_{u_{t-}, u_t} + V(t, G_t, \bar{C}_t(u), u_t)\right].$$

Operating Regimes

Because payoffs are *linear* in u_t, controls are necessarily of bang-bang type, so the only choices are $u_t \in \mathcal{I} \triangleq \{a_{min}, 0, a_{Max}\}$.
So a priori have a finite-dimensional control space.
$\Longrightarrow$ Three possible operational states

$$\begin{array}{lll} \text{inject} & dC_t = a_{inj}(C_t)dt, & \psi_{-1}(G_t, C_t) = -b_{-1}(C_t) - a_{inj}(C_t) \cdot G_t \\ \text{store} & dC_t = 0, & \psi_0(G_t, C_t) = -b_0(C_t) \\ \text{withdraw} & dC_t = -a_{wdr}(C_t)dt, & \psi_1(G_t, C_t) = -b_1(C_t) + a_{wdr}(C_t) \cdot G_t \end{array}$$

b_i's are the O&M costs, storage costs, transmission inefficiencies, etc.

Example

- 1-d Exponential OU model:

$$dG_t = \kappa(\bar{g} - \log G_t)G_t\,dt + \sigma_G G_t\,dW_t$$

- Mean-reverting, log-normal, non-negative (Jaillet et al. 2004).
- Terminal condition reflects stipulations for final inventory:

$$V(T, g, c, i) = -2g \cdot \max(\underline{c} - c, 0).$$

- Gas pressure laws:

$$a_{wdr}(c) = k_0\sqrt{c}, \quad a_{inj}(c) = k_1\sqrt{\frac{1}{c + k_2} - \frac{1}{k_3}}.$$

Related Literature

- Real Options: Brennan and Schwartz (1985), Dixit and Pindyck (1994), Insley (2003).
- Approaches based on pde methods: Ahn et al. (2002), de Jong and Walet (2003).
- Stochastic Programming: Jacobs et al. (1995), Doege et al. (2006).
- Optimal Switching (w/out inventory): Zervos (2003), L. and Carmona (2005), Barrera-Esteve et al. (2006).

- The operational flexibility of the manager is a compound timing option. Under mild assumptions can show that will only make a finite number of changes in the optimal policy.

⇒ Recursively define $V^k(t, g, c, i)$ for $k = 0, 1, \ldots, 0 \leq t \leq T$, $g \in \mathbb{R}^d$, $c \in [c_{min}, c_{max}]$ and $i \in \{-1, 0, 1\}$:

$$V^0(t, g, c, i) \triangleq \mathbb{E}\Big[\int_t^T \psi_i(s, G_s, \bar{C}_s(c, i))\, ds \Big| G_t = g\Big],$$

$$V^k(t, g, c, i) \triangleq \sup_{\tau \in \mathcal{S}_t} \mathbb{E}\Big[\int_t^\tau \mathrm{e}^{-r(s-t)} \psi_i(s, G_s, \bar{C}_s(c, i))\, ds + \max_{j \neq i}\{-K_{i,j} + V^{k-1}(\tau, G_\tau, \bar{C}_\tau(c, i), j)\}\mathrm{e}^{-r(\tau - t)} \Big| G_t = g\Big].$$

- Iterative Optimal Stopping Problems.

Proposition

1. V^k *is equal to the value function for the storage problem with at most k regime switches allowed.*
2. *An optimal finite strategy* $u^* = (\tau_1^*, \xi_1^*, \tau_2^*, \xi_2^*, \cdots)$ *for* $V^k(0, g, c, i)$ *exists and is:* $\tau_0^* = 0, \xi_0^* = i$, *and for* $\ell = 1, \ldots, k$

$$\begin{cases} \tau_\ell^* & \triangleq & \inf\Big\{ s \geq \tau_{\ell-1}^* : V^\ell(s, G_s, C_s(u^*), i) \\ & & \qquad = \max_{j \neq i}\big(-K_{i,j} + V^{\ell-1}(s, G_s, C_s(u^*), j)\big)\Big\} \wedge T, \\ \xi_\ell^* & \triangleq & \arg\max_{j \neq i}\{-K_{i,j} + V^{\ell-1}(\tau_\ell^*-, G_{\tau_\ell^*-}, C_{\tau_\ell^*-}(u^*), i).\} \end{cases}$$

3. $\lim_{k\to\infty} V^k(t, g, c, i) = V(t, g, c, i)$ *pointwise, uniformly on compacts.*

QVI Approach

The classical analytic theory (Øksendal and Sulem, 2005) implies that the value function is also the (unique viscosity) solution of the Quasi-Variational Inequality

$$\begin{cases} \min\Big\{ -V_t - \mathcal{L}_G V(t,g,c,i) + a_i(c)\cdot\partial_c V(t,g,c,i) \\ \qquad -\psi_i(g,c) + rV(t,g,c,i),\ V(t,g,c,i) - \max_{j\neq i}(V(t,g,c,j) - K_{i,j})\Big\} = 0. \\ \qquad V(T,g,c,j) \quad \text{given.} \end{cases} \tag{1}$$

Assuming a smooth V this can then be implemented with a free boundary pde solver.
The problem (1) is degenerate (convection-dominated).

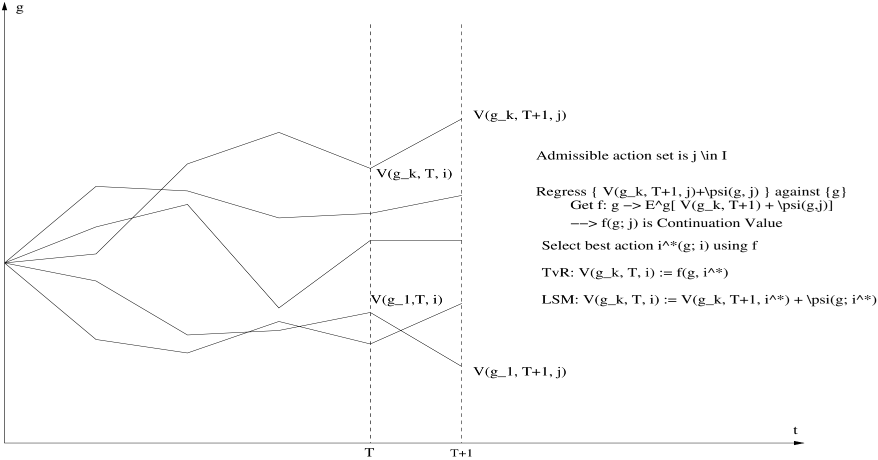
Numerical Algorithm

Bermudization

- Discretize time: decisions are now made only at $t = m\Delta t$.
- Then to make a decision today compute

$$V(t, G_t, C_t, i) = \max_j \Big(-K_{i,j} + \mathbb{E}\big[\psi_{t,t+1}(G_t, C_t, j) \\ + e^{-r\Delta t} \cdot V(t+1, G_{t+1}, \bar{C}_{\Delta t}(C_t, j), j) \,|\, \mathcal{F}_t\big]\Big).$$

- Can apply Dynamic Programming backward in time once can do the conditional expectation against the Markov state G.



Tsitsiklis-van Roy and Longstaff-Schwartz Pricing Methods for Optimal Switching

Simulation Methods

- Simulate $\{g_t^n\}_{n=1}^N$ and work with path values $v(s, g_s^n)$.
- Given future path values $\{v(t+1, g_{t+1}^n; j)\}$ and associated rewards $\{\psi_{t,t+1}(g_t^n; j)\}$, regress their sum onto $\{g_t^n\}$ to find out the continuation value $\tilde{E}(g_t^n; j)$ for each action j.
- Find best action i^* for each path.
- TvR then sets $v(t, g_t^n; i) = \tilde{E}(g_t^n; i^*)$.
- LSM propagates back $v(t, g_t^n; i) = v(t+1, g_{t+1}^n; i^*) + \psi_{t,t+1}(g_t^n, i^*)$.
- In LSM the value function is computed *exactly* as long as policy decisions are made *correctly* along the path.

First Attempt

- With storage have inventory C_t which depends on the *past* control u_s, $s \leq t$. Dynamic Programming proceeds backwards.
- Suppose that $v(t+1, g^n_{t+1}, c; i)$ were known *for all* c. Then can find $v(t, g^n_t, c, i)$ as above.
- Interpolate to construct the new $v(t, g, \cdot; i)$ as function of c.
- Make a grid in the C-variable.
- If the grid size is N^c, then have N^c optimal switching problems. $\Rightarrow$ Expensive.
- Can no longer propagate back like in LSM.

Second Attempt

- Instead do quasi-simulation of C_t.
- If can *guess* correctly today's action $\tilde{i}$ and know inventory tomorrow, then have inventory today and can propagate.
- Perform bivariate regression of path values $\{v(t, g^n_{t+1}, c^n_{t+1}; i)\}$ against (g^n_t, c^n_{t+1}).
- Try to back-out c^n_t such that $\bar{C}_{\Delta t}(c^n_t, \tilde{i}) = c^n_{t+1}$.
- Attempt to do LSM and fall back onto TvR when cannot.
- One large bivariate optimal switching problem: BLSM scheme.

Overall BLSM Algorithm I

1. Select a set of bivariate basis functions $(\bar{B}_j)$ and algorithm parameters $\Delta t, M = T/\Delta t, N, N_b$.
2. Generate N paths of the price process: $\{g^n_{m\Delta t}, m = 0, 1, \ldots, M, n = 1, 2, \ldots, N\}$ with fixed $g^n_0 = g_0$. Generate a random terminal $c^n_T(i)$.
3. Initialize the pathwise values $v(T, g^n_T, c^n_T(i), i)$.
4. Moving backward with $t = m\Delta t$, $m = M, \ldots, 0$ repeat:
   - Guess Current C: generate $(c^n_{m\Delta t}(i))$ by guessing the optimal decision $\hat{\jmath}^n(m\Delta t, i)$ and solving $\bar{C}_{\Delta t}(c^n_{m\Delta t}(i), \hat{\jmath}^n(m\Delta t, i)) = c^n_{(m+1)\Delta t}(\hat{\jmath}^n(m\Delta t, i))$.
   - Regression Step: do the bivariate regression to find

$$\tilde{E} : (g, c, k) \mapsto \sum_{j=1}^{N_b} \bar{\alpha}_j \bar{B}_j(g, c; m\Delta t, k)$$

$$\simeq \mathbb{E}\left[\psi_{m\Delta t}(m\Delta t, g, c) + \mathrm{e}^{-r\Delta t} \cdot v((m+1)\Delta t, G_{(m+1)\Delta t}, c, k) \middle| G_{m\Delta t} = g\right]$$

of the value tomorrow given today's prices and *tomorrow's* inventory.

- Optimal Decision Step: find the optimal decision by evaluating $\tilde{E}(g^n_{m\Delta t}, \bar{C}_{\Delta t}(c^n_{m\Delta t}(i), j))$ above for different j's.
- Update Step: compute $v(m\Delta t, g^n_{m\Delta t}, c^n_{m\Delta t}(i), i)$ via (LSM) if correctly guessed $\hat{\jmath}^n(m\Delta t, i)$ or via (TvR) if not.
- Switching Sets: the points

$$\{(g^n_{m\Delta t}, c^n_{m\Delta t}) : n \text{ is such that } \hat{\jmath}^n(m\Delta t, i) = i\}$$

define the empirical action set for policy i.

5. end Loop
6. Interpolate $V(0, g_0, c, i)$ from the N values $v(0, g^0_n, c^0_n(i), i)$.

Performance

- Complexity is $\mathcal{O}(M \cdot N \cdot N_b^3)$.
- Quite fast on "toy problems", speed comparable to 1-d pde solvers.
- Much faster than the first Mixed Interpolation TvR attempt.
- No results on convergence rate. Expect algorithm variance of $\mathcal{O}(N^{-1/2})$.
- Variance strongly affected by choice of basis functions (need intuition about the shape of V).
- Number of paths N needed is exponential in number of basis functions N_b used.
- Computing resources: $40,000$ paths, 15 basis functions, 400 time-steps takes 30 minutes in Matlab on a desktop.
- Within 2% of "true" value (from pde).

Example from de Jong and Walet (2003):

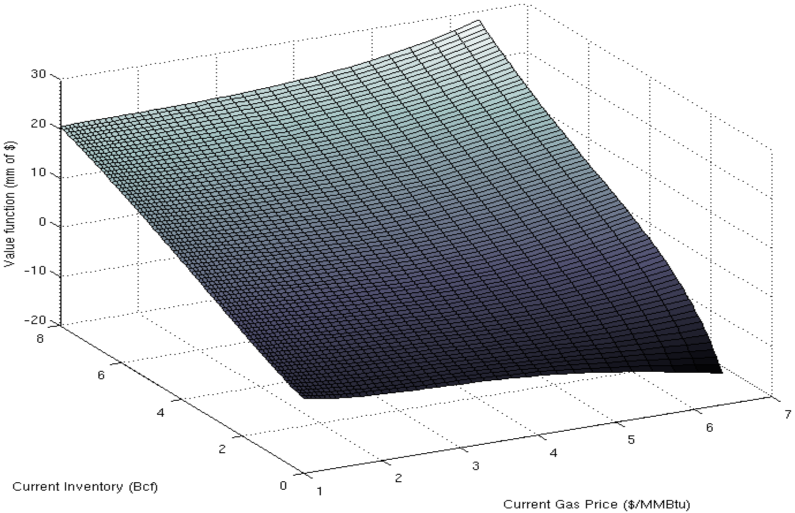
- $d\log G_t = 17.1 \cdot (\log 3 - \log G_t)\,dt + 1.33\,dW_t$.
- 8 Bcf capacity: $0 \le C_t \le 8$.
- $V(T, g, c, i) = -2 \cdot g \cdot \max(4 - c, 0)$.
- $$\begin{aligned} a_{in} &= 0.06 \cdot 365, & r &= 0.06, T = 1 \\ a_{out} &= 0.25 \cdot 365, & b_i &\equiv 0.1, K_{i,j} \equiv 0.25 \end{aligned}$$
- Thus, it takes about $8/0.06 = 133$ days to fill the facility and $8/0.25 = 32$ days to empty it.
- $g_0 = 3$, $c_0 = 4$.

Algorithm Variance

Table: Variance of the BLSM scheme as a function of N. Standard deviations were obtained by running the algorithm 50 times.

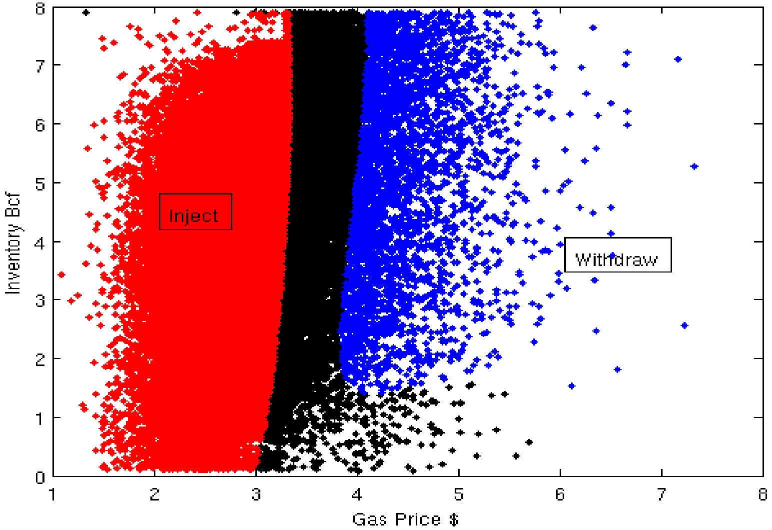
No. Paths N	Mean	Std. Dev
8000	14.24	4.81
16000	11.03	2.08
24000	10.42	1.48
32000	10.03	0.940
40000	10.01	0.698
pde	9.86	–

Value Surface



function surface showing $V(0.5, g, c, 0; T = 1)$ as a function of current gas price $G_t = g$ and current inventory $C_t = c$.

Optimal Policy Regions



Best Policy showing $i^*(0.5, g, c, 0; T = 1)$ as a function of current gas price $G_t = g$ and current inventory $C_t = c$.

Value of Flexibility

Effect of Storage Flexibility on the Value Function. Results obtained using the BLSM algorithm with $40,000$ paths.

Daily a_{in}	Daily a_{out}	$V(0, g_0, c_0, 0)$
0.06	0.25	9.86
0.03	0.125	6.41
0.12	0.5	12.96
0.18	0.75	14.63
0.12	0.25	12.95

Add-Ons

- Can easily incorporate jumps/seasonality in the model.
- Can add other constraints.
- Use the computed optimal policy in a new simulation to obtain a less biased estimate of V.
- Iterate the method to successively improve guesses of optimal policy.
- Can use different bases for different t's, i's.
- Can use other regression tools besides L^2: kernel, etc.

PDE Methods vs. Simulation Schemes I

PDE Methods:

- Extensive Literature
- Known error rate/stability conditions
- Many speed-ups possible
- Guaranteed structure of optimal policy regions

But:

- hard to handle degenerate C-variable
- Changes to price model may require extensive modification
- Impossible to consider multiple factors

PDE Methods vs. Simulation Schemes II

Simulation Schemes:

- Very flexible off-the-shelf capability
- Much easier to scale/add constraints
- Can be easily combined with other simulation engines
- Better probabilistic interpretation

Unfortunately:

- No error analysis
- May be unstable – must fine-tune basis functions
- No structure of optimal policy regions

Beyond 1-factor Models

The major limitation of pde method is curse of dimensionality:

- It is likely that gas prices are described by a factor model (stochastic mean-reversion level, or regime-switching or pure-jump factors).
- In hydroelectric applications, river run-off and precipitation cause exogenous stochastic fluctuations in inventory levels.
- Power supply guarantees: combine a gas storage problem with the need to serve a client base with stochastic demand.
- Margin constraints: loan for buying commodity to store is marked-to-market and subject to margin calls if prices fall too low.
- A lot remains to be done...

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